**INSTITUTO SUPERIOR DE AGRONOMIA**

**Exam of Applied Operations Research - Module 2 - 12 June 2016/17**

Number: Name:

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**1.** (10val.) A furniture company manufactures tables, chairs, desks and bookcases. Table 1 displays, for one unit of each product, the amounts of resources (softwood, hardwood and labor) consumed and the revenue obtained.

| | Softwood (feet/unit) | Hardwood (feet/unit) | Labor (hours/unit) | Revenue (euros/unit) |
|---|---|---|---|---|
| Table | 5 | 2 | 3 | 62 |
| Chair | 1 | 3 | 2 | 42 |
| Desk | 9 | 4 | 5 | 103 |
| Bookcase | 12 | 1 | 10 | 139 |

Table 1:

The company has available 1500 feet of softwood and 1000 feet of hardwood. The factory employs 10 people, each of which works 8 hours per day. The firm plans its production for a 10-day period. Softwood costs 2€ per foot, hardwood 5€ per foot and labor 10€ per hour.

The following LP model translates the problem that the company would like to solve.

$$\text{Max} \quad Z = 12x_1 + 5x_2 + 15x_3 + 10x_4 \tag{1}$$

$$\text{s.t.} \quad 5x_1 + x_2 + 9x_3 + 12x_4 \leq 1500 \tag{2}$$

$$2x_1 + 3x_2 + 4x_3 + x_4 \leq 1000 \tag{3}$$

$$3x_1 + 2x_2 + 5x_3 + 10x_4 \leq 800 \tag{4}$$

$$x_1 \geq 40 \tag{5}$$

$$x_2 \geq 130 \tag{6}$$

$$x_3 \geq 30 \tag{7}$$

$$x_4 \leq 10 \tag{8}$$

$$x_1, \quad x_2, \quad x_3, \quad x_4 \in \mathbb{N}_0. \tag{9}$$

a) What can be the meaning of the decision variables $x_1$, $x_2$, $x_3$ and $x_4$, the objective function (1) and constraints (2) to (8)?

b) Table 2 displays the answer report provided by the Excel Solver concerning constraints (4), (5), (6) and (7). Complete the gray boxes in Table 2 and find the optimal solution.

| Name | Cell value | Status | Slack |
|---|---|---|---|
| (4) | | Binding | 0 |
| (5) | | Not Binding | 90 |
| (6) | | Binding | 0 |
| (7) | | Binding | 0 |

Table 2:

**2.** (5val.) Consider the following LP problem (P1):

$$\min z = 4x_1 + 4x_2 + x_3$$
$$\begin{cases} x_1 + x_2 + x_3 \leq 2 \\ 2x_1 + x_2 \leq 3 \\ 2x_1 + x_2 + 3x_3 \geq 3 \\ x_1, \quad x_2, \quad x_3 \geq 0. \end{cases}$$

Use the Big M method to obtain a starting basic feasible solution for (P1). Identify the first tableau for the simplex method and the corresponding basic feasible solution.

**3.** (5val.) Consider the following LP problem (P2):

$$\min z = 5x_1 + 3x_2 + 8x_3$$
$$\begin{cases} x_1 - x_2 + 4x_3 = 5 \\ 2x_1 + 5x_2 + 7x_3 \geq 6 \\ x_1 \in R, \quad x_2 \leq 0, \quad x_3 \geq 0. \end{cases}$$

a) Write the dual problem of (P2).

b) The primal optimal solution for (P2) is $x_1 = -11$, $x_2 = 0$ and $x_3 = 4$. Use complementary slackness conditions to obtain the dual optimal solution.