Applied Operations Research

Simplex method at a glance

One resolution of some exercises

- 2018/2019 -

1. A farmer owns 35 acres of land and is going to plant wheat or corn. Each acre planted with wheat yields \$200 profit; each acre planted with corn yields \$300 profit. The labor and fertilizer used for each acre are given in the table below:

	Wheat	Corn
Labor	3 workers	2 workers
Fertilizer	2 tons	4 tons

One hundred workers and 120 tons of fertilizer are available. The problem is to determine how the farmer can maximize the profit from his land.

(a) Formulate the problem in linear programming.

Answer.

Decision variables:

x_1 - area assigned to wheat (acres)

x_2 - area assigned to corn (acres)

$$
\begin{array}{lrl}
\text{Max} & Z = 200x_1 + 300x_2 & \\
\text{s.t.} & x_1 + x_2 & \leq 35 \\
 & 3x_1 + 2x_2 & \leq 100 \\
 & 2x_1 + 4x_2 & \leq 120 \\
 & x_1, \quad x_2 & \geq 0
\end{array}
$$

(b) Graphically represent the feasible region.

Answer.

Vertex	(x_1, x_2)
A	(0,0)
B	(0,30)
C	(10,25)
D	(30,5)
E	$(\frac{100}{3}, 0)$

(c) Determine an optimal solution and the corresponding optimal value.

Answer.

Vertex	(x_1, x_2)	$Z = 200x_1 + 300x_2$
A	(0,0)	0
B	(0,30)	9000
C	(10,25)	9500
D	(30,5)	7500
E	$(\frac{100}{3}, 0)$	6666.(6)

The optimal solution is vertex C with the objective function value equal to 9500.

(d) Use Excel Solver to solve the problem.

(e) The Excel Solver provides a Sensitivity Report. For the objective function coefficients it gives the following values:

Variable Cells

...	Objective Coefficient	Allowable Increase	Allowable Decrease
...	200	100	50
...	300	100	100

For each objective function coefficient, show how to obtain the values in columns Allowable Increase and Allowable Decrease.

Answer. Columns Allowable Increase and Allowable Decrease give the amount by which each objective function coefficient can be increased or decreased, respectively, without changing the optimal activity levels.

$200x_1 + 300x_2 = k$ is the linear equation defining the set level C_k of the objective function (set of feasible solutions with the objective function value equal to k).

To answer this question with respect to x_1, one have to replace the coefficient of x_1 in $200x_1 + 300x_2 = k$ by a. The slope of the line that represents C_k ($ax_1 + 300x_2 = k$) is $-\frac{a}{300}$. This slope should be between the slope of the line $x_1 + x_2 = 35$ (-1) and the slope of the line $2x_1 + 4x_2 = 120$ ($-\frac{1}{2}$). Then, $-1 \leq -\frac{a}{300} \leq -\frac{1}{2} \Leftrightarrow a \in [150, 300]$. Thus, the coefficient of x_1 can increase 100 at the most (Allowable Increase) and decrease 50 at the most (Allowable Decrease) without changing the optimal activity levels.

With respect to x_2, one have to replace the coefficient of x_2 in $200x_1 + 300x_2 = k$ by a. The slope of the line that represents C_k ($200x_1 + ax_2 = k$) is $-\frac{200}{a}$ for $a \neq 0$. This slope should be between -1 and $-\frac{1}{2}$. Then, $-1 \leq -\frac{200}{a} \leq -\frac{1}{2} \Leftrightarrow a \in [200, 400]$. Thus, the coefficient of x_2 can increase and decrease 100 at the most without changing the optimal optimal activity levels. Notice that for $a = 0$, the optimal solution will be E.

(f) Fix the wheat profit to \$200. For which value(s) of corn profit does the LP problem have optimal solutions alternative to the one found in b)?

Answer. As previously, one have to replace the coefficient of x_2 in $200x_1+300x_2 = k$ by a. The slope of the line that represents C_k ($200x_1+ax_2 = k$) is $-\frac{200}{a}$ for $a \neq 0$. This slope should be equal to the slope of the line $x_1+x_2 = 35$ (-1) or the slope of the line $2x_1 + 4x_2 = 120$ ($-\frac{1}{2}$). Thus, $a = 200$ or $a = 400$. For $a = 0$, the optimal solution will be E. So, the answer is $a = 200$ or $a = 400$.

