Fitting Growth Functions

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Theoretical growth functions

- **Allow interpretation of the function parameters and helps to impose restrictions on the values that the parameters can take to be biologically consistente**

- **Theoretical growth functions are grouped according to their functional form in:**

Lundqvist-Korf type

$$Y = A\, e^{-k \frac{1}{t^m}}$$

Richards type

$$Y = A\left(1 - c\, e^{-k\,t}\right)^{\frac{1}{1-m}}$$

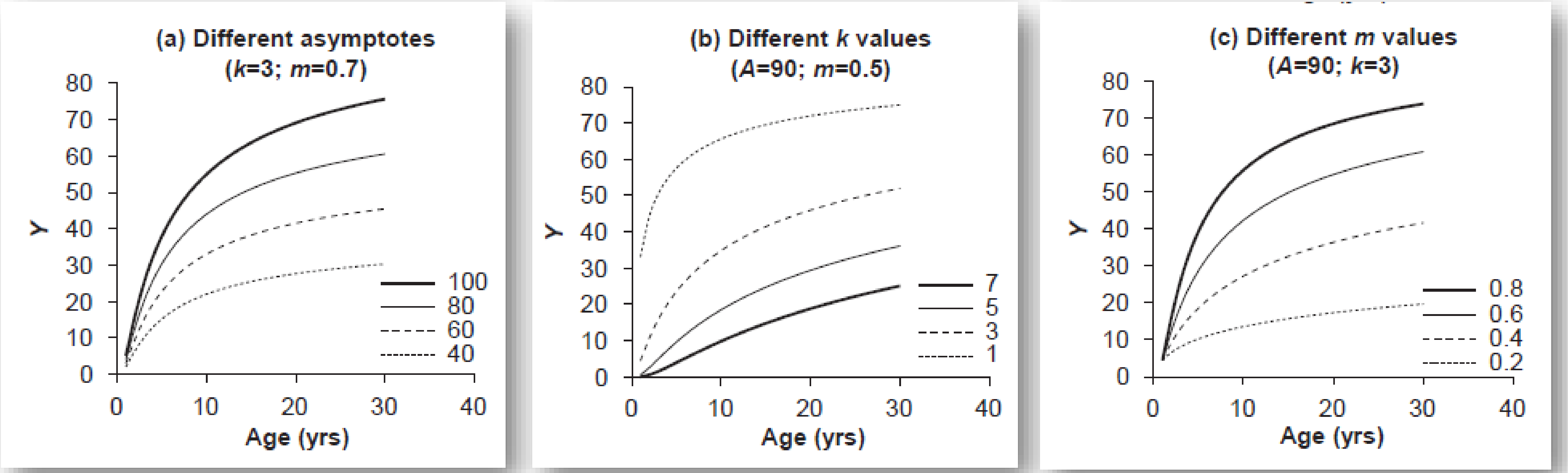
Hossfeld IV type

$$Y = \frac{A}{1 - \left(1 - \frac{A}{Y_0}\right)\left(\frac{t_0}{t}\right)^k}$$

▪ Lundqvist-Korf type

$$Y = A\, e^{-k\frac{1}{t^m}}$$

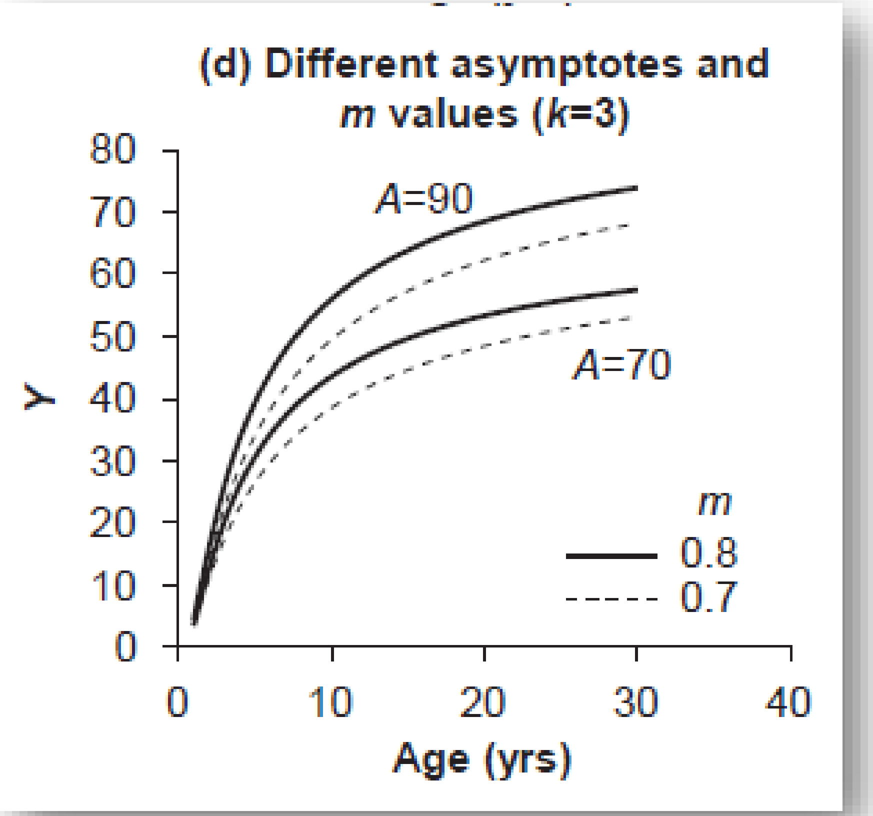
A **- assymptote**
k **– inversely related to growth rate**
m **- shape parameters**

