INSTITUTO SUPERIOR DE AGRONOMIA

$1^{\underline{st}}$ **Exam of Applied Operations Research** - June 17, 2022

Name: ____

1. A certain university maintains a powerful mainframe computer for research use. There are four operators (two graduate students and two undergraduate students) to operate and maintain the computer, as well as to perform some programming services, from Wednesday to Friday (in the other days the computer is to be operated by staff outside the university). They all have different wage rates because of differences in their experience with computers and in their programming ability. Table 1 shows their wage rates, along with the maximum number of hours that each can work each day. Each operator is guaranteed a certain minimum number of hours per week that will maintain an adequate knowledge of the operation. This level is set arbitrarily at 7 hours per week for the graduate students (A and B) and 8 hours per week for the undergraduate students (C and D). [3v]

Operators	Wage Rate (€/hour)	Maximum Hours of Availability		
		Wednesday	Thursday	Friday
A	10.00	6	0	6
B	10.10	6	6	0
C	9.90	0	4	4
D	9.80	5	0	5

Table 1:

The computer facility is to be open for operation from 8 A.M. to 6 P.M. every day. The director of the computer facility wishes to determine the **number of hours she should assign to each operator on each day** in order to minimize cost.

   (a) Formulate a linear programming model for this problem.

   (b) Determine a feasible solution.

2. Consider projects P_1, P_2 and P_3 and the binary variables x_i, with $i = 1, 2, 3$, that assume the unitary value if P_i is selected and the null value otherwise. Assume that these variables can only assume the values in the table below. Determine just one constraint that ensures those (and only) those values, and explain its meaning. [2v]

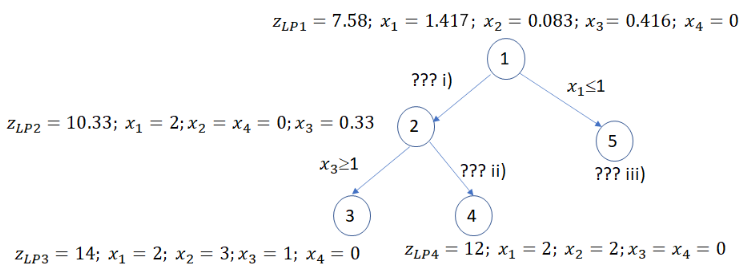
x_1	x_2	x_3
1	0	0
0	0	1
0	0	0

3. Consider the following mixed integer linear programming problem, further denoted by (MILP): [4v]

$$\min\ z = 5x_1 + x_2 + x_3 + 2x_4$$

$$\begin{cases} 5x_1 - x_2 \geq 7 \\ x_1 + x_2 + 6x_3 + x_4 \geq 4 \\ -x_2 + 5x_3 - x_4 \leq 2 \\ x_1\ ,\ x_2\ ,\ x_3\ ,\ x_4 \geq 0 \\ x_1\ ,\ x_2\ ,\ x_3\ \ integer \end{cases}$$

The figure below shows the branch-and-bound for solving the (MILP). Information concerning the linear programming relaxation solution of each subproblem is displayed near the corresponding node, where z_{LPi} is the optimal solution value of subproblem i.



(a) Nodes 2 and 4 are obtained by adding which constraints to the linear programming relaxation of the (MILP)? Specify the constraint in i) and the constraint in ii).

(b) Complete node 5 iii). Justify your answer. Can you prune this node?

(c) Can you identify an optimal solution for the (MILP)? Justify your answer and, if yes, display the optimal solution.

INSTITUTO SUPERIOR DE AGRONOMIA

Applied Operations Research - Linear Programming

Full exam

17 June 2022

[5v] LPP

I. Consider the following linear programming problem (LPP):

$$\text{Max } Z = 40\,y1 + 30\,y2$$

subject to:

$$y1 + y2 \leq 12$$
$$2\,y1 + y2 \leq 16$$
$$Y2 \leq 10$$

$$y1 \quad y2 \geq 0$$

a) Display the feasible region in a graphic and highlight the corner points.

b) Write the LP problem in the standard form.

c) Apply the Simplex Method and display all Simplex tableaus

d) For each tableau, make sure you indicate the:

- feasible solution and objective function value
- the basic and non-basic variables
- corresponding corner point in the graphic
- operations carried out to update each row/table

e) Define shadow price. Without resolving the simplex, indicate the impact on the objective function that a unitary change in the RHS of constraint 2 would have

[6v] MOLP (answer 1 and choose between 2 and 3)

1. Consider the following multiple-objective linear programming problem with 2 objective functions:

$$\max Z_1 = 4x_1 - 5x_2$$
$$\min Z_2 = 4x_1 - 2x_2$$

Subject to:

$$-6x_1 + 3x_2 \leq 12 \qquad X_2 \leq 6$$
$$4x_1 + 2x_2 \leq 24 \qquad x_1, x_2 \geq 0$$
$$3x_1 + 2x_2 \leq 30$$

a) Plot the graph for the decision space (x_1, x_2) identifying the feasible region and the corner points for this problem.

b) Build the table for the decision variables and their corresponding objective values for all corner point solutions.

c) Plot the graph for the objective space (Z_1, Z_2) making the correspondence between the corner points in the solution space and the corner points in the decision space.

d) Explain what you understand by "non-dominated solutions' set"

e) For each corner point indicate if it is part of the non-dominated solutions' set (analyzing the objective space) and in case it is not indicate which solutions dominate it (use the graphic to illustrate it)

2. Preemptive optimization is one way of solving multiple objective linear programing (MOLP) problems. In this classical approach, optimization is performed by solving single objective linear programming problems one at a time prioritizing the different objective functions.

   f.1) formulate the second stage of your problem assuming you used preemptive optimization and prioritized Z_2 (min $Z_2 = 4x_1 - 2x_2$) finding an optimal value of -8 in the first stage.

3. Goal programming: consider it was possible to set target values to both objective functions; that both positive and negative deviations from the goal value for Z_1 (of approximately 20) should be penalized with 2 and 5, respectively; while a penalty weight of 3 should be applied for Z_2 greater than -2.

	Goals	penalties
Z_1	$4x_1 - 5x_2 = 20$	2(+), 5(-)
Z_2	$4x_1 - 2x_2 \leq -2$	3(+)

   g.1) Use the deviational variables (d_i^+ and d_i^-) to write the constraints that represent Z_1 and Z_2 as goals

   g.2) Write the GP objective function using the deviational variables and corresponding penalties.

Monte Carlo

1. Explain by your own words the principles of Monte Carlo Simulation, focusing on the use of theoretical and empirical distributions, pseudo-random numbers and sample size. Present a practical application of Monte Carlo simulation in Forestry.