Margarida Tomé & Susana Barreiro, last revised 2025

INDIVIDUAL TREE MODELS

Individual tree models

competition indices

Factors affecting tree growth

- Growth of individual trees on particular sites is influenced by a number of factors such as:
  - tree age
  - micro-environment conditions (mainly soil)
  - tree size
  - competitive status
- Past growing conditions and genetic potential to grow account for actual characteristics of the tree, such as size and vigor, which are usually introduced in a tree growth model by initial tree size and age

Factors affecting tree growth

- Other factors influencing tree growth may be separated into the following three components:
  - General environment of competition, which is usually taken into account using **stand density**
  - Micro-environmental and genetic influences, represented by a **ratio of** some **dimension of the tree** to **the mean** or **maximum** value of this dimension in the stand
  - Influence of **local neighbors**

Competition indices

- The effect of local neighbors may be expressed by some mathematical formulation - commonly referred to as a "competition index" - representing how much each tree is affected by its neighbors

I Distance-independent or non spatially-explicit indices

- simple formulations expressing the hierarchical position of the tree within the stand or plot (d)

II Distance-dependent or spatially explicit-indices

- to more complex indices that express the size of, distance to, and number of local neighbors

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I - Distance independent competition indices

- Distance-independent indices are simple functions of stand level variables and/or dimensions of the subject tree in relation to the average or maximum tree value of the stand without taking into account inter-tree distances
  - I.1 Relative dimensions
  - I.2 Area proportional to tree basal area
  - I.3 Crown ratio
  - I.4 Measures based on trees larger than the subject tree (e.g. G>d)
  - I.5 Measures based on crown variables evaluated at a certain percentage of tree height

I - Distance independent competition indices

I.1 Relative dimensions

- Measure the hierarchical position of the tree within the stand by comparison with the size of the average, maximum or dominant trees

$$Rx_m = \frac{x_i}{x_m} \qquad Rx_{max} = \frac{x_i}{x_{max}} \qquad Rx_{dom} = \frac{x_i}{x_{dom}}$$

where x is a tree variable such as diameter, height or some crown variable and the subscripts *m*, *max* and *dom* indicate, respectively, the stand average, maximum and the average size of dominant trees

I - Distance independent competition indices

I.2 Area proportional to relative tree basal area

- Consists on dividing the plot area among the individual trees according to their dimension (for instance basal area) in relation to the dimension of the average tree of the stand

$$\frac{APgi}{10000} = \frac{gi}{\sum gi}$$

$$APg_i = \frac{10{,}000}{N}\frac{g_i}{\bar{g}}$$

I - Distance independent competition indices

I.3 Crown ratio

- Crown ratio (crown length divided by total tree height) has also been used to express the past competition undergone by each individual tree

$$c_r = c_l / h$$

I - Distance independent competition indices

- I.4 Measures based on trees larger than the subject tree
  - Basal area of the trees larger than the subject tree ($G_{>di}$)
  - Modified version of the previous that combines it with relative spacing (R_s)

$$G_{>dimod} = \frac{1}{R_s}\left(1-\left(\frac{G_{>di}}{G}\right)\right)$$

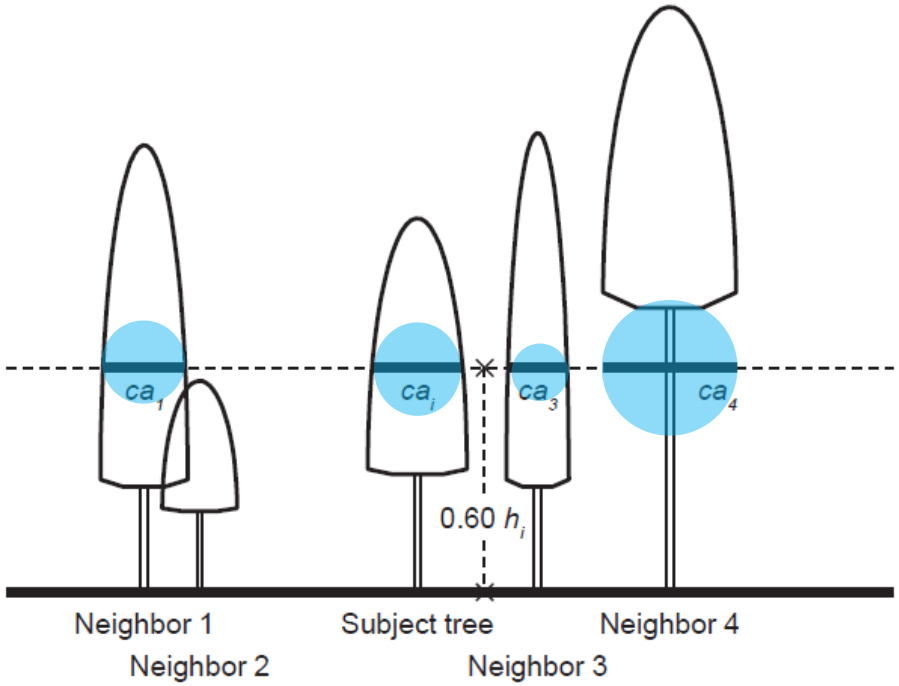
  - Crown competition factor of trees larger than the subject tree