2. Consider the following linear programming model

$$\begin{array}{llcl}
\text{Max} & Z = x_1 - x_2 + x_3 & & \\
\text{s.t.} & 2x_1 - x_2 + 2x_3 & \leq & 6 \\
 & -2x_1 + 4x_2 - x_3 & \geq & \alpha \qquad (\alpha \in \mathbb{R}) \\
 & x_1 - x_2 + 2x_3 & \geq & 4 \\
 & x_1, \quad x_2, \quad x_3 & \geq & 0
\end{array}$$

and the point $P = (0, 2, 4)$.

(a) Write the problem in the standard form.

(b) Find, if any, a value for α such that P is the vertex of the feasible region and give the corresponding value of the objective function.

(c) Consider that P is an optimal solution of the problem for the α value found previously. Comment the following sentence: "The plan $x_1 - x_2 + x_3 = 3$ intercepts the feasible region of the problem."

3. Consider the following linear programming model

$$\begin{array}{lrcl}
\text{Max} & Z = x_1 + 2x_2 & - & x_3 \\
\text{s.t.} & 2x_1 + 4x_2 & + & 3x_3 \geq 8 \\
 & x_1 + x_2 & & \leq 6 \\
 & -x_1 + x_2 & & \leq 4 \\
 & x_1 & + & x_3 \leq 4 \\
 & x_1, \quad x_2, & & x_3 \geq 0
\end{array}$$

(a) Write the problem in the standard form.

(b) Find an optimal solution of the problem that is obtained from the initial problem by adding the restriction $x_3 = 0$ and indicate the corresponding binding constraints.

(c) Does the solution of the previous question correspond to a vertex of the feasible region of the initial problem?

4. A farmer has the following resource endowments: 500 acres of land, 1500 hours of family labour and 120000€ of capital investment. She can use these resources to grow the following crops: corn, sorghum, wheat and soybeans. Any crop should not occupy more than 250 acres. Assume that the farmer works to maximize profit (revenue minus costs) from the production of these crops. She expects the following in terms of prices, crop yields, costs and labor requirements.

Crop	Price (€/bushel)	Yield (bushel/acre)	Cost (€/acre)	Labor Requirement (hours/acre)
Corn	2.75	120	250	3.25
Sorghum	2.65	100	200	3.00
Wheat	3.15	105	245	3.15
Soybeans	6.75	45	230	3.30

(a) Formulate the problem as a linear program.

Answer. Decision variables:

x_1 - area assigned to corn (acres)

x_2 - area assigned to sorghum (acres)

x_3 - area assigned to wheat (acres)

x_1 - area assigned to soybeans (acres)

$$
\begin{array}{llllllll}
\text{Max} & Z = 80x_1 + 65x_2 & + 85.75x_3 + 73.75x_4 & & & & & \\
\text{s.t.} & x_1 + x_2 & + & x_3 & + & x_4 & \leq & 500 \\
 & 3.25x_1 + 3x_2 & + & 3.15x_3 & + & 3.3x_4 & \leq & 1500 \\
 & 250x_1 + 200x_2 & + & 245x_3 & + & 230x_4 & \leq & 120000 \\
 & x_1 & & & & & \leq & 250 \\
 & x_2 & & & & & \leq & 250 \\
 & & & x_3 & & & \leq & 250 \\
 & & & & & x_4 & \leq & 250 \\
 & x_1, \quad x_2, & & x_3, & & x_4 & \geq & 0
\end{array}
$$

(b) Solve the problem using the Excel Solver. What is the optimal solution to the problem?

Answer.

$x_1^* = 219.23$ (acres)

$x_2^* = 0$ (acres)

$x_3^* = 250$ (acres)

$x_4^* = 0$ (acres)

with the objective function value $Z^* = 38975.96$ (€).

(c) Indicate a basic feasible solution of the problem in the standard form.

Answer. (219.23,0,250,0,30.77,0,3042.31,30.77,250,0,250), where the 5th and further coordinates are relative to the slack variables associated to the constraints in the standard form (by the same order).

5. Consider the problem P in linear programming associated to the following standard formulation:

Max $Z = 2x_1 + 3x_2$ given the feasible region

$$\mathcal{F} = \{(x_1, x_2, x_3, x_4, x_5) : \left| \begin{array}{rcrcrcrcrcr} x_1 & + & x_2 & + & x_3 & & & & & = & 11 \\ x_1 & - & x_2 & & & - & x_4 & & & = & -2 \\ -2x_1 & + & x_2 & & & & & - & x_5 & = & -10 \\ x_1, & & x_2, & & x_3 & & x_4, & & x_5 & \geq & 0 \end{array} \right. \}$$

where x_1 and x_2 are the decision variables of P.

(a) Write P.

Answer.

$$\begin{array}{llcl} \text{Max} & Z = 2x_1 + 3x_2 & & \\ \text{s.t.} & x_1 + x_2 & \leq & 11 \\ & x_1 - x_2 & \geq & -2 \\ & -2x_1 + x_2 & \geq & -10 \\ & x_1, \quad x_2 & \geq & 0 \end{array}$$

(b) Graph the feasible region of P.

Answer.

Vertex	(x_1, x_2)
A	(0,0)
B	(0,2)
C	$(\frac{9}{2}, \frac{13}{2})$
D	(7,4)
B	(5,0)

(c) Find an optimal solution of P and indicate the corresponding basic feasible solution.

Answer.

Vertex	(x_1, x_2)	$Z = 2x_1 + 3x_2$
A	(0,0)	0
B	(0,2)	6
C	$(\frac{9}{2}, \frac{13}{2})$	28.5
D	(7,4)	26
B	(5,0)	10

The optimal solution is C with the objective function value equal to 28.5. The corresponding basic feasible solution is $(\frac{9}{2}, \frac{13}{2}, 0, 0, \frac{15}{2})$ where the 3rd and further coordinates are relative to the slack variables associated to the constraints in the standard form (by the same order).

(d) Consider set $S = \{(x_1, x_2, x_3, x_4, x_5) \in \mathcal{F} : x_3,\ x_4,\ x_5 > 0\}$. How many vertices of the feasible region of P correspond to elements of S?

Answer. Only A. The other vertices have at least one slack variable equal to zero.

(e) Consider that the Simplex method is applied to P, starting at a basic feasible solution from S. Indicate which vertices the algorithm passes through.

Answer. $A \rightarrow E(E$ is better than $B) \rightarrow D \rightarrow C$.

6. Consider the following problem in linear programming:

$$\begin{array}{llcl}
\text{Min} & Z = 2x_1 + x_2 & + 2x_3 & \\
\text{s.t.} & x_1 + x_2 & + \ x_3 & \leq 4 \\
 & x_1 + x_2 & + \ x_3 & \geq 1 \\
 & x_1 & & \leq 2 \\
 & & x_3 & \leq 3 \\
 & 3x_2 & + \ x_3 & \leq 6 \\
 & x_1, \quad x_2, & x_3 & \geq 0
\end{array}$$

(a) Write the problem in the standard form.

(b) Show that (1,0,3) is a vertex of the feasible region of the problem.

(c) Consider that constraint $x_1 + x_2 + x_3 \leq 4$ is replaced by $x_1 + x_2 + x_3 = 4$. Will (1,0,3) be an optimal solution of the problem?

7. Consider the following problem in linear programming:

$$\begin{array}{ll}
\text{Max} & Z = 2x_1 + x_2 - x_3 + 3x_4 \\
\text{s.t.} & x_2 - 2x_3 + x_4 \geq 3
\end{array}$$

$$
\begin{aligned}
x_1 \qquad\quad - 2x_3 + x_4 &\geq 2 \\
x_1 \qquad\quad + \ x_3 \qquad &\leq 3 \\
x_1 + x_2 - 2x_3 + x_4 &= 5 \\
x_1, \quad x_2, \quad x_3, \quad x_4 &\geq 0
\end{aligned}
$$

(a) Write the problem in the standard form.

(b) Verify that $(2, 3, 0, 0)$ is a vertex of the feasible region of the problem and indicate the corresponding value of the objective function value.

8. Consider the following problem in linear programming:

$$
\begin{aligned}
\text{Max} \quad & Z = 4x_1 + x_2 + 2x_3 + 3x_4 \\
\text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 \leq \frac{3}{2} \\
& x_1 + x_2 \geq x_3 \\
& x_2 + 2x_4 = 2 \\
& x_1, \quad x_2, \quad x_3, \quad x_4 \geq 0
\end{aligned}
$$

(a) Write the problem in the standard form.

Answer.

$$
\begin{aligned}
\text{Max} \quad & Z = 4x_1 + x_2 + 2x_3 + 3x_4 \\
\text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 + s_1 = \frac{3}{2} \\
& x_1 + x_2 - x_3 - s_2 = 0
\end{aligned}
$$

$$\begin{array}{lllllll} & x_2 & & + 2x_4 & & & = 2 \\ x_1, & x_2, & x_3, & x_4, & s_1, & s_2 & \geq 0 \end{array}$$

(b) Verify that $(0,\ 1,\ 0,\ \frac{1}{2})$ is a vertex of the feasible region of the problem.

Answer. $(0,\ 1,\ 0,\ \frac{1}{2})$ corresponds to point $u = (0, 1, 0, \frac{1}{2}, 0, 1)$ in the standard form.

u is a basic feasible solution because

all components are greater or equal than 0

has 6-3=3 null components

the solution set of the above system of linear equations with $x_1 = x_3 = s_1 = 0$

is $\{u\}$

$$\left\{ \begin{array}{lllll} x_2 & + & x_4 & & = \frac{3}{2} \\ x_2 & & & - \ s_2 & = 0 \\ x_2 & + & 2x_4 & & = 2 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{lll} x_2 & = & 1 \\ s_2 & = & 1 \\ x_4 & = & \frac{1}{2} \end{array} \right. .$$

As u is a basic feasible solution, $(0,\ 1,\ 0,\ \frac{1}{2})$ is a vertex of the feasible region of the problem.

(c) Find an optimal solution of the problem that is obtained from the initial problem by adding the restriction $x_4 = 1$ and indicate the corresponding objective function value.

Answer. Adding $x_4 = 1$, the initial problem becomes

$$\begin{array}{ll} \text{Max} & Z = 4x_1 + 2x_3 \ + \ 3 \\ \text{s.t.} & x_1 + x_3 \ \leq \dfrac{1}{2} \end{array}$$

$$x_1 - x_3 \geq 0$$

$$x_1, \quad x_3 \geq 0$$

Graphing the feasible region of this problem (with two variables)

Vertex	(x_1, x_3)	$Z = 4x_1 + 2x_3 + 3$
A	(0,0)	3
B	$(\frac{1}{4},\frac{1}{4})$	4.5
C	$(\frac{1}{2},0)$	5

The optimal solution is C with the objective function value equal to 5.

(d) Justify that solution $(0, 1, 0, \frac{1}{2})$ is not an optimal solution of the initial problem.

Answer. The feasible solution $(\frac{1}{2}, 0, 0, 1)$ (obtained from the previous question) is better than $(0, 1, 0, \frac{1}{2})$. The objective function value of the first solution is 5 while that of the second solution is 2.5.

9. Consider the problem in linear programming

$$\begin{aligned} \text{Max} \quad & Z = 20x_1 + 30x_2 \\ \text{s.t.} \quad & x_1 + 2x_2 \le 120 \\ & x_1 \le 60 \\ & x_2 \le 50 \\ & x_1, \quad x_2 \ge 0 \end{aligned}$$

(a) Graph the feasible region and the feasible solutions with the objective function value equal to 600.

Answer.

The red line is the set level C_{600} of the objective function (set of feasible solutions with the objective function value equal to 600).

Vertex	(x_1, x_2)
A	(0,0)
B	(0,50)
C	(20,50)
D	(60,30)
E	(60,0)

(b) Find an optimal solution and indicate the corresponding basic feasible solution. Which variables are basic in this basic feasible solution?

Answer. Moving the red line $20x_1 + 30x_2 = 600$ in the direction that improves the objective function values (up), the last feasible solution one can reach is D. Thus, D is the optimal solution with the objective function value equal to 2100. The corresponding basic feasible solution is (60,30,0,0,20), where the 3rd and further coordinates are relative to the slack variables associated to the constraints in the standard form (by the same order).

(c) Indicate two adjacent basic feasible solutions.

Answer. (60,30,0,0,20) and (60,0,60,0,50), as they correspond to adjacent vertices.