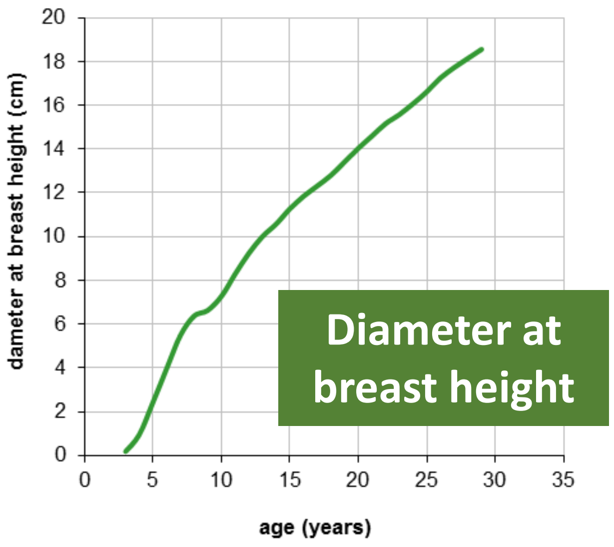


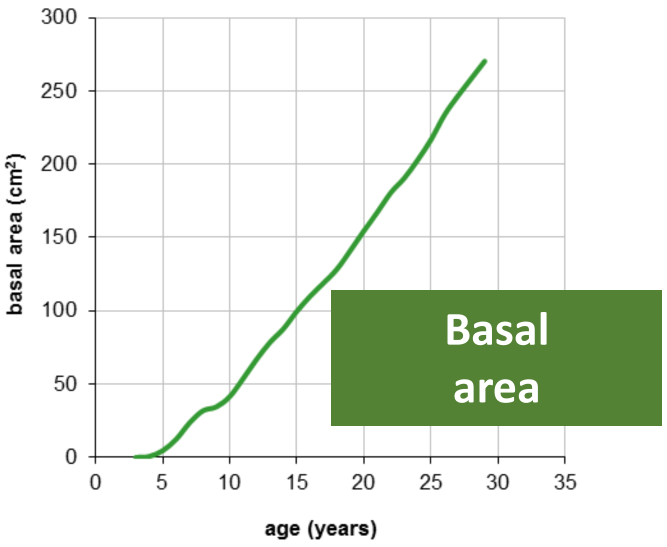
▪ Lundqvist-Korf type

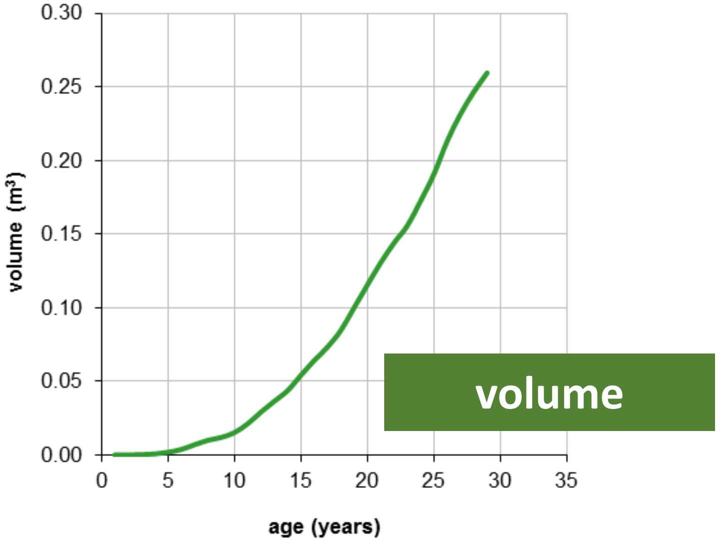
$$Y = A\, e^{-k \frac{1}{t^m}}$$

A **- assymptot**
k **– inversely**
m **- shape pa**



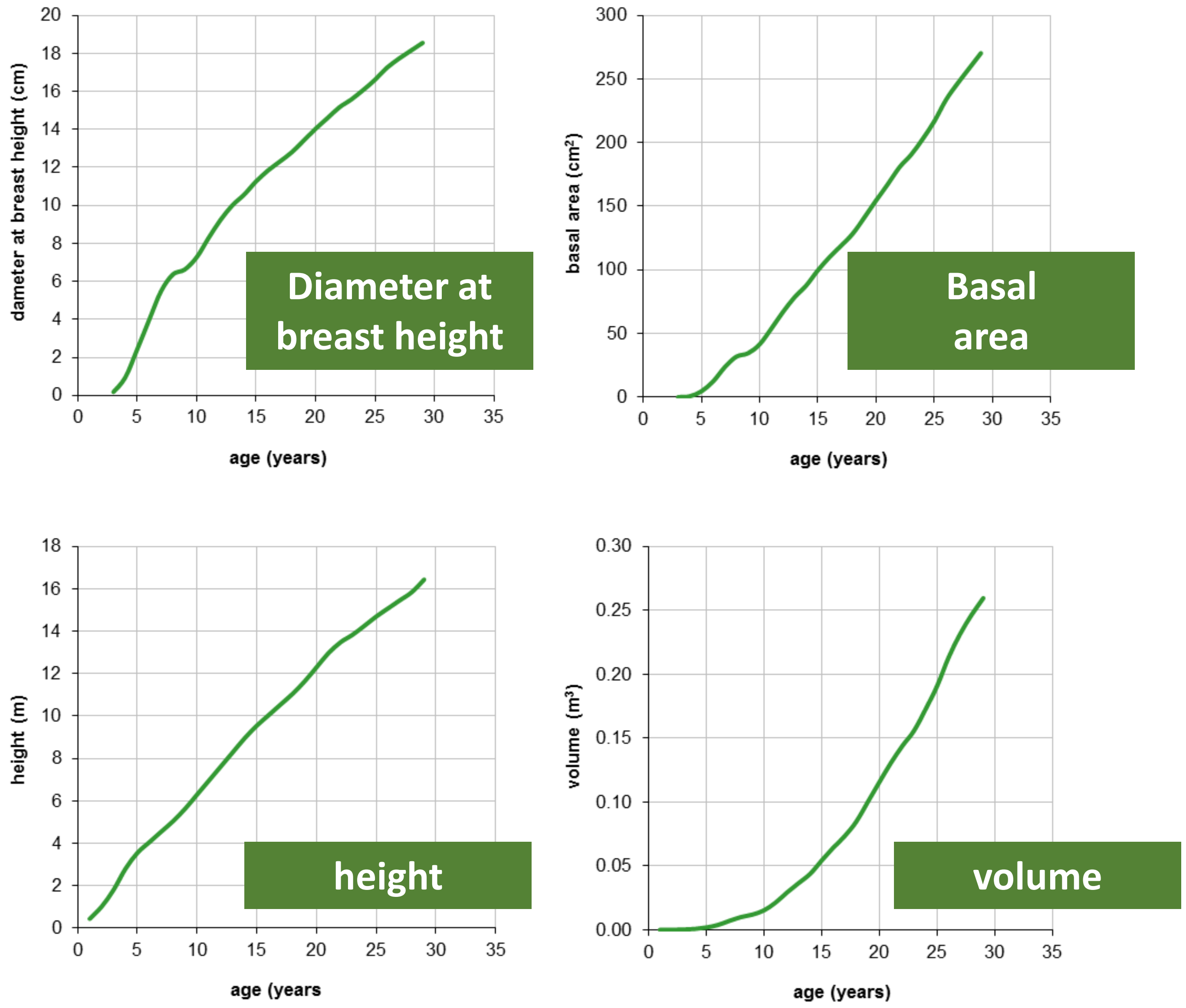






Growth curves for different tree variables

- All tree variables "grow" according to a **sigmoid curve**
- However, the length of the 4 stages is different leading to different shapes



- # Richards type

$$Y = A\left(1 - c\,e^{-k\,t}\right)^{\frac{1}{1-m}}$$

***A* - assymptote**

***k* – rate of decrease of the absolute growth rate**

(higher values of K produce higher growth rates)

***m* - shape parameter (usually 2/3)**

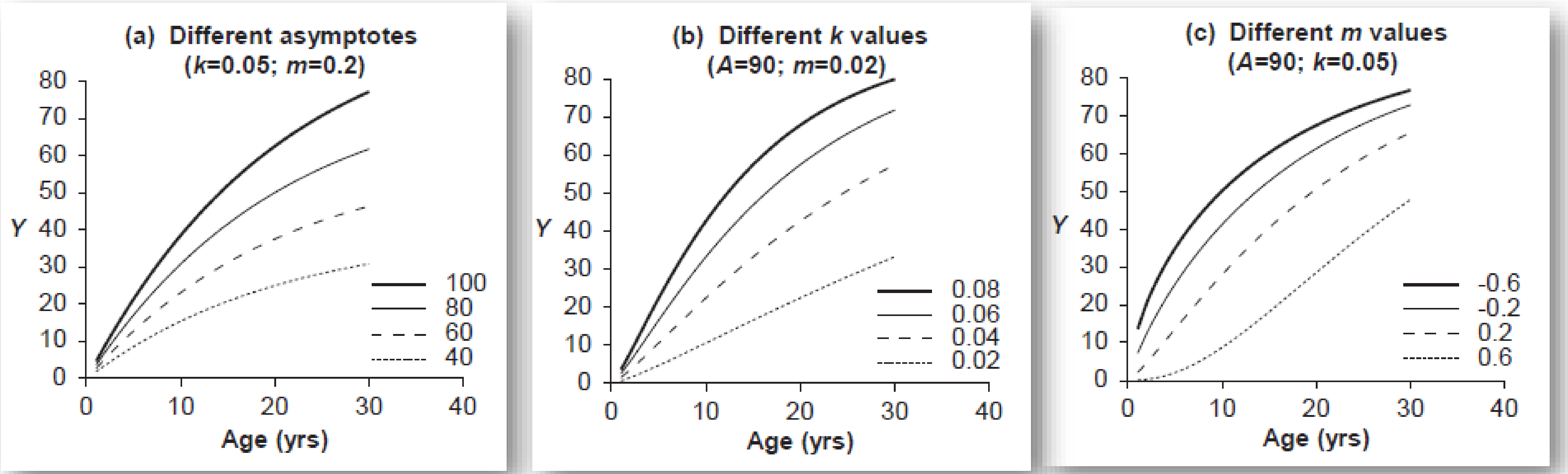
(smaller values of m result in higher growth rates)

