Some topics related to tree and stand growth modelling

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Outline

- Tree and stand growth
- The sigmoid curve, periodic, current and mean annual increments
- Components of growth: net growth of the standing volume, net growth, gross growth and ingrowth
- Components of volume: standing volume, thinned volume and total volume
- Productivity of the ecosystem: GPP, NPP and NEP
- Growth and yield relationships
- Allometric relationships

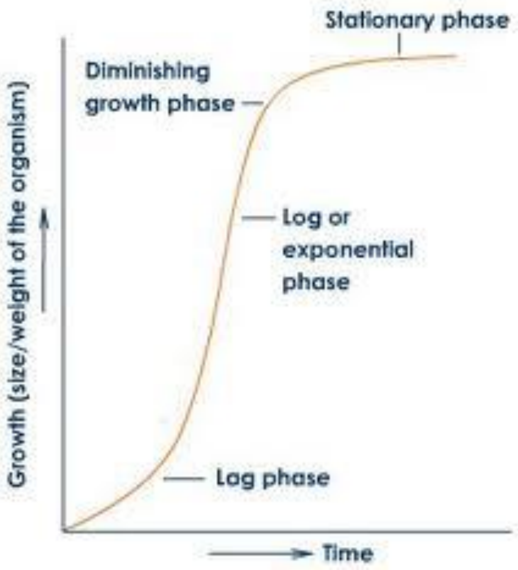
Tree growth

- The growth of a tree is expressed by the modification of the different variables that characterize a tree: diameter at breast height, total height, height to the base of the crown, stem profile, total and partial volumes, etc
- When studying tree growth it is very important to define unambiguously which is the variable of interest
- The shape of the growth curve is not the same for all the tree variables

Shape of the growth curves

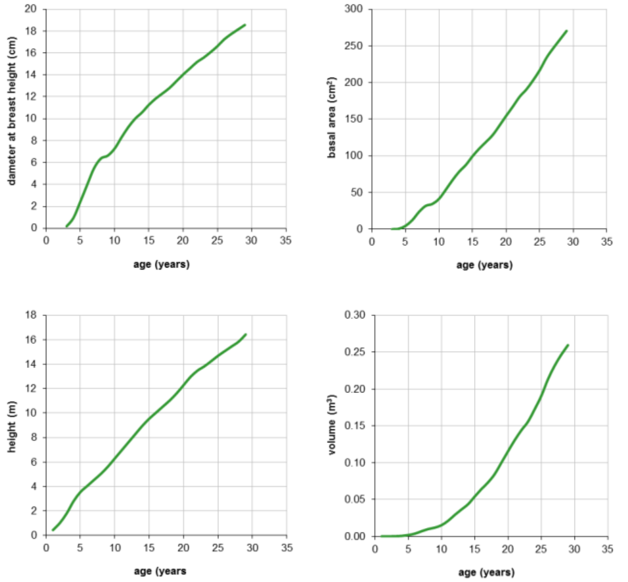
- The evolution of all the tree variables corresponds to a S-shaped curve (sigmoidal). In a S-shaped curve there are several stages:

  - ➔ **Lag stage** - initial stage of establishment
  - ➔ **Juvenil stage** - rapid initial growth, exponential, concavity upward
  - ➔ **Straight line stage**
  - ➔ **Senescense stage** - curve turns downward and tends to a plateau, the asymptote



Growth curves for different tree variables

- All tree variables “grow” according to a sigmoid curve
- However, the length of the 4 stages is different leading to curves of different shapes



Tree growth
assessment of growth intensity

- Tree growth intensity is usually assessed by the increments:
  - ➔ Periodic or current increment

$$i_{xn} = X_{t+n} - X_t$$

  - ➔ Current annual increment (CAI)

$$cai = \frac{X_{t+n} - X_t}{n}$$

  - ➔ Mean annual increment (MAI)

$$mai = \frac{X_t}{t}$$

Tree growth
assessment of growth intensity

- When is *mai* at a maximum?

