INSTITUTO SUPERIOR DE AGRONOMIA

Exam of Applied Operations Research (Module 3) - 3 June 2016

Number: Name:

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1. (10val.) An industrialist manufactures a certain product, which results in the emission of three pollutants into the air: A, B and C. She is required to reduce A by at least 50 units, B by at least 100 units and C by at least 75 units. There are two pollution abatement methods available to the industrialist: (1) to use filters or (2) to use cleaning additive for fuel. For each m^2 of filter that is used, the emission of the three pollutants can be reduced by 1/2, 4/5 and 1 units, respectively, for A, B and C. For each ton of cleaning additive added to the fuel, the three pollutants can be reduced by 1, 5/2 and 1 units, respectively, for A, B and C. The filter costs 500 € per m^2 and the cleaning additive costs 750 € per ton. The industrialist's objective is to adopt the lowest cost method of reducing pollution according to the requirements.

This problem is formulated as a linear program as follows:

$$\min Z = ax + by \tag{1}$$
$$\textit{subject to}$$
$$cx + dy \geq 50 \tag{2}$$
$$ex + fy \geq 100 \tag{3}$$
$$gx + hy \geq 75 \tag{4}$$
$$x, y \geq 0, \tag{5}$$

where a, b, c, d, e, f, g and h are parameters of the problem.

a) Explain what the decision variables x and y represent and determine a, b, c, d, e, f, g and h.

b) The sensitivity report created by the Excel Solver for the model (1)-(5), with a, b, c, d, e, f, g and h replaced by their values, is displayed in Tables 1 and 2.

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
x	50	0	500	250	125
y	25	0	750	250	250

Table 1: Variable cells (sensitivity report by the Excel Solver).

Name	Final Value	Shadow Price	Constant R.H. Side	Allowable Increase	Allowable Decrease
(1)	50	500	50	25	0.735294118
(2)	102.5	0	100	2.5	1E+30
(3)	75	250	75	2.777777778	25

Table 2: Constraints (sensitivity report by the Excel Solver).

i) What is the optimal solution to the problem?

ii) Which constraints are binding? How do you interpret each binding constraint?

iii) By how much should filter's current cost per m^2 increase in order for the optimal solution to change?

iv) What is the shadow price for each constraint? How do you interpret this value?

v) What is the range of feasibility for the minimum number of units of B to reduce with respect to the shadow price? How do you interpret this range?

Resolution: