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Topic: Overview of fundamentals of neural networks

Consider the following expressions:

$$g(z) = \frac{1}{1 + e^{-z}}$$
(1)

$$\hat{y} = g\left(w_0 + \sum_{j=1}^k x_j \, w_j\right) \tag{2}$$

$$L_i(\mathbf{w}) = \mathcal{L}(y_i, \hat{y}_i), \text{ for } i = 1, \dots, n$$
(3)

$$L(\mathbf{w}) = \frac{1}{B} \sum_{i=1}^{B} L_i(\mathbf{w})$$
(4)

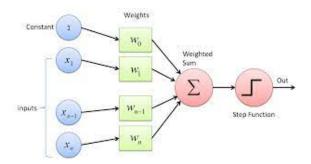
where  $y_i$  is the label of the *i*th example. Each example is described by a vector  $(x_1, \ldots, x_k)$ , where each x is an explanatory variable.  $\mathbf{w} = (w_0, w_1, \ldots, w_k)$  is a vector of weights.  $\mathcal{L}$  is a function that takes two numbers and returns a measure of dissimilarity between them.

1. Equation 2 describes the a perceptron model. Is this a model for a regression problem or for a classification problem? How do you interpret the output?

Response: The output of the model is a value between 0 and 1 (since function g returns a value between 0 and 1) that can be interpreted as the probability of the example belonging to class "1". This model is therefore designed for a classification problem.

2. Draw a diagram that describes that perceptron model and label it with the symbols in Equation 2.

Response: The following diagram where instead of the step function we have g.



- **3.** Indicate the symbols in the equations above that correspond to the following concepts, with a brief explanation,
  - a) Bias:

Response:  $w_0$ , is the constant term.

b) Batch size:

<u>Response</u>: *B*, is the nember of examples that are grouped in order to compute the loss. c) Activation function:

Response: The activation function is represented by g. It is non linear.

d) Predicted label:

<u>Response</u>: The predicted label is the output the value returned by the model  $\hat{y}$ 

e) Loss of a batch of examples:

Response:  $L(\mathbf{w})$  is the loss computed for B examples.

- 4. What is the optimal set of weights for the perceptron model? Choose one option and give a brief explanation for the ones you did not choose.
  - a) The set of weights  $\mathbf{w}^*$  that minimize  $L(\mathbf{w})$

Response: correct answer

b)  $\mathbf{w}^*$  such that the gradient  $\nabla L(\mathbf{w}^*) = \frac{\partial L}{\partial \mathbf{w}}(\mathbf{w}^*)$  is zero

Response: A gradient converging to zero is an indication that the gradient descent algorithm could be approaching a local optimum but it does not guarantee that it is the best overall solution.

c) Given a current set of weights  $\mathbf{w}^*$ , and a learning rate  $\eta$ , it is the new set of weights defined by  $\mathbf{w}^* - \eta \nabla L(\mathbf{w}^*)$ 

Response: This is just the updating rule of the gradient descent technique.