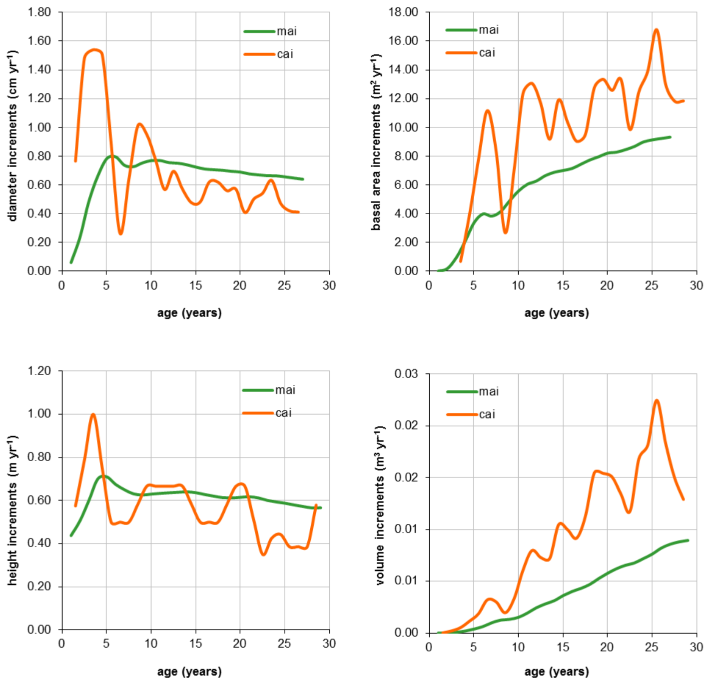
$$\text{maximum} \quad \Rightarrow \quad \frac{d}{dt} mai = 0$$

$$\frac{d}{dt} mai = \frac{d}{dt}\frac{X}{t} = -\frac{X}{t^2} + \frac{1}{t}\frac{dX}{dt} = 0$$

$$\frac{X}{t^2} = \frac{1}{t}\frac{dX}{dt}$$

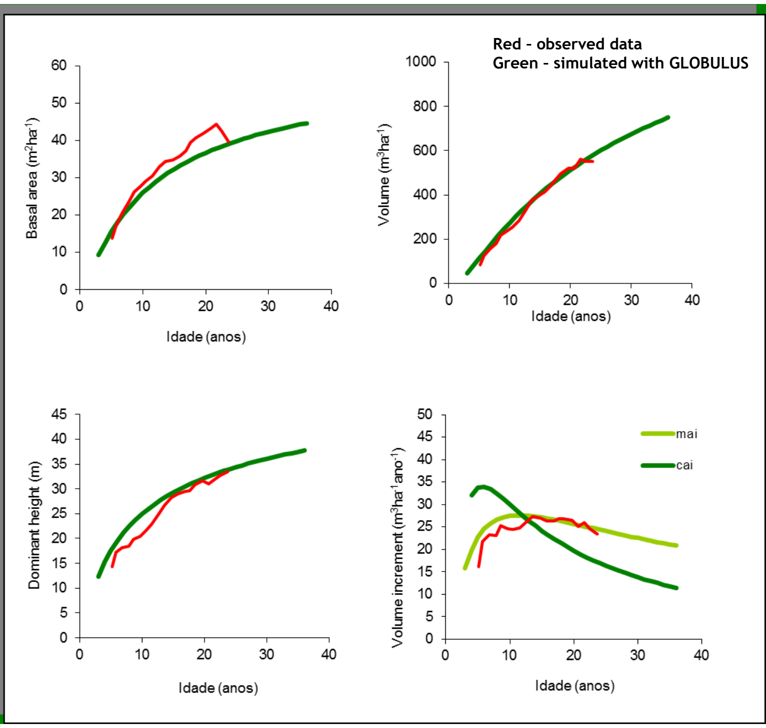
$$\frac{X}{t} = \frac{dX}{dt} \quad \Rightarrow \quad mai = cai$$

- *cai* and *mai* are equal at the maximum of *mai*!



Stand growth

- When dealing with stand growth it is very important, as was the case for tree growth, to identify the variable of interest
- The shape of the growth curves also varies among stand variables
- Stand growth intensity is assessed using mean and current annual increments



Stand growth
components of growth

- **Net growth of the standing stand (NGS_V)**

  difference between the volume in two successive forest inventories, is the difference between the standing volumes in two points in time (volume after thinning)

- **Net growth (NG_V)**

  net growth of the standing stand plus the volume lost due to thinnings that occurred during the period

- **Gross growth (GG_V)**

  net growth plus the volume lost through mortality

Stand growth - ingrowth

- During a forest inventory there is always a diameter threshold below which trees are not measured
- When a plot is remeasured there is a set of the smaller trees that were not measured at the first inventory – those trees are known as “ingrowth”
- Ingrowth has to be taken into account when studying stand growth, namely in uneven-aged stands (continuous cover management)

Stand growth
components of volume growth

- In a certain point in time (age t), we can consider three components of the stand volume:

  → Standing volume – volume at age t

  → Thinned volume – volume that has been thinned till the age t (accumulated)