I - Distance independent competition indices

- I.5 Measures based on crown variables evaluated at a certain percentage of crown length or tree height

Crown cover computed at h_p

$$CC_p = \frac{1}{\text{Plot area}} \sum_{i=1}^{n} ca_{p_i}$$



Competition indices

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I Distance-independent or non spatially-explicit indices

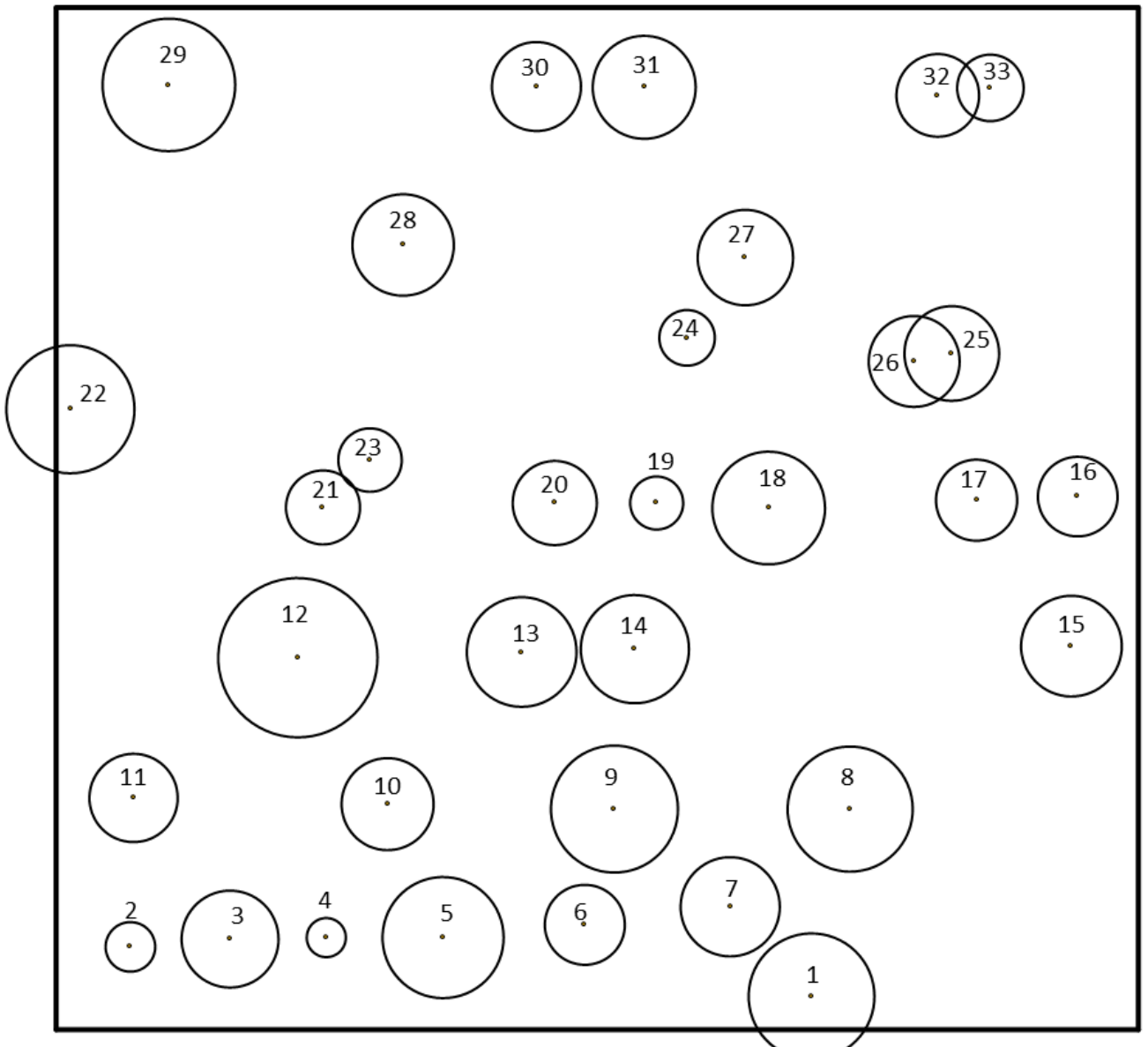
- simple formulations expressing the hierarchical position of the tree within the stand or plot (d)

II Distance-dependent or spatially explicit-indices

- to more complex indices that express the size of, distance to, and number of local neighbors

II - Distance dependent competition indices

- Distance dependent competition indices quantify the local inter-tree competition and therefore require coordinates of every tree (stem-map)



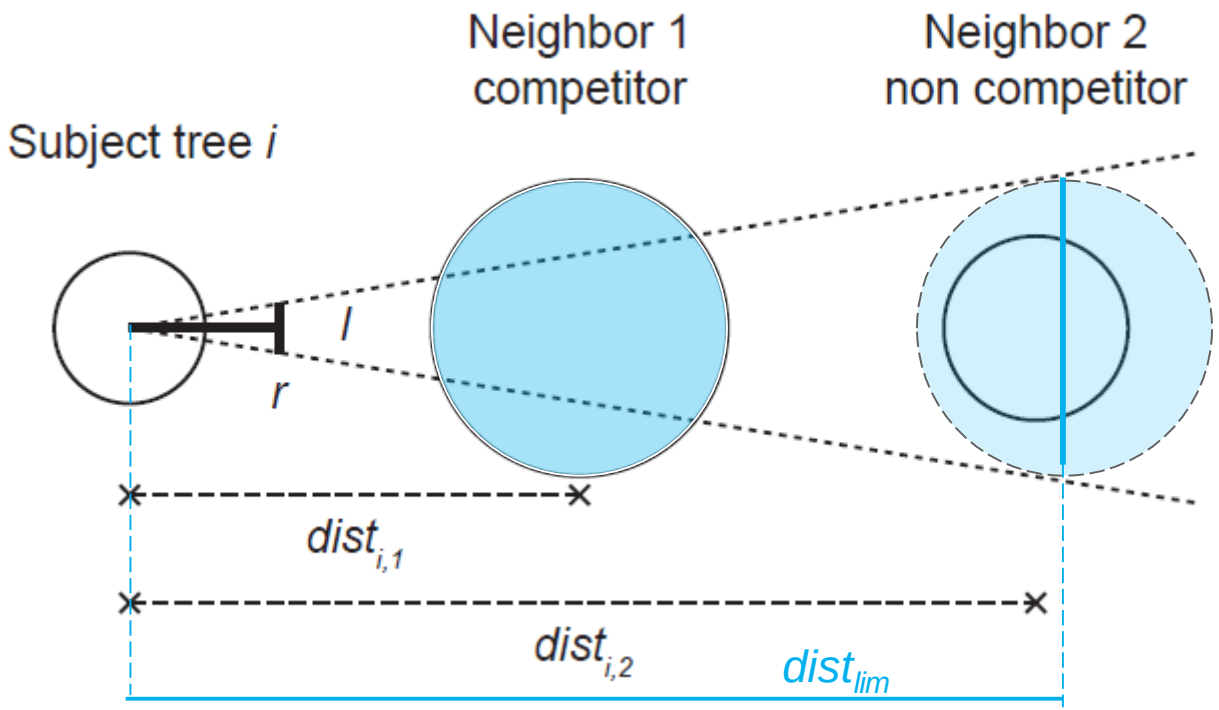
II - Distance dependent competition indices

- The computation of distance-dependent measures of point density, or competition indices, involves two main steps
  - II.1 selection of competitors
  - II.2 computation of an index that synthetize the degree to which the subject tree has to share resources with its competitors

II - Distance dependent competition indices

II.1 selection of competitors

II.1.1 Trees selected by angle count sampling (Bitterlich's method)