***c*- for initial conditions such as Y0=0, leads to c=1**

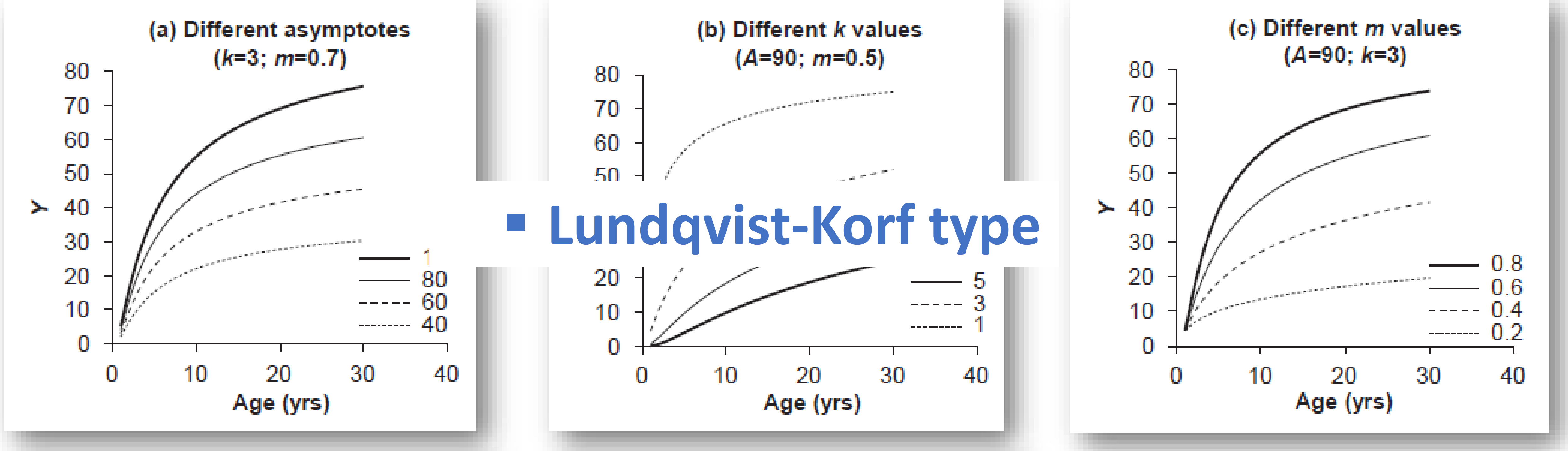
→ **Monomolecular (m = 0)**

→ **Gompertz (m -->1)**

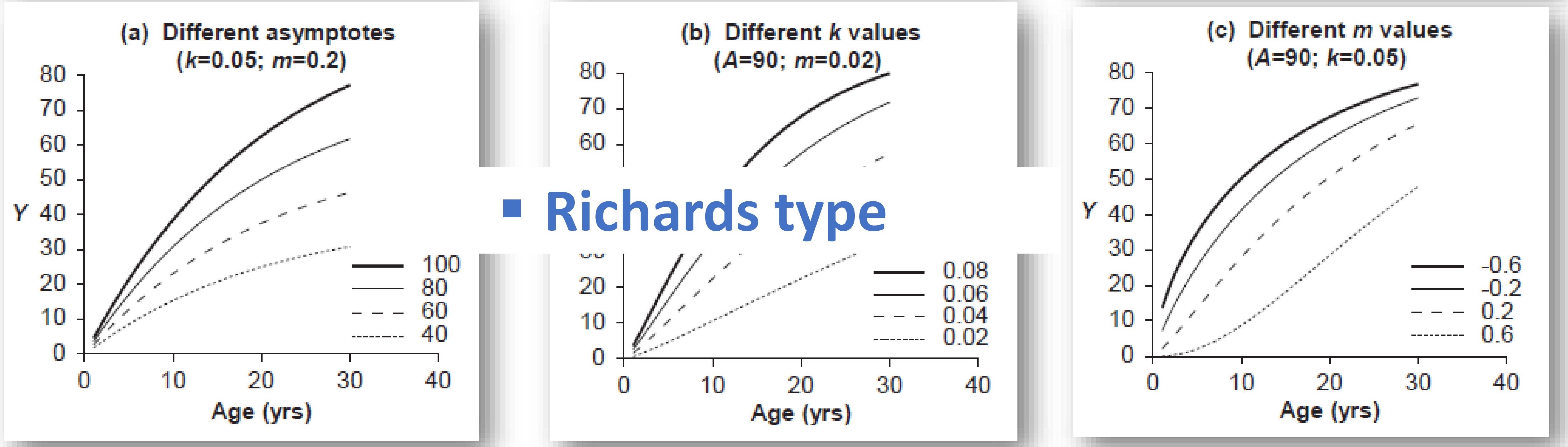
→ **Logistic (m = 2)**



- Lundqvist-Korf type



- Richards type



- **Lundqvist-Korf type**
- **Richards type**

Diameter at breast height | Basal area | Height | Volume

Forest Functions Playground

http://home.isa.utl.pt/~joaopalma/modelos/fgfp/

check the Forest Functions playground to have na idea about the range of logical values for the parameters before solver

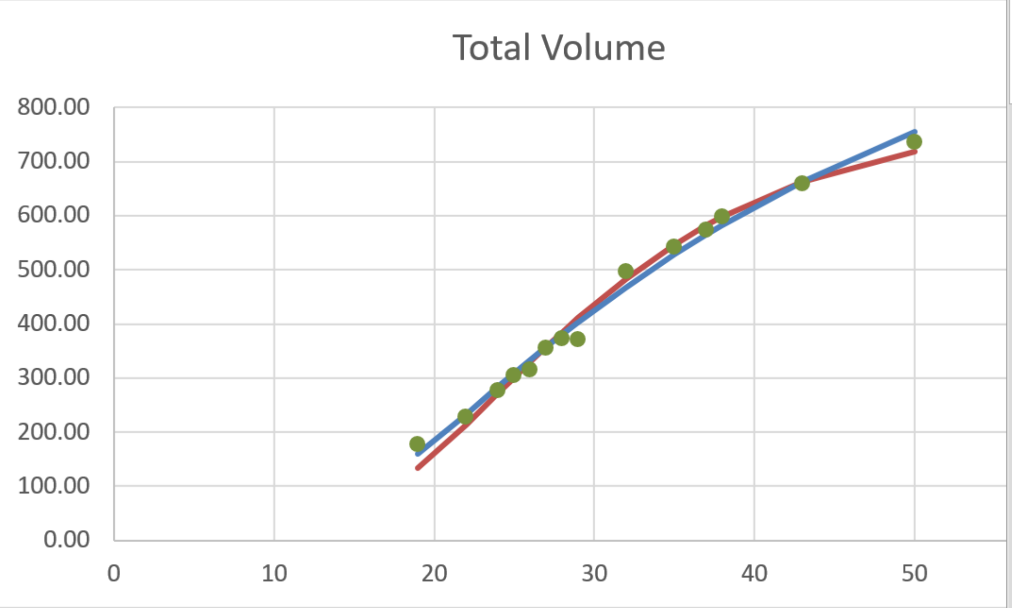
id_stand	year	t	V (m3 ha-1)	Sum Vthin (m3 ha-1)	Vtotal (m3 ha-1)	Vest_R	QSS	Vest_L	QSS
B	1981	19	159.05	18.05	177.10	133.6747	1885.458	160.1754	286.3248
B	1984	22	210.53	18.05	228.58	213.8126	218.1578	234.223	31.81279
B	1986	24	222.80	53.35	276.15	271.1522	24.98353	284.028	62.05381
B	1987	25	252.29	53.35	305.64	299.9959	31.83093	308.6158	8.868336
B	1988	26	261.53	53.35	314.88	328.5765	187.6078	332.8585	323.2425
B	1989	27	301.98	53.35	355.33	356.6548	1.7668	356.6811	1.837247
B	1990	28	319.22	53.35	372.57	384.0293	131.3085	380.026	55.58649
B	1991	29	317.44	53.35	370.79	410.5356	1579.554	402.8502	1027.73
B	1994	32	442.86	53.35	496.21	483.7076	156.1971	467.9373	799.0917
B	1997	35	488.93	53.35	542.27	546.1542	15.05316	527.7311	211.5059
B	1999	37	350.65	222.26	572.91	581.7006	77.25198	564.6751	67.83418
B	2000	38	375.84	222.26	598.10	597.7203	0.143726	582.2991	249.6503
B	2005	43	437.37	222.26	659.63	662.0885	6.038021	662.483	8.132569
B	2012	50	513.36	222.26	735.62	717.7688	318.6659	755.324	388.2479
							4634.017		3521.919

Richards		
A	780.86	
k	0.095	
m	0.899	
cc	1	

$$V = A\left(1 - c\,e^{-kt}\right)^{\frac{1}{1-m}}$$

Lundqvist		
AA	1360.586	
KK	108.6491	
mm	1.333903	

$$V = A\,e^{-k\frac{1}{t^m}}$$



- **Lundqvist-Korf type**
- **Richards type**

Diameter at breast height | Basal area | Height | Volume

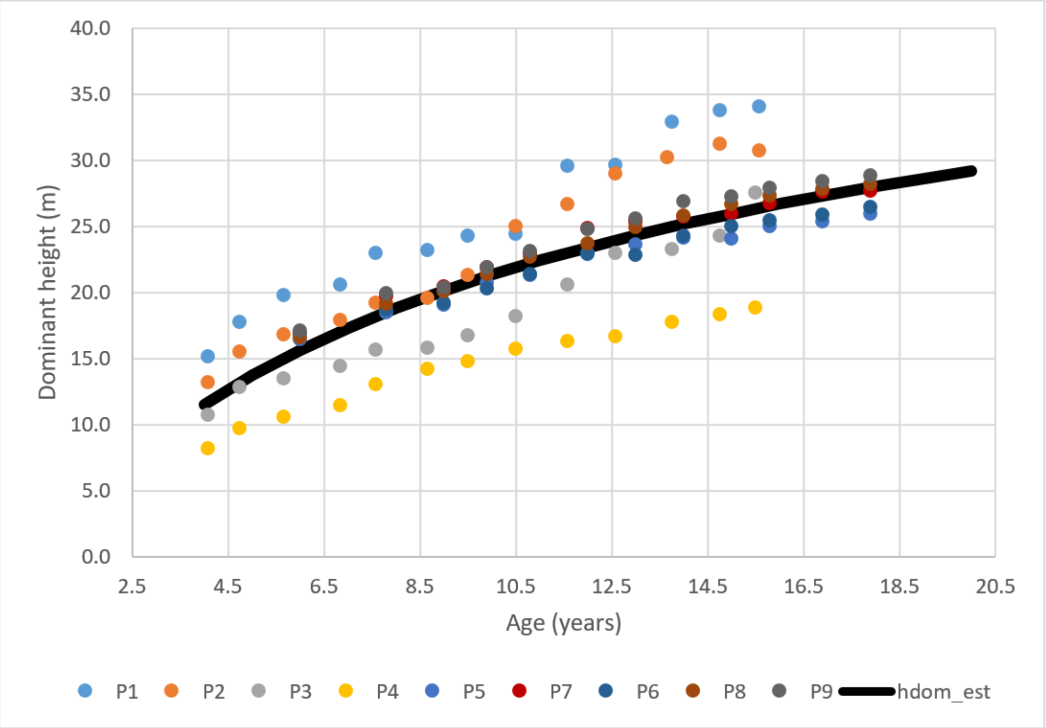
	Plot	t	hdom	N	G	ddom	dg	V	hdom_est	res^2
2	P1	4.1	15.1	1125	13.30	17.90	12.27	77.72	11.70	11.69
3	P1	4.8	17.8	1125	17.11	20.24	13.92	117.06	13.21	20.73
4	P1	5.7	19.8	1114	21.10	22.65	15.53	158.49	15.05	22.42
5	P1	6.8	20.6	1114	25.08	25.03	16.93	196.73	17.07	12.17
6	P1	7.6	22.9	1114	27.06	25.96	17.59	233.37	18.23	22.23
7	P1	8.7	23.2	1114	29.34	26.90	18.31	260.81	19.73	12.21
8	P1	9.5	24.3	1114	31.04	27.81	18.84	297.91	20.77	12.35
9	P1	10.5	24.4	1114	32.31	28.31	19.22	324.82	21.91	6.42
10	P1	11.6	29.6	1114	34.77	29.42	19.94	399.26	23.03	43.33
11	P1	12.6	29.7	1114	36.85	29.08	19.89	437.25	23.97	32.45
12	P1	13.8	32.9	1102	38.42	31.87	21.06	487.86	24.99	63.28
13	P1	14.8	33.8	1102	39.46	32.42	21.35	513.30	25.78	63.94
14	P1	15.6	34.1	1080	41.18	33.15	22.04	527.64	26.41	59.34
15	P2	4.1	13.2	1081	5.20	13.12	7.80	23.68	11.70	2.25
16	P2	4.8	15.5	1081	7.45	15.17	9.37	39.68	13.21	5.21
17	P2	5.7	16.8	1070	10.41	17.05	11.13	61.41	15.05	3.15
18	P2	6.8	17.9	1048	14.03	19.27	13.05	90.83	17.07	0.67
19	P2	7.6	19.2	1048	16.26	20.62	14.05	115.38	18.23	0.89
20	P2	8.7	19.6	1048	19.08	22.05	15.22	142.64	19.73	0.03
21	P2	9.5	21.3	1048	21.24	23.19	16.06	177.70	20.77	0.32
22	P2	10.5	25.0	1037	22.87	24.15	16.76	219.22	21.91	9.57
23	P2	11.6	26.7	1037	25.43	25.59	17.67	261.11	23.03	13.27
24	P2	12.6	29.0	1037	24.50	27.60	17.82	299.95	23.97	25.26
25	P2	13.7	30.2	1037	29.20	27.89	18.93	338.79	24.92	28.13
26	P2	14.8	31.2	1026	30.65	28.62	19.50	366.52	25.78	29.55
27	P2	15.6	30.7	1026	32.40	29.20	20.05	387.67	26.41	18.60
28	P3	4.1	10.7	1092	4.32	10.91	7.10	16.31	11.70	0.92
29	P3	4.8	12.8	1092	5.84	12.59	8.25	27.30	13.21	0.14
30	P3	5.7	13.5	1092	7.47	14.19	9.34	36.57	15.05	2.45
31	P3	6.8	14.4	1092	9.21	15.81	10.37	47.75	17.07	6.93

A	62.8445
k	3.369699
m	0.494316
SSres	1128.514

hdom=A*exp(-k*1/t^m)

$$hdom = A\, e^{-k \frac{1}{t^m}}$$

use the solver function from EXCEL to fit the Lundqvist function to the data and plot the estimated values together with the original data



SUM | =$M$2*EXP(-$M$3*1/U13^$M$4)

	Plot	dg	V	hdom_est	res^2
2	P1	12.27	77.72	11.70	11.69
3	P1	13.92	117.06	13.21	20.73
4	P1	15.53	158.49	15.05	22.42
5	P1	16.93	196.73	17.07	12.17
6	P1	17.59	233.37	18.23	22.23
7	P1	18.31	260.81	19.73	12.21
8	P1	18.84	297.91	20.77	12.35
9	P1	19.22	324.82	21.91	6.42
10	P1	19.94	399.26	23.03	43.33
11	P1	19.89	437.25	23.97	32.45
12	P1	21.06	487.86	24.99	63.28
13	P1	21.35	513.30	25.78	63.94
14	P1	22.04	527.64	26.41	59.34
15	P2	7.80	23.68	11.70	2.25
16	P2	9.37	39.68	13.21	5.21
17	P2	11.13	61.41	15.05	3.15
18	P2	13.05	90.83	17.07	0.67
19	P2	14.05	115.38	18.23	0.89
20	P2	15.22	142.64	19.73	0.03
21	P2	16.06	177.70	20.77	0.32
22	P2	16.76	219.22	21.91	9.57
23	P2	17.67	261.11	23.03	13.27
24	P2	17.82	299.95	23.97	25.26
25	P2	18.93	338.79	24.92	28.13
26	P2	19.50	366.52	25.78	29.55
27	P2	20.05	387.67	26.41	18.60
28	P3	7.10	16.31	11.70	0.92
29	P3	8.25	27.30	13.21	0.14
30	P3	9.34	36.57	15.05	2.45

A	62.8445
k	3.369699
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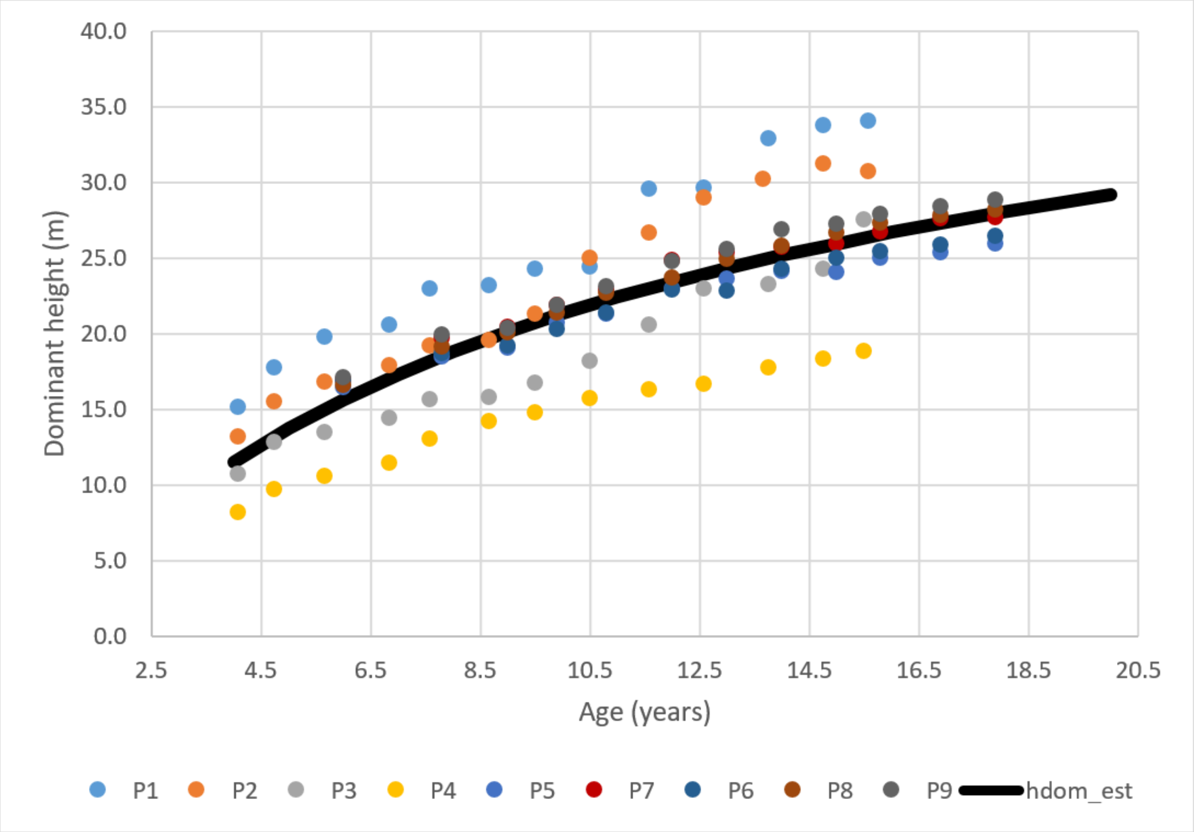
hdom=A*exp(-k*1/t^m)

$$Y = A\,e^{-k\frac{1}{t^m}}$$

use the solver function from EXCEL to fit the Lundqvist function to the data and plot the estimated values together with the original data



t	hdom_est
4	=$M$2*EXP(-$M
5	13.73
6	15.66
7	17.34
8	18.82
9	20.15
10	21.35
11	22.44
12	23.43
13	24.35
14	25.19
15	25.97
16	26.71
17	27.39
18	28.03
19	28.63
20	29.20

But how to model the growth of several plots?

- There are several methods to simultaneously model the growth of several plots:

1. Expressing the parameters as a function of site and/or tree/stand variables
2. Using growth functions formulated as difference equations – ADA

But how to model the growth of several plots?

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1. **Expressing the parameters as a function of site and/or tree/stand variables**
2. Using growth functions formulated as difference equations – ADA

1. Expressing the parameters as a function of site and/or tree/stand variables

- **Lundqvist-Korf type**

Height | **A= A0 + A1*S**

SUM | =($M$2+$M$3*V$10)*EXP(-$M$4*1/$U12^$M$5)

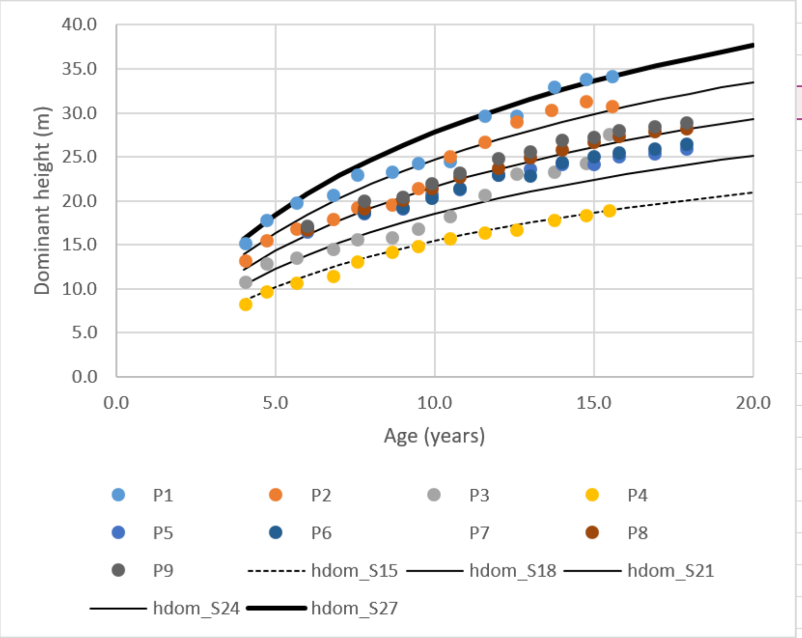
	A	G	H	I	J	K
1	Plot	dg	V	S	hdom_e	res^2
2	P1	12.27	77.72	24.36	14.4	0.6
3	P1	13.92	117.06	24.36	16.0	3.0
4	P1	15.53	158.49	24.36	18.1	3.0
5	P1	16.93	196.73	24.36	20.3	0.1
6	P1	17.59	233.37	24.36	21.6	1.9
7	P1	18.31	260.81	24.36	23.2	0.0
8	P1	18.84	297.91	24.36	24.4	0.0
9	P1	19.22	324.82	24.36	25.7	1.5
10	P1	19.94	399.26	24.36	26.9	7.1
11	P1	19.89	437.25	24.36	28.0	2.7
12	P1	21.06	487.86	24.36	29.2	14.3
13	P1	21.35	513.30	24.36	30.1	13.7
14	P1	22.04	527.64	24.36	30.8	11.1
15	P2	7.80	23.68	23.17	13.7	0.2
16	P2	9.37	39.68	23.17	15.2	0.1
17	P2	11.13	61.41	23.17	17.2	0.1
18	P2	13.05	90.83	23.17	19.3	2.0
19	P2	14.05	115.38	23.17	20.5	1.8
20	P2	15.22	142.64	23.17	22.1	6.5
21	P2	16.06	177.70	23.17	23.2	3.5
22	P2	16.76	219.22	23.17	24.4	0.3
23	P2	17.67	261.11	23.17	25.6	1.1
24	P2	17.82	299.95	23.17	26.6	5.6
25	P2	18.93	338.79	23.17	27.7	6.6
26	P2	19.50	366.52	23.17	28.6	6.9
27	P2	20.05	387.67	23.17	29.3	2.1
28	P3	7.10	16.31	17.45	10.3	0.2
29	P3	8.25	27.30	17.45	11.5	1.9
30	P3	9.34	36.57	17.45	12.9	0.3
31	P3	10.37	47.75	17.45	14.5	0.0

A0	-0.09344
A1	3.356889
k	3.189638
m	0.431302
SSres	195.714
S_P1	24.36

Plot	S
P1	24.36
P2	23.17
P3	17.45
P4	15.24
P5	20.81
P6	20.39
P7	21.98
P8	21.53
P9	22.04

hdom=(A0+A1*S)*exp(-k*1/t^m)

estimate de site index (S) for each plot and fit the Lundqvist function with the A parameter expressed as a linear function of the site index and plot the estimated values together with the original data



	S				
	15	18	21	24	27
t	hdom_S15	hdom_S18	hdom_S21	hdom_S24	hdom_S27
4	=($M$2+$M	10.44	12.18	13.93	15.67
5	10.22	12.26	14.31	16.36	18.40
6	11.52	13.83	16.14	18.45	20.76
7	12.67	15.21	17.75	20.29	22.82
8	13.69	16.43	19.17	21.91	24.65
9	14.60	17.52	20.45	23.37	26.29
10	15.42	18.51	21.60	24.69	27.78
11	16.17	19.41	22.65	25.89	29.13
12	16.86	20.24	23.62	27.00	30.38
13	17.50	21.00	24.51	28.02	31.52
14	18.09	21.71	25.34	28.96	32.59
15	18.64	22.37	26.11	29.84	33.58
16	19.15	22.99	26.83	30.67	34.50
17	19.64	23.57	27.51	31.44	35.37
18	20.09	24.12	28.14	32.17	36.19
19	20.52	24.63	28.74	32.86	36.97
20	20.93	25.12	29.31	33.51	37.70

But how to model the growth of several plots?

- There are several methods to simultaneously model the growth of several plots:

1. Expressing the parameters as a function of site and/or tree/stand variables
2. **Using growth functions formulated as difference equations - ADA**

2. Using growth functions formulated as difference equations – ADA

- Lundqvist-Korf type

Height

- Lundqvist-A: *with A as free parameter*

$$Y = A\, e^{-k\frac{1}{t^m}} \Rightarrow A = \frac{Y}{e^{-k\frac{1}{t^m}}} \longrightarrow \frac{Y_2}{e^{-k\frac{1}{t_2^m}}} = \frac{Y_1}{e^{-k\frac{1}{t_1^m}}}$$

The height (hdom) at a given age must be expressed as a function of the given age (t), an initial age (t0) and an initial height (hdom0), that is: hdom = *f* (t , hdom0 , t0).

The height (hdom2) at a future age must be expressed as a function of the future age (t2), a current age (t1) and a current height (hdom1), that is: hdom2 = *f* (t2 , hdom1, t1).

$$Y = Y_0\, e^{-k\left(\frac{1}{t^m} - \frac{1}{t_0^m}\right)} \longleftrightarrow Y_2 = Y_1\, e^{-k\left(\frac{1}{t_2^m} - \frac{1}{t_1^m}\right)}$$

2. Using growth functions formulated as difference equations – ADA

	A	B	C	D	E	F	G	H	I	J	K
1	Plot	t	hdom	N	G	ddom	dg	V	S	hdom_e	res^2
2	P1	4.1	15.1	1125	13.30	17.90	12.27	77.72	24.36	14.4	0.6
3	P1	4.8	17.8	1125	17.11	20.24	13.92	117.06	24.36	16.0	3.0
4	P1	5.7	19.8	1114	21.10	22.65	15.53	158.49	24.36	18.1	3.0
5	P1	6.8	20.6	1114	25.08	25.03	16.93	196.73	24.36	20.3	0.1
6	P1	7.6	22.9	1114	27.06	25.96	17.59	233.37	24.36	21.6	1.9
7	P1	8.7	23.2	1114	29.34	26.90	18.31	260.81	24.36	23.2	0.0
8	P1	9.5	24.3	1114	31.04	27.81	18.84	297.91	24.36	24.4	0.0
9	P1	10.5	24.4	1114	32.31	28.31	19.22	324.82	24.36	25.7	1.5
10	P1	11.6	29.6	1114	34.77	29.42	19.94	399.26	24.36	26.9	7.1
11	P1	12.6	29.7	1114	36.85	29.08	19.89	437.25	24.36	28.0	2.7
12	P1	13.8	32.9	1102	38.42	31.87	21.06	487.86	24.36	29.2	14.3
13	P1	14.8	33.8	1102	39.46	32.42	21.35	513.30	24.36	30.1	13.7
14	P1	15.6	34.1	1080	41.18	33.15	22.04	527.64	24.36	30.8	11.1
15	P2	4.1	13.2	1081	5.20	13.12	7.80	23.68	23.17	13.7	0.2
16	P2	4.8	15.5	1081	7.45	15.17	9.37	39.68	23.17	15.2	0.1
17	P2	5.7	16.8	1070	10.41	17.05	11.13	61.41	23.17	17.2	0.1
18	P2	6.8	17.9	1048	14.03	19.27	13.05	90.83	23.17	19.3	2.0
19	P2	7.6	19.2	1048	16.26	20.62	14.05	115.38	23.17	20.5	1.8
20	P2	8.7	19.6	1048	19.08	22.05	15.22	142.64	23.17	22.1	6.5
21	P2	9.5	21.3	1048	21.24	23.19	16.06	177.70	23.17	23.2	3.5
22	P2	10.5	25.0	1037	22.87	24.15	16.76	219.22	23.17	24.4	0.3
23	P2	11.6	26.7	1037	25.43	25.59	17.67	261.11	23.17	25.6	1.1
24	P2	12.6	29.0	1037	24.50	27.60	17.82	299.95	23.17	26.6	5.6
25	P2	13.7	30.2	1037	29.20	27.89	18.93	338.79	23.17	27.7	6.6
26	P2	14.8	31.2	1026	30.65	28.62	19.50	366.52	23.17	28.6	6.9
27	P2	15.6	30.7	1026	32.40	29.20	20.05	387.67	23.17	29.3	2.1
28	P3	4.1	10.7	1092	4.32	10.91	7.10	16.31	17.45	10.3	0.2
29	P3	4.8	12.8	1092	5.84	12.59	8.25	27.30	17.45	11.5	1.9
30	P3	5.7	13.5	1092	7.47	14.19	9.34	36.57	17.45	12.9	0.3
31	P3	6.8	14.4	1092	9.21	15.81	10.37	47.75	17.45	14.5	0.0
32	P3	7.6	15.6	1080	10.73	16.99	11.25	62.53	17.45	15.4	0.0
33	P3	8.7	15.8	1080	12.25	18.03	12.01	73.34	17.45	16.6	0.7
34	P3	9.5	16.7	1080	13.24	18.81	12.49	86.35	17.45	17.5	0.6
35	P3	10.5	18.2	1069	14.18	19.29	12.99	99.66	17.45	18.4	0.0

Sheet1 | a) | b) | c) | d) | d) (2) | e) | e) +S | e) +S (2) | f) +S +N | f) +S +N (2) | g)

- **Lundqvist-Korf type** — Height

Lundqvist-A: *with A as free parameter*

	A	B	C	D	E	F	G	H	I	J	K	L
1	Plot	t1	t2	hdom1	hdom2	N1	N2	G1	G2	S	hdom_est	res^2
2	P1	4.1	4.8	15.1	17.8	1125	1125	13.30	17.11	24.36	16.5	1.
3	P1	4.8	5.7	17.8	19.8	1125	1114	17.11	21.10	24.36	19.5	0.
4	P1	5.7	6.8	19.8	20.6	1114	1114	21.10	25.08	24.36	21.8	1.5
5	P1	6.8	7.6	20.6	22.9	1114	1114	25.08	27.06	24.36	21.7	1.6
6	P1	7.6	8.7	22.9	23.2	1114	1114	27.06	29.34	24.36	24.5	1.6
7	P1	8.7	9.5	23.2	24.3	1114	1114	29.34	31.04	24.36	24.3	0.0
8	P1	9.5	10.5	24.3	24.4	1114	1114	31.04	32.31	24.36	25.5	1.
9	P1	10.5	11.6	24.4	29.6	1114	1114	32.31	34.77	24.36	25.6	16.
10	P1	11.6	12.6	29.6	29.7	1114	1114	34.77	36.85	24.36	30.7	1.
11	P1	12.6	13.8	29.7	32.9	1114	1102	36.85	38.42	24.36	30.9	4.4
12	P1	13.8	14.8	32.9	33.8	1102	1102	38.42	39.46	24.36	33.9	0.0
13	P1	14.8	15.6	33.8	34.1	1102	1080	39.46	41.18	24.36	34.6	0.2
14	P2	4.1	4.8	13.2	15.5	1081	1081	5.20	7.45	23.165	14.4	1.
15	P2	4.8	5.7	15.5	16.8	1081	1070	7.45	10.41	23.165	17.1	0.
16	P2	5.7	6.8	16.8	17.9	1070	1048	10.41	14.03	23.165	18.6	0.5
17	P2	6.8	7.6	17.9	19.2	1048	1048	14.03	16.26	23.165	18.9	0.
18	P2	7.6	8.7	19.2	19.6	1048	1048	16.26	19.08	23.165	20.6	1.0
19	P2	8.7	9.5	19.6	21.3	1048	1048	19.08	21.24	23.165	20.5	0.7
20	P2	9.5	10.5	21.3	25.0	1048	1037	21.24	22.87	23.165	22.4	6.6
21	P2	10.5	11.6	25.0	26.7	1037	1037	22.87	25.43	23.165	26.2	0.2
22	P2	11.6	12.6	26.7	29.0	1037	1037	25.43	24.50	23.165	27.7	1.7
23	P2	12.6	13.7	29.0	30.2	1037	1037	24.50	29.20	23.165	30.1	0.0
24	P2	13.7	14.8	30.2	31.2	1037	1026	29.20	30.65	23.165	31.2	0.0
25	P2	14.8	15.6	31.2	30.7	1026	1026	30.65	32.40	23.165	32.0	1.0
26	P3	4.1	4.8	10.7	12.8	1092	1092	4.32	5.84	17.445	11.8	1.
27	P3	4.8	5.7	12.8	13.5	1092	1092	5.84	7.47	17.445	14.3	0.6
28	P3	5.7	6.8	13.5	14.4	1092	1092	7.47	9.21	17.445	15.0	0.4
29	P3	6.8	7.6	14.4	15.6	1092	1080	9.21	10.73	17.445	15.3	0.
30	P3	7.6	8.7	15.6	15.8	1080	1080	10.73	12.25	17.445	16.8	1.

Sheet1 | a) | b) | c) | d) | d) (2) | e) | e) +S | e) +S (2) | f) +S +N | f) +S +N (2) | g)

2. Using growth functions formulated as difference equations – ADA

- **Lundqvist-Korf type**

Height

Lundqvist-k: *with k as free parameter*

$$hdom_2 = A\left(\frac{hdom_1}{A}\right)^{\left(\frac{t_1}{t_2}\right)^n}$$

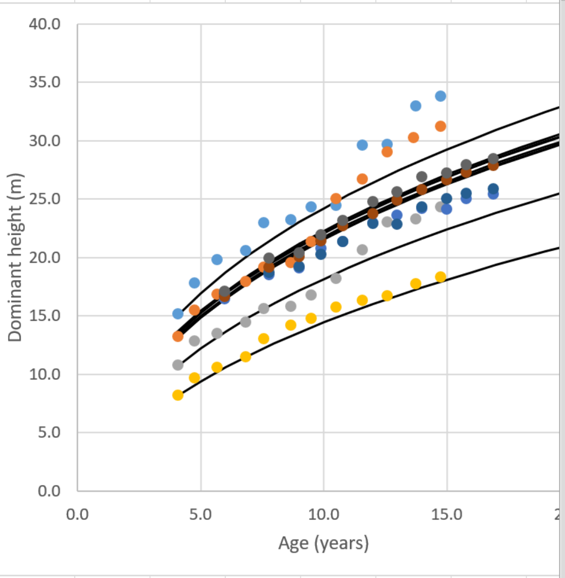
SUM | =$N$2*(D2/$N$2)^((B2/C2)^$N$3)

	Plot	t1	t2	hdom1	hdom2	N1	N2	G1	G2	S	hdom_est	res^2
2	P1	4.1	4.8	15.1	17.8	1125	1125	13.30	17.11	24.36	)^$N$3)	1.7
3	P1	4.8	5.7	17.8	19.8	1125	1114	17.11	21.10	24.36	19.5	0.1
4	P1	5.7	6.8	19.8	20.6	1114	1114	21.10	25.08	24.36	21.8	1.5
5	P1	6.8	7.6	20.6	22.9	1114	1114	25.08	27.06	24.36	21.7	1.6
6	P1	7.6	8.7	22.9	23.2	1114	1114	27.06	29.34	24.36	24.5	1.6
7	P1	8.7	9.5	23.2	24.3	1114	1114	29.34	31.04	24.36	24.3	0.0
8	P1	9.5	10.5	24.3	24.4	1114	1114	31.04	32.31	24.36	25.5	1.1
9	P1	10.5	11.6	24.4	29.6	1114	1114	32.31	34.77	24.36	25.6	16.0
10	P1	11.6	12.6	29.6	29.7	1114	1114	34.77	36.85	24.36	30.7	1.1
11	P1	12.6	13.8	29.7	32.9	1114	1102	36.85	38.42	24.36	30.9	4.4
12	P1	13.8	14.8	32.9	33.8	1102	1102	38.42	39.46	24.36	33.9	0.0
13	P1	14.8	15.6	33.8	34.1	1102	1080	39.46	41.18	24.36	34.6	0.2
14	P2	4.1	4.8	13.2	15.5	1081	1081	5.20	7.45	23.165	14.4	1.1
15	P2	4.8	5.7	15.5	16.8	1081	1070	7.45	10.41	23.165	17.1	0.1
16	P2	5.7	6.8	16.8	17.9	1070	1048	10.41	14.03	23.165	18.6	0.5
17	P2	6.8	7.6	17.9	19.2	1048	1048	14.03	16.26	23.165	18.9	0.1
18	P2	7.6	8.7	19.2	19.6	1048	1048	16.26	19.08	23.165	20.6	1.0
19	P2	8.7	9.5	19.6	21.3	1048	1048	19.08	21.24	23.165	20.5	0.7
20	P2	9.5	10.5	21.3	25.0	1048	1037	21.24	22.87	23.165	22.4	6.6
21	P2	10.5	11.6	25.0	26.7	1037	1037	22.87	25.43	23.165	26.2	0.2
22	P2	11.6	12.6	26.7	29.0	1037	1037	25.43	24.50	23.165	27.7	1.7
23	P2	12.6	13.7	29.0	30.2	1037	1037	24.50	29.20	23.165	30.1	0.0
24	P2	13.7	14.8	30.2	31.2	1037	1026	29.20	30.65	23.165	31.2	0.0
25	P2	14.8	15.6	31.2	30.7	1026	1026	30.65	32.40	23.165	32.0	1.6
26	P3	4.1	4.8	10.7	12.8	1092	1092	4.32	5.84	17.445	11.8	1.1
27	P3	4.8	5.7	12.8	13.5	1092	1092	5.84	7.47	17.445	14.3	0.6
28	P3	5.7	6.8	13.5	14.4	1092	1092	7.47	9.21	17.445	15.0	0.4
29	P3	6.8	7.6	14.4	15.6	1092	1080	9.21	10.73	17.445	15.3	0.1
30	P3	7.6	8.7	15.6	15.8	1080	1080	10.73	12.25	17.445	16.8	1.1

A	309.1166
m	0.189678
SSres	72.22658

hdom2=A*(hdom1/A)^((t1/t2)^m)

use the solver function from EXCEL to fit the difference equatic
derived from Lundqvist
function with k as the free parameter and plot the estimated



Sheet1 | a) | b) | c) | d) | d) (2) | e) | e) +S | e) +S (2) | f) +S +N | f) +S +N (2) | g)

2. Using growth functions formulated as difference equations – ADA

- **Lundqvist-Korf type**

Height

Lundqvist-k: *with k as free parameter*

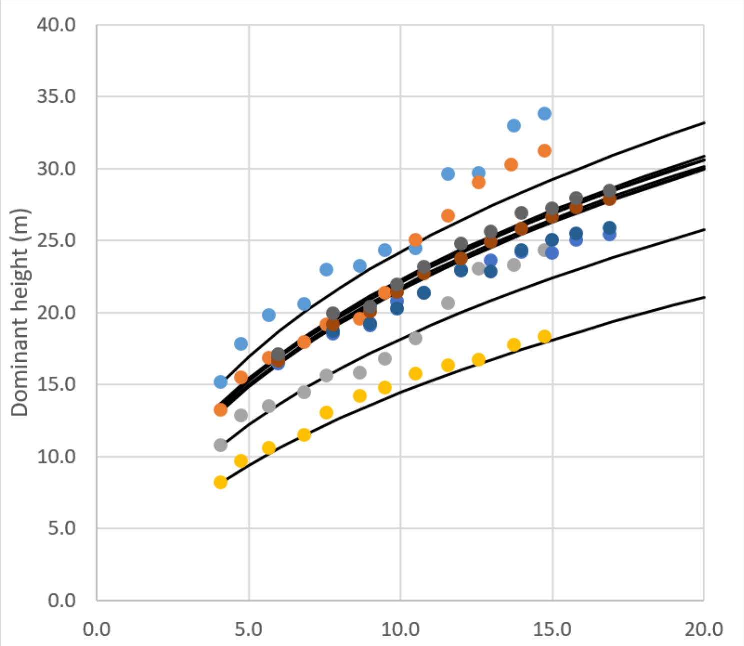
$$hdom_2 = A\left(\frac{hdom_1}{A}\right)^{\left(\frac{t_1}{t_2}\right)^n}$$

SUM | fx | =$N$2*(Z$3/$N$2)^((Z$4/$Y6)^$N$3)

	A	M	N
1	Plot		
2	P1	A	309.1166
3	P1	m	0.189678
4	P1		
5	P1	SSres	72.22658
6	P1		
7	P1		
8	P1		
9	P1		
10	P1		
11	P1		
12	P1		
13	P1		
14	P2		
15	P2		
16	P2		
17	P2		
18	P2		
19	P2		
20	P2		
21	P2		
22	P2		
23	P2		
24	P2		
25	P2		
26	P3		

hdom2=A*(hdom1/A)^((t1/t2)^m)

use the solver function from EXCEL to fit the difference equation derived from Lundqvist function with k as the free parameter and plot the estimated



Y	Z	AA	AB	AC	AD	AE	AF	AG	AH
	hdom1,t1								
	P1	P2	P3	P4	P5	P6	P7	P8	P9
hdom1	15.1	13.2	10.7	8.2	16.4	16.9	16.9	16.6	17.1
t1	4.1	4.1	4.1	4.1	6.0	6.0	6.0	6.0	6.0
t2	hdom_P1	hdom_P2	hdom_P3	hdom_P4	hdom_P5	hdom_P6	hdom_P7	hdom_P8	hdom_P9
4.0	=$N$2*(Z$	13.04	10.60	8.06	13.00	13.38	13.41	13.15	13.54
5.0	16.94	14.87	12.19	9.38	14.83	15.24	15.27	14.99	15.41
6.0	18.70	16.48	13.61	10.56	16.44	16.88	16.92	16.61	17.06
7.0	20.27	17.93	14.89	11.64	17.89	18.36	18.39	18.07	18.55
8.0	21.70	19.26	16.06	12.64	19.21	19.70	19.74	19.40	19.90
9.0	23.01	20.48	17.14	13.56	20.43	20.94	20.98	20.63	21.14
10.0	24.23	21.61	18.15	14.43	21.55	22.08	22.12	21.76	22.30
11.0	25.36	22.66	19.10	15.24	22.61	23.15	23.19	22.82	23.37
12.0	26.42	23.65	19.99	16.01	23.60	24.16	24.20	23.82	24.38
13.0	27.42	24.58	20.83	16.74	24.53	25.10	25.14	24.75	25.33
14.0	28.36	25.47	21.63	17.44	25.41	26.00	26.04	25.64	26.23
15.0	29.25	26.31	22.39	18.10	26.25	26.85	26.89	26.49	27.09
16.0	30.10	27.11	23.12	18.74	27.05	27.66	27.70	27.29	27.90
17.0	30.92	27.87	23.82	19.35	27.81	28.43	28.48	28.06	28.68
18.0	31.69	28.61	24.48	19.94	28.55	29.17	29.22	28.79	29.42
19.0	32.44	29.31	25.12	20.50	29.25	29.89	29.93	29.50	30.14
20.0	33.16	29.99	25.74	21.05	29.92	30.57	30.62	30.18	30.82

2. Using growth functions formulated as difference equations – ADA

- **Lundqvist-Korf type**

Height

Lundqvist-k: *with k as free parameter*

$$hdom_2 = A\left(\frac{hdom_1}{A}\right)^{\left(\frac{t_1}{t_2}\right)^n}$$

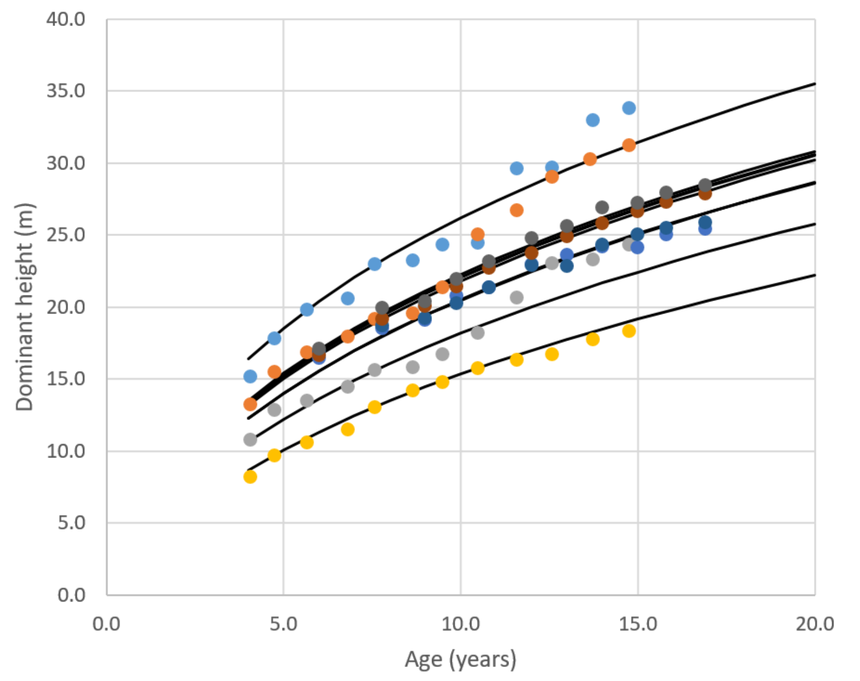
SUM | =$N$2*(Z$3/$N$2)^((Z$4/$Y6)^$N$3)

	A	N
1	Plot	
2	P1	309.1166
3	P1	0.189678
4	P1	
5	P1	72.22658
6	P1	
7	P1	
8	P1	
9	P1	
10	P1	
11	P1	
12	P1	
13	P1	
14	P2	
15	P2	
16	P2	
17	P2	
18	P2	
19	P2	
20	P2	
21	P2	
22	P2	
23	P2	
24	P2	
25	P2	
26	P3	

4th measurement as initial value

hdom2=A*(hdom1/A)^((t1/t2)^m)

use the solver function from EXCEL to fit the difference equation derived from Lundqvist function with k as the free parameter and plot the estimated values together



	hdom1,t1								
	P1	P2	P3	P4	P5	P6	P7	P8	P9
hdom1	22.9	19.2	15.6	13.0	21.3	21.3	22.9	22.7	23.1
t1	7.6	7.6	7.6	7.6	10.8	10.8	10.8	10.8	10.8
t2	hdom_P1	hdom_P2	hdom_P3	hdom_P4	hdom_P5	hdom_P6	hdom_P7	hdom_P8	hdom_P9
4	=$N$2*(Z$	13.39	10.62	8.67	12.24	12.26	13.35	13.18	13.52
5	18.52	15.25	12.21	10.05	13.99	14.02	15.21	15.02	15.40
6	20.38	16.90	13.63	11.29	15.55	15.57	16.85	16.65	17.05
7	22.05	18.37	14.91	12.42	16.95	16.97	18.32	18.11	18.53
8	23.55	19.72	16.09	13.46	18.22	18.25	19.67	19.44	19.88
9	24.93	20.95	17.17	14.43	19.40	19.43	20.90	20.67	21.13
10	26.20	22.10	18.18	15.33	20.49	20.52	22.04	21.80	22.28
11	27.39	23.17	19.13	16.18	21.51	21.54	23.11	22.87	23.35
12	28.49	24.17	20.02	16.98	22.47	22.50	24.12	23.86	24.36
13	29.54	25.12	20.87	17.74	23.38	23.41	25.06	24.80	25.31
14	30.52	26.01	21.67	18.46	24.23	24.27	25.95	25.69	26.21
15	31.45	26.86	22.43	19.15	25.05	25.08	26.80	26.53	27.07
16	32.34	27.67	23.15	19.81	25.83	25.86	27.61	27.34	27.88
17	33.19	28.45	23.85	20.44	26.57	26.60	28.39	28.11	28.66
18	33.99	29.19	24.52	21.05	27.28	27.32	29.13	28.84	29.40
19	34.77	29.90	25.16	21.63	27.97	28.00	29.84	29.55	30.12
20	35.51	30.58	25.78	22.20	28.63	28.66	30.52	30.23	30.80

But how to model the growth of several plots?

