# INSTITUTO SUPERIOR DE AGRONOMIA 

1 ${ }^{\text {st }}$ Assessment of Applied Operations Research - March 31, 2022 - Part I

Name: $\qquad$

1. You are the manager of two paper mills, Mill A and Mill B, that manufacture three grades of paper, low, medium and high. You have contracts to supply at least 1600 tons of low-grade paper, 500 tons of medium-grade, and 2000 tons of high-grade. It costs $1000 €$ /day to operate Mill A, and $2000 € /$ day to operate Mill B. Mill A produces, per day, 8 tons of low-grade paper, 1 ton of medium-grade, and 2 tons of high-grade paper. Mill B produces, per day, 2 tons of low-grade, 1 ton of medium-grade, and 10 tons of high-grade paper. How many days each mill should operate to satisfy the order at least cost?
(a) Formulate this problem as a linear program (do not solve it).
(b) If you signed a labor contract that specifies that both mills must operate the same number of days, how would this change the model described in (a)? Find the optimal solution of the problem with this additional constraint.
2. Consider the following linear programming problem:

$$
\begin{array}{crl}
\text { Max } \quad Z=0.12 x_{1}+0.2 x_{2}+0.3 x_{3} & \\
\text { s.t. } & 0.35 x_{1}+0.4 x_{2}+0.5 x_{3} & \geq 0.9 \\
& 0.5 x_{1}+0.75 x_{2}+1 x_{3} & \geq 1 \\
& x_{1}+x_{2} & \leq 0.8  \tag{4}\\
& x_{1}, \quad x_{2}, & x_{3}
\end{array} \geq 0 .
$$

The optimal solution obtained with the Solver of Excel is $x_{1}=0.8, x_{2}=$ $0, x_{3}=1.24$.
(a) Complete the gray boxes in Tables 1 and 2.

| - | - | Original Value |
| :---: | :---: | :---: |
| - | - | 0 |

Table 1: Answer report by the Excel Solver - Objective cell (Min).

| - | Name | Cell value | Formula | Status | Slack |
| :---: | :---: | :---: | :---: | :---: | :---: |
| - | $(2)$ |  | - |  |  |
| - | $(3)$ |  | - |  |  |
| - | $(4)$ |  | - |  |  |

Table 2: Answer report by the Excel Solver - Constraints.
(b) Consider the following information about the sensitivity analysis provided by the Excel Solver:
i. As long as $x_{1}$ 's objective function coefficient is not greater than $0.125, x_{1}$ will remain equal to 0.8 in the optimal solution;
ii. If $x_{2}$ 's objective function coefficient is decreased by infinity, $x_{2}$ will remain equal to 0 in the optimal solution.
Complete the gray boxes in Table 3

|  |  | Final | Reduced | Objective | Allowable | Allowable |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | Name | Value | Cost | Coefficient | Increase | Decrease |
| - | $x_{1}$ |  | - |  |  | - |
| - | $x_{2}$ |  | - |  | - | - |
| - | $x_{3}$ |  | - |  | - | - |

Table 3: Sensitivity report by the Excel Solver - Variable cells.
(c) Consider Table 4.

|  | Name | Final <br> Value | Shadow <br> Price | Constant <br> R.H. Side | Allowable <br> Increase | Allowable <br> Decrease |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | $(2)$ | 0.9 | 0.6 | 0.9 | $1 \mathrm{E}+30$ | 0.32 |
| - | $(3)$ | 1.64 | 0 | 1 | 0.64 | $1 \mathrm{E}+30$ |
| - | $(4)$ | 0.8 | -0.09 | 0.8 | 1.77 | 0.8 |

Table 4: Sensitivity report by the Excel Solver - Constraints.
i. How do you interpret the shadow price for each constraint?
ii. What is the range of feasibility for each RHS value? How do you interpret this range?

