INSTITUTO SUPERIOR DE AGRONOMIA

Exam of Applied Operations Research - Module 2 - 28 June 2016/17

Number: Name:

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1. (10val.) The government intends to build fire stations in the counties of a district to fight the fires. The district has five counties, C1 to C5. Each county may have one station or have none. Any county where a station is built should be assigned to counties of the district so that the station is responsible for fighting the fires that occur in these counties. The following table shows the average response time of a station to a fire, depending on the locations of the station and the fire.

County where a station can be built i	Counties where the fires may occur j				
	C1	C2	C3	C4	C5
C1	5	12	30	20	15
C2	20	4	15	10	25
C3	15	20	6	15	12
C4	25	15	25	4	10
C5	10	25	15	12	5

Table 1: Average response time (in minutes) of a station to a fire, depending on the locations of the station and the fire.

The following integer programming model translates the problem that the government would like to solve.

$$\min\ Z = \sum_{i=1}^{5}\sum_{j=1}^{5} a_{ij}x_{ij} \tag{1}$$

$$y_1 + y_2 + y_3 + y_4 + y_5 = 2 \tag{2}$$

$$x_{1j} + x_{2j} + x_{3j} + x_{4j} + x_{5j} = 1 \quad j = 1, ..., 5 \tag{3}$$

$$x_{ij} \leq y_i \quad i = 1, ..., 5; j = 1, ..., 5 \tag{4}$$

$$y_i \in \{0, 1\} \quad i = 1, ..., 5 \tag{5}$$

$$x_{ij} \in \{0, 1\} \quad i = 1, ..., 5; j = 1, ..., 5, \tag{6}$$

where a_{ij} denote the average response time (in minutes) of a station located in C_i to a fire located in C_j.

a) What can be the meaning of the decision variables y_i, for $i = 1, ..., 5$, and x_{ij}, for $i = 1, ..., 5$ and $j = 1, ..., 5$, the objective function (1) and constraints (2) to (4)?

b) In the optimal solution obtained by the Excel Solver, the variables with non-zero values are y_1, y_5, x_{11}, x_{12}, x_{53}, x_{54} and x_{55}. Complete the gray boxes in Table 2 and calculate the optimal objective function value.
NOTE: Do not forget that the RHS of a constraint in the Excel Solver should be a constant.

Name	Cell value	Status	Slack
(2)			
(3) for $j = 1$			
(3) for $j = 3$			
(4) for $i = 1$ and $j = 3$			
(4) for $i = 3$ and $j = 4$			

Table 2: The answer report provided by the Excel Solver concerning constraint (2), constraints (3) for $j = 1$ and $j = 3$, constraint (4) for $i = 1$ and $j = 3$ and constraint (4) for $i = 3$ and $j = 4$.

2. (5val.) Consider the following LP problem (P1):
max $z = 5x_1 - x_2$

$$\begin{cases} 2x_1 & + & x_2 & = & 6 \\ x_1 & + & x_2 & \leq & 4 \\ x_1 & + & 2x_2 & \leq & 5 \\ x_1 & , & x_2 & \geq & 0 \end{cases}$$

a) Use the Big M method to find a starting basic feasible solution for simplex. Identify the basic feasible solution.

b) Is the basic feasible solution you got in point a) optimal for (P1)? Justify your answer.

3. (5val.) Consider the following LP problem (P2):
max $z = 3x_1 + x_2$

$$\begin{cases} 2x_1 & + & x_2 & \leq & 4 \\ 3x_1 & + & 2x_2 & \geq & 6 \\ 4x_1 & + & 2x_2 & = & 7 \\ x_1 & , & x_2 & \geq & 0 \end{cases}$$

a) Write the dual problem of (P2).

b) The primal optimal solution for (P2) is $x_1 = 1$ and $x_2 = \frac{3}{2}$. Use complementary slackness conditions to obtain the dual optimal solution.