- There are several methods to simultaneously model the growth of several plots:

1. Expressing the parameters as a function of site and/or tree/stand variables
2. Using growth functions formulated as difference equations – ADA
3. **Combining parameters and ADA**

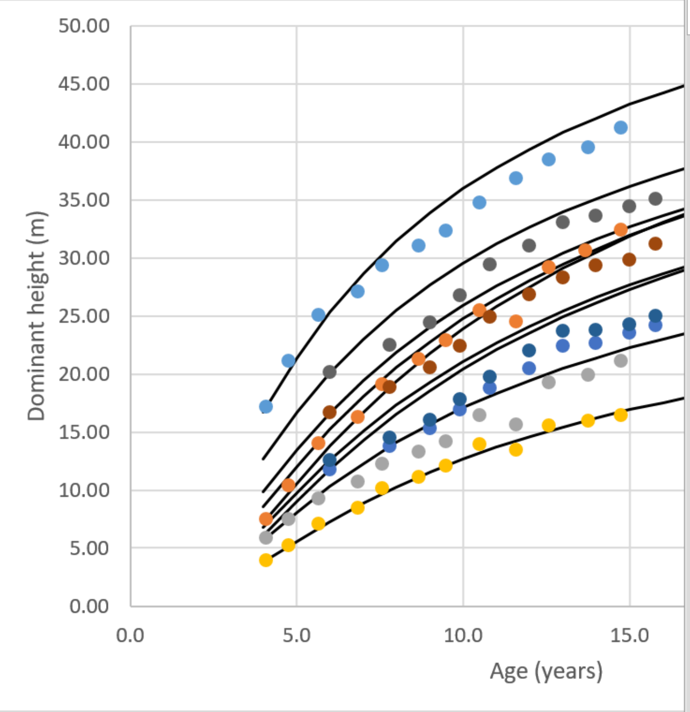
3. Combining parameters and ADA

$$G_2 = A\left(\frac{G_1}{A}\right)^{\left(\frac{t_1}{t_2}\right)^n} \quad ; \quad A = A_0 + A_1 S$$

SUM | fx | =($N$2+$N$3*J2)*(H2/($N$2+$N$3*J2))^((B2/C2)^$N$4)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Plot	t1	t2	hdom1	hdom2	N1	N2	G1	G2	S	G_est	res^2		
2	P1	4.1	4.8	15.1	17.8	1125	1125	13.30	17.11	24.36	=($N$2+$N	0.67	A0	-22.346
3	P1	4.8	5.7	17.8	19.8	1125	1114	17.11	21.10	24.36	20.84	0.07	AS	3.610724
4	P1	5.7	6.8	19.8	20.6	1114	1114	21.10	25.08	24.36	25.15	0.00	m	0.898288
5	P1	6.8	7.6	20.6	22.9	1114	1114	25.08	27.06	24.36	27.33	0.07		
6	P1	7.6	8.7	22.9	23.2	1114	1114	27.06	29.34	24.36	29.91	0.32	SSres	47.66513
7	P1	8.7	9.5	23.2	24.3	1114	1114	29.34	31.04	24.36	31.27	0.05		
8	P1	9.5	10.5	24.3	24.4	1114	1114	31.04	32.31	24.36	33.10	0.63		
9	P1	10.5	11.6	24.4	29.6	1114	1114	32.31	34.77	24.36	34.30	0.22		
10	P1	11.6	12.6	29.6	29.7	1114	1114	34.77	36.85	24.36	36.39	0.21		
11	P1	12.6	13.8	29.7	32.9	1114	1102	36.85	38.42	24.36	38.51	0.01		
12	P1	13.8	14.8	32.9	33.8	1102	1102	38.42	39.46	24.36	39.70	0.06		
13	P1	14.8	15.6	33.8	34.1	1102	1080	39.46	41.18	24.36	40.44	0.55		
14	P2	4.1	4.8	13.2	15.5	1081	1081	5.20	7.45	23.165	7.11	0.11		
15	P2	4.8	5.7	15.5	16.8	1081	1070	7.45	10.41	23.165	10.15	0.07		
16	P2	5.7	6.8	16.8	17.9	1070	1048	10.41	14.03	23.165	13.70	0.11		
17	P2	6.8	7.6	17.9	19.2	1048	1048	14.03	16.26	23.165	16.00	0.07		
18	P2	7.6	8.7	19.2	19.6	1048	1048	16.26	19.08	23.165	18.89	0.04		
19	P2	8.7	9.5	19.6	21.3	1048	1048	19.08	21.24	23.165	20.93	0.10		
20	P2	9.5	10.5	21.3	25.0	1048	1037	21.24	22.87	23.165	23.27	0.16		
21	P2	10.5	11.6	25.0	26.7	1037	1037	22.87	25.43	23.165	24.86	0.33		
22	P2	11.6	12.6	26.7	29.0	1037	1037	25.43	24.50	23.165	27.09	6.68		
23	P2	12.6	13.7	29.0	30.2	1037	1037	24.50	29.20	23.165	26.16	9.24		
24	P2	13.7	14.8	30.2	31.2	1037	1026	29.20	30.65	23.165	30.67	0.00		
25	P2	14.8	15.6	31.2	30.7	1026	1026	30.65	32.40	23.165	31.69	0.50		
26	P3	4.1	4.8	10.7	12.8	1092	1092	4.32	5.84	17.445	5.74	0.01		
27	P3	4.8	5.7	12.8	13.5	1092	1092	5.84	7.47	17.445	7.76	0.08		
28	P3	5.7	6.8	13.5	14.4	1092	1092	7.47	9.21	17.445	9.71	0.25		
29	P3	6.8	7.6	14.4	15.6	1092	1080	9.21	10.73	17.445	10.52	0.05		

G2=(A0+As*S)*(G1/(A0+As*S))^((t1/



3. Combining parameters and ADA

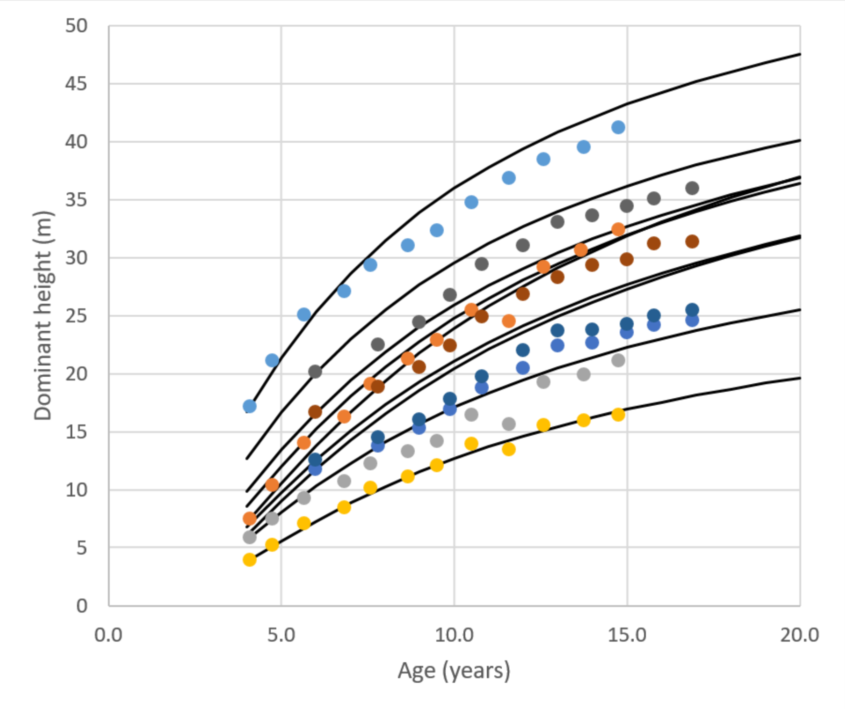
$$G_2 = A\left(\frac{G_1}{A}\right)^{\left(\frac{t_1}{t_2}\right)^n} \quad ; \quad A = A_0 + A_1 S$$

Z7 =($N$2+$N$3*Z$3)*(Z$4/($N$2+$N$3*Z$3))^((Z$5/$Y7)^$N$4)

G2=(A0+As*S)*(G1/(A0+As*S))^((t1/t2)^m)

	A
1	Plot
2	P1
3	P1
4	P1
5	P1
6	P1
7	P1
8	P1
9	P1
10	P1
11	P1
12	P1
13	P1
14	P2
15	P2
16	P2
17	P2
18	P2
19	P2
20	P2
21	P2
22	P2
23	P2
24	P2
25	P2
26	P3
27	P3
28	P3

M	N
A0	-22.346
AS	3.610724
m	0.898288
SSres	47.66513



	G1,t1								
	P1	P2	P3	P4	P5	P6	P7	P8	P9
S	24.36	23.17	17.45	15.24	20.81	20.39	21.98	21.53	22.04
G1	17.1	7.5	5.8	3.9	11.8	12.6	15.2	16.7	20.1
t1	4.1	4.1	4.1	4.1	6.0	6.0	6.0	6.0	6.0
t2	Gest_P1	Gest_P2	Gest_P3	Gest_P4	Gest_P5	Gest_P6	Gest_P7	Gest_P8	Gest_P9
4	=($N$2+$	7.2	5.6	3.8	6.1	6.8	8.5	9.8	12.7
5	21.4	10.6	8.1	5.6	9.0	9.8	12.0	13.4	16.7
6	25.3	13.8	10.3	7.3	11.8	12.6	15.2	16.7	20.1
7	28.7	16.7	12.3	8.8	14.3	15.1	18.1	19.5	23.0
8	31.5	19.4	14.1	10.3	16.5	17.3	20.6	21.9	25.5
9	33.9	21.7	15.7	11.5	18.6	19.3	22.8	24.0	27.7
10	36.0	23.9	17.1	12.7	20.4	21.1	24.7	25.9	29.5
11	37.8	25.8	18.3	13.7	22.1	22.7	26.5	27.6	31.2
12	39.4	27.5	19.5	14.6	23.6	24.1	28.1	29.1	32.6
13	40.8	29.1	20.5	15.4	24.9	25.4	29.5	30.4	33.9
14	42.1	30.5	21.4	16.2	26.2	26.6	30.8	31.6	35.1
15	43.2	31.8	22.2	16.9	27.3	27.7	31.9	32.7	36.2
16	44.2	33.0	23.0	17.5	28.3	28.6	33.0	33.7	37.1
17	45.2	34.1	23.7	18.1	29.3	29.5	34.0	34.6	38.0
18	46.0	35.2	24.4	18.7	30.2	30.4	34.8	35.4	38.8
19	46.8	36.1	25.0	19.2	31.0	31.1	35.7	36.2	39.5
20	47.5	37.0	25.5	19.6	31.7	31.8	36.4	36.9	40.1