INSTITUTO SUPERIOR DE AGRONOMIA

$2^{\underline{nd}}$ **Assessment of Applied Operations Research** - May 4, 2022

Name: ____

1. One post office needs a different number of employees during each day of the week, according to the following table. [2.5v]

	Minimum number of employees
Monday	17
Tuesday	13
Wednesday	16
Thursday	19
Friday	14
Saturday	16
Sunday	11

The labor rules impose that each employee must have 2 days off (rest period) after 5 consecutive days of work (shift). The post office's chief would like to know the **number of employees assigned to each shift** in order to minimize the total number of employees. Note: One shift is Monday through Friday, other shift is Tuesday through Saturday and so on.

(a) Write an integer programming model for this problem.

(b) Indicate a feasible solution to the problem that is not optimal.

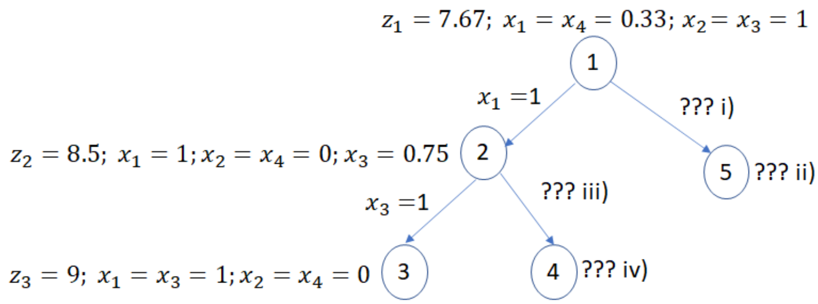
2. Consider projects P_1, P_2 and P_3 and the binary variables x_i, with $i = 1, 2, 3$, that assume the unitary value if P_i is selected and the null value otherwise. Assume that these variables can only assume the values in the table below. Determine just one constraint that ensures those (and only) those values, and explain its meaning. [0.5v]

x_1	x_2	x_3
0	0	0
0	0	1
1	0	1
0	1	1

3. Consider the following integer linear programming problem, further denoted by (IP): [3v]

$$\min z = 7x_1 + 3x_2 + 2x_3 + x_4$$
$$\left\{ \begin{array}{ccccccccc} 4x_1 & + & 2x_2 & - & x_3 & + & 2x_4 & \geq & 3 \\ 4x_1 & + & 2x_2 & + & 4x_3 & - & x_4 & \geq & 7 \\ x_1, & & x_2, & & x_3, & & x_4 & \in & \{0,1\} \end{array} \right.$$

The figure below shows the branch-and-bound for solving the (IP). Information concerning the linear programming relaxation solution of each subproblem is displayed near the corresponding node, where z_i is the optimal solution value of subproblem i.



(a) Write the linear programming relaxation problem that gave rise to the solution obtained in node 1.

(b) In node 1, z_1=7.67 is a lower bound or an upper bound on the optimal value of (IP)?

(c) Nodes 4 and 5 are obtained by adding which constraints to the linear programming relaxation of (IP)? Specify the constraint in i) and the constraint in iii).

(d) Complete node 4 iv) and node 5 ii). Justify your answer. Can you prune any of these nodes?

(e) Can you identify an optimal solution for (IP)? Justify your answer and, if yes, display the optimal solution to (IP).