(d) What is the largest range of variation in the RHS of the third constraint that maintains optimal the solution referred to in question b)?

Answer. $[30, +\infty[$. With $x_2 \leq b$ with $b \in [30, +\infty[$, moving up the red line $20x_1 +$

$30x_2 = 600$, the last feasible solution one can reach is still D. For $b < 30$, D is no longer a feasible solution.

(e) What is the range of optimality for the coefficient of x_1 of the objective function?

Answer. $[15, +\infty[$. $20x_1 + 30x_2 = k$ is the linear equation defining the set level C_k of the objective function. To answer this question, one have to replace the coefficient of x_1 by a. The direction of the line that represents C_k ($ax_1 + 30x_2 = k$) should be between the directions of the lines $x_1 + 2x_2 = 120$ and $x_1 = 60$ (vertical line). The slope of $ax_1 + 30x_2 = k$ is $-\frac{a}{30}$. This slope should be less than or equal to the slope of $x_1 + 2x_2 = 120$, $-\frac{1}{2}$ (the slope of the line $x_1 = 60$ is not defined). Then, $-\frac{a}{30} \leq -\frac{1}{2} \Leftrightarrow a \geq 15$.

(f) If the coefficients of x_1 and x_2 in the objective function were equal and positive, what would be the optimal solutions?

Answer. D. To answer this question, one have to replace the coefficients of x_1 and x_2 by a with $a > 0$. The slope of the line that represents C_k ($ax_1 + ax_2 = k$) is -1. The red line in the figure below is a set level of the objective function. Moving this line in the direction that improves the objective function values (up), the last feasible solution one can reach is D. Thus, D is the optimal solution.

0

10. Consider the problem P in linear programming associated to the following standard formulation:

$$
\begin{array}{llcl}
\text{Max} & Z = 50000x_1 + 120000x_2 + 150000x_3 & & \\
\text{s.t.} & x_1 + x_2 + x_3 & = & 100 \\
 & x_1 - x_4 & = & 20 \\
 & x_2 - x_5 & = & 30 \\
 & x_3 - x_6 & = & 10 \\
 & 30000x_1 + 50000x_2 + 40000x_3 + x_7 & = & 65000000 \\
 & 240x_1 + 480x_2 + 300x_3 + x_8 & = & 40000 \\
 & x_1, \ x_2, \ x_3, \ x_4, \ x_5, \ x_6, \ x_7, \ x_8 & \geq & 0
\end{array}
$$

where x_1, x_2 and x_3 are the decision variables of P. Let $u = (20, 30, 50, 0, 0, 40, 24000000, 5800)$

and $v = (60, 30, 10, 40, 0, 0, 28000000, 8200)$ be two basic feasible solutions.

(a) Indicate the solution set of the system

$$\begin{cases} x_1 + x_2 + x_3 = 100 \\ x_1 = 20 \\ x_2 = 30 \\ x_3 - x_6 = 10 \\ 30000x_1 + 50000x_2 + 40000x_3 + x_7 = 65000000 \\ 240x_1 + 480x_2 + 300x_3 + x_8 = 40000 \end{cases}.$$

Answer. A basic feasible solution of problem P in the standard form has 8-6=2 null components. Concerning u, the non-basic variables are x_4 and x_5 and the basic variables are x_1, x_2, x_3, x_6, x_7 and x_8. As u is a basic feasible solution, then the first system of linear equations with $x_4 = x_5 = 0$ (the previous system) provides

the unique solution $\begin{cases} x_1 = 20 \\ x_2 = 30 \\ x_3 = 50 \\ x_6 = 40 \\ x_7 = 24000000 \\ x_8 = 5800. \end{cases}$.

(b) Do u and v correspond to adjacent vertices of the feasible region of P?

Answer. Yes, because u and v have all non-basic variables (or all basic variables) in common except one. The non-basic variables of u are x_4 and x_5 and those of v are x_5 and x_6. They have in common, 2-1=1 non-basic variable. Or, the basic variables of u are x_1, x_2, x_3, x_6, x_7 and x_8 and those of v are x_1, x_2, x_3, x_4, x_7 and x_8. They have in common, 6-1=5 basic variables.

(c) Write P.