  → Total volume – standing volume plus the thinned volume; under certain limits of stand density, it represents the potential of the site

Stand growth
productivity of the ecosystem

- **Gross primary production (GPP)**

  total amount of CO_2 fixed by a stand due to photosynthesis (the same concept applies to trees)

- **Net primary productivity (NPP)**

  net amount of CO_2 fixed by the stand after plant respiration is subtracted from GPP

  $NPP = GPP - R_p$

- **Net ecosystem productivity (NEP)**

  the net primary production after all respiration from plants, heterotrophs, and decomposers are included

  $NEP = GPP - (R_p + R_h + R_d)$

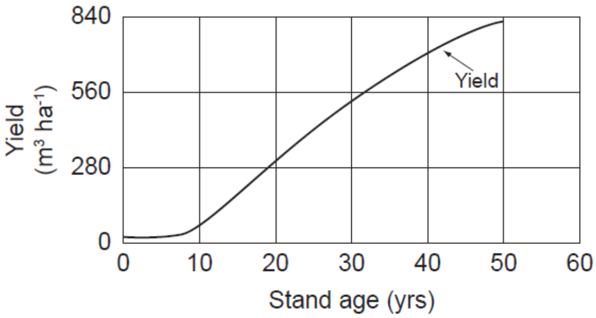
Growth and Yield relationships

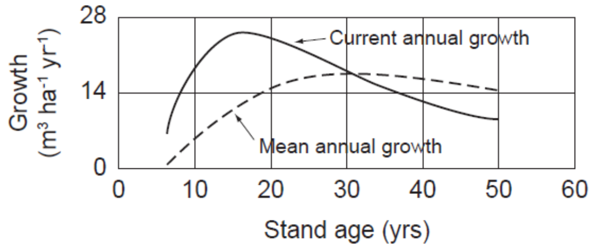
- **Yield**

  total amount available at a given time, yield can be regarded as the summation of the annual increments

- **Growth**

  the increase (increment) over a given period of time





Growth and Yield relationships

- Let's consider the evolution of stand volume as an example to show the relationships between Growth and Yield

  → Yield – stand volume plotted over stand age results in a sigmoid curve

  → The growth curve (current annual increment), which is the first derivative of the yield function, increases up to the inflection point of the yield curve and decreases thereafter

  → The current annual growth curve crosses the mean annual growth curve at its highest value

Compatibility between growth and yield

- Although growth and yield are biologically and mathematically related, this relationship has not always been considered in growth and yield studies (what is not the case nowadays...)
- It is essential that forest growth models exhibit this property:
  - →If one estimates annual growth during the first 10 consecutive years of a stand, this value must be equal to one that will be directly obtained by estimating yield at 10 years of age

Compatibility between growth and yield

- Being *Y* the yield (accumulated growth) and *t* the time (age), growth is represented by

$$\frac{dY}{dt} = f(t)$$

- Yield at age t will be

$$Y = \int f(t)\ dt = F(t) + c,$$

- **where *c* is obtained from the yield *Y*0 at time *t*0 (initial condition)**

Allometric relationships

- The allometric equation describes one tree variable as a function of other tree variable following the model:

$$Y = k\ X^a \quad \Leftrightarrow \quad \ln\ Y = \ln\ k + a\ \ln\ X$$

➔ *a* - allometric constant, characterizes the individual in a certain environment

➔ *k* - depends on the initial conditions and units of *Y* and *X*

- Taking logarithms and then differentiating, it can be shown that the allometric equation assumes that the relative growth rate of one plant component is proportional to that of another plant component

Allometric relationships

- The parameter *a* – allometric constant - is the coefficient of proportionality between the relative growth rates of the two plant parts

$$\frac{1}{Y}\frac{dY}{dt} = a\frac{1}{X}\frac{dX}{dt}$$

$$\int \frac{1}{Y}\frac{dY}{dt} = a\int \frac{1}{X}\frac{dX}{dt} \quad \Rightarrow \quad \ln\ Y = k + a\ \ln\ X$$

- *a* can, therefore, provide direct information on the partitioning of assimilates between plant parts

Allometric relationships

- The multiple allometric relationship is established between one variable and other variables (more than one):

$$Y = k\ X^{a} Z^{b}$$

- Allometric relationships can also be established between stand variables
- The existence of allometric relationships between tree variables (or between stand variables) is very important for growth and yield modelling of trees and stands
- It is one of the biologic hypothesis that can be used in the formulation of models

- Recommended Bibliography:

  → Avery and Burkhart (1994), Forest Measurements, 5th Edition, pages 352-355