INSTITUTO SUPERIOR DE AGRONOMIA

Exam of Applied Operations Research - 11 July 2016/17

Number: Name:

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1. (10val.) Consider a ponderosa pine forest that could be managed either as multiple-use area for recreation and timber or as a wilderness that would allow only for recreational activities. The forest consists of 1600 ha of high-site (*i.e.*, high-productivity) land and 2400 ha of low-site land. The expected outputs from the forest, by site and management option, are given in the following table.

Table 1: Outputs per hectare, by site and management options.

	High-site land		Low-site land	
Output	Wilderness	Multiple use	Wilderness	Multiple use
Timber (m^3/ha/y)		3.5		1.2
Sediment (m^3/ha/y)	0.06	0.12	0.03	0.06
Recreation (vd/ha/y)	1	0.25	0.6	0.15

The following LP model translates the problem that the forest manager would like to solve.

$$\text{Max} \quad Z = x_1 + 0.25x_2 + 0.6x_3 + 0.15x_4 \qquad (1)$$
$$\text{s.t.} \quad x_1 + x_2 \leq 1600 \qquad (2)$$
$$x_3 + x_4 \leq 2400 \qquad (3)$$
$$3.5x_2 + 1.2x_4 \geq 1400 \qquad (4)$$
$$0.06x_1 + 0.12x_2 + 0.03x_3 + 0.06x_4 \leq 200 \qquad (5)$$
$$x_1, \quad x_2, \quad x_3, \quad x_4 \geq 0. \qquad (6)$$

a) What can be the meaning of the decision variables x_i, for $i = 1, ..., 4$, the objective function (1) and constraints (2) to (5)?

b) Tables 2 and 3 display partially the answer report provided by the Excel Solver concerning variables and constraints. Complete the gray boxes and find the optimal objective function value.

Table 2: The answer report provided by the Excel Solver concerning variables.

Name	Original Value	Final value
x_1	0	
x_2	0	400
x_3	0	2400
x_4	0	

Table 3: The answer report provided by the Excel Solver concerning constraints.

Name	Cell value	Status	Slack
(2)		Binding	
(3)			
(4)			
(5)			

2. (10val.) Consider the following LP problem (P):

$$\max z = 4x_1 + 2x_2$$
$$\begin{cases} 2x_1 + x_2 \leq 100 & (1) \\ x_1 + x_2 \leq 80 & (2) \\ x_1 \leq 40 & (3) \\ x_1 , x_2 \geq 0 & (4) \end{cases}$$

The simplex method yields the following optimal tableau, in which s_1, s_2 and s_3 are slack variables related to constraints (1), (2) and (3), respectively,

z	x_1	x_2	s_1	s_2	s_3	rhs
1	0	0	2	0	0	200
	0	1	1	0	-2	20
	0	0	-1	1	1	20
	1	0	0	0	1	40

a) Write the optimal solution for (P).

b) Use the simplex method to obtain an alternative optimal solution, different from the solution determined in a).

c) How many optimal solutions does (P) have? Justify your answer.

d) Write the dual problem of (P).

e) Use complementary slackness conditions to obtain the dual optimal solution.