Answer.

$$
\begin{array}{llcl}
\text{Max} & Z = 50000x_1 + 120000x_2 & + 150000x_3 & \\
\text{s.t.} & x_1 + x_2 & + & x_3 = 100 \\
 & x_1 & & \geq 20 \\
 & \quad x_2 & & \geq 30 \\
 & & & x_3 \geq 10 \\
 & 30000x_1 + 50000x_2 & + & 40000x_3 \leq 65000000 \\
 & 240x_1 + 480x_2 & + & 300x_3 \leq 40000 \\
 & x_1, \quad x_2, & & x_3 \geq 0
\end{array}
$$

11. Consider the problem P in linear programming associated to the following standard formulation:

$$\text{Max} \quad Z = 29x_1 + 45x_2$$

$$\begin{array}{llr} \text{s.t.} & 2x_1 + 8x_2 \quad + \; x_3 \qquad\quad & = 60 \\ \text{(s)} & 4x_1 + 4x_2 \qquad\quad + x_4 & = 60 \\ & x_1, \quad x_2, \qquad x_3, \quad x_4 & \geq \; 0 \end{array}$$

where x_1 and x_2 are the decision variables of P.

Consider that the Simplex algorithm is applied to P, starting at the basic feasible solution where x_3 and x_4 are the basic variables. Let $\begin{cases} x_3 & = & 60 - 2x_1 - 8x_2 \\ x_4 & = & 60 - 4x_1 - 4x_2 \end{cases}$ be the system (s) solved for these variables.

(a) Indicate the vertex where the Simplex algorithm starts.

(b) Find which vertex the algorithm goes next and indicate the corresponding objective function value.

12. Consider that the Simplex algorithm is applied to a maximization problem with two variables, x_1 and x_2, and two inequalities. Let x_3 and x_4 denote the slack variables associated to these constraints. Suppose that the current basic feasible solution has x_1 and x_3 as basic variables. Let $\begin{cases} x_1 & = & 15 - x_2 - \frac{1}{4}x_4 \\ x_3 & = & 30 - 6x_2 + \frac{1}{2}x_4 \end{cases}$ be the system of constraints in the standard

form and $Z = 435 + 16x_2 - 7.25x_4$ the objective function, both expressed in terms of the non-basic variables.

(a) Indicate the current vertex.

Answer. (15,30). As x_2 and x_4 are non-basic variables, $x_2 = x_4 = 0$. The current basic feasible solution is (15,0,30,0), which corresponds to vertex (15,0).

(b) Find which vertex the algorithm goes next and indicate the corresponding objective function value.

Answer. $\begin{cases} Z &= 435 + 16x_2 - 7.25x_4 \\ \\ x_1 &= 15 - x_2 - \frac{1}{4}x_4 \\ x_3 &= 30 - 6x_2 + \frac{1}{2}x_4 \end{cases}$ $\quad \Delta x_2 = 15$ $\quad \boldsymbol{\Delta x_2 = \frac{30}{6} = 5} \quad \boldsymbol{\Delta Z = 16(5) = 80}$

Increasing the value of x_4 will decrease the objective function value.

Therefore, $\begin{cases} x_2 &= 5 \\ x_4 &= 0 \end{cases} \Rightarrow \begin{cases} x_1 &= 10 \\ x_3 &= 0 \end{cases}$. Then, the algorithm goes to vertex (10,5) with the objective function value equal to 515.

13. Consider that the Simplex algorithm is applied to a maximization problem in linear programming with two variables, x_1 and x_2, and two inequalities. Let x_3 and x_4 denote the

slack variables associated to these constraints. Suppose that the current basic feasible solution has x_1 and x_2 as basic variables. Let $\begin{cases} x_1 &= 10 + \frac{1}{6}x_3 - \frac{1}{3}x_4 \\ x_2 &= 5 - \frac{1}{6}x_3 + \frac{1}{12}x_4 \end{cases}$ be the system of constraints in the standard form and $Z = 515 - 2.667x_3 - 5.917x_4$ the objective function, both expressed in terms of the non-basic variables.

(a) Indicate the current vertex.

Answer. (10,5). As x_3 and x_4 are non-basic variables, $x_3 = x_4 = 0$. The current basic feasible solution is (10,5,0,0), which corresponds to vertex (10,5).

(b) Is it possible that the algorithm goes to another vertex?

Answer. No, because increasing either the value of x_3 or that of x_4 will decrease the objective function value.