INSTITUTO SUPERIOR DE AGRONOMIA

Applied Operations Research
Multi-Objective Linear Programming and Monte Carlo Simulation

Part 3
26 May 2022

MULTI-OBJECTIVE LINEAR PROGRAMMING

1. Consider the following multiple-objective linear problem with 2 objective functions:

$$\max Z_1 = 4x_1 + 2x_2$$
$$\min Z_2 = x_1 + 3x_2$$

Subject to:

$$x_1 + 1.5\,x_2 \le 15 \qquad x_1 + x_2 \ge 6$$
$$2x_1 + x_2 \le 18 \qquad x_1 \le 8$$
$$x_1 + 4x_2 \ge 12 \qquad x_1, x_2 \ge 0$$

a) Plot the graph for the decision space (x_1, x_2) identifying the feasible region and the corner points for this problem.

b) Build the table for the decision variables and their corresponding objective values for all corner point solutions.

c) Plot the graph for the objective space (Z_1, Z_2) making the correspondence between the corner points in the solution space and the corner points in the decision space.

d) Explain what you understand by "non-dominated solutions' set"

e) Analyze the objective space and indicate for each corner point if this is part of the non-dominated solutions' set and in case it is not indicate which solutions dominate it (use the graphic to illustrate it)

f) One way of solving MOLP problems is to create a composite objective function as a linear combination of all objectives in the problem. Note that Minimizing is equivalent to (- Max).

f1) Write the composite objective function: weight the first objective by 0.75 and the second objective by 0.25.

f2) What problem, if any, do you see with the use of this approach?

g) Another way of solving MOLP problems is using Goal Programming (GP). Suppose the first objective ($Z_1 = 4x_1 + 2x_2$) is a goal in a GP problem and it is possible to make it approximately equal to 25.

g.1) Using the deviational variables (d_1^+ and d_1^-) write the constraint that can be used to express this goal

g.2) If the solution (x_1, X_2) = (4, 3) was found, what values would the deviational variables assume?

h) Finally, use the weighting method to generate a set of composite objective functions considering 6 combinations of weights as well distributed as possible within the 0-1 range.

h.1) Write the composite objective functions in order to produce an approximation to the non-dominated set.

h.2) Would you expect a different set of non-dominated solutions' if a higher number of weight combinations was used? Justify you answer.

MONTE CARLO SIMULATION

"Monte Carlo methods are a broad class of computational algorithms that rely on repeated random sampling from a probability distribution, which can be empirical or theoretical, being especially useful for simulating phenomena with significant uncertainty. In general, the method requires many samples to get a good approximation which may result in extremely high computational cost."

1) *Random numbers are the core ingredient to any Monte Carlo study.*

1.a) Discuss the use of seeds in experimental design to remain in control of the generated random numbers and give an example of when you would choose to use pseudorandom numbers and true random numbers. Justify

1.b) Build a pseudorandom number generator expression of your choice and generate 5 numbers (keep it simple)

2) *PDFs, essential for implementing Monte Carlo methods, define the relative probability of an event to take place. A PDF must be a non-negative real-valued function, and its integral over its range must be 1.*

2.a) Explain the processes of building Cumulative Distribution Functions when using a theoretical distribution and when using an empirical distribution.

2.b) All the trees in a small stand were measured and distributed by diameter class. Build a cumulative distribution based on these measurements and use the random numbers previously generated in 1.b) to assign a diameter class (central d) to 5 trees.

diameter classes	central d	nr trees
2.5-7.4	5	404
7.5-12.4	10	404
12.5-17.4	15	808
17.5-22.4	20	1820
22.5-27.4	25	3033
27.5-32.4	30	606

3) *"Monte Carlo simulation methods consist of generating random draws from the relevant distribution and replacing expectations by arithmetic means with the accuracy of Monte Carlo estimates being inversely proportional to the number of draws"*.

Comment the sentence mentioning the different alternatives that can be used to determine the size

of samples. You can use the *example in 2.b) to explain which alternative you would select for determining the sample size.*

4) Now that you know Monte Carlo simulation basic concepts and how it can be implemented. Imagine and describe a practical situation in which Monte Carlo simulation could be applied (apart from examples presented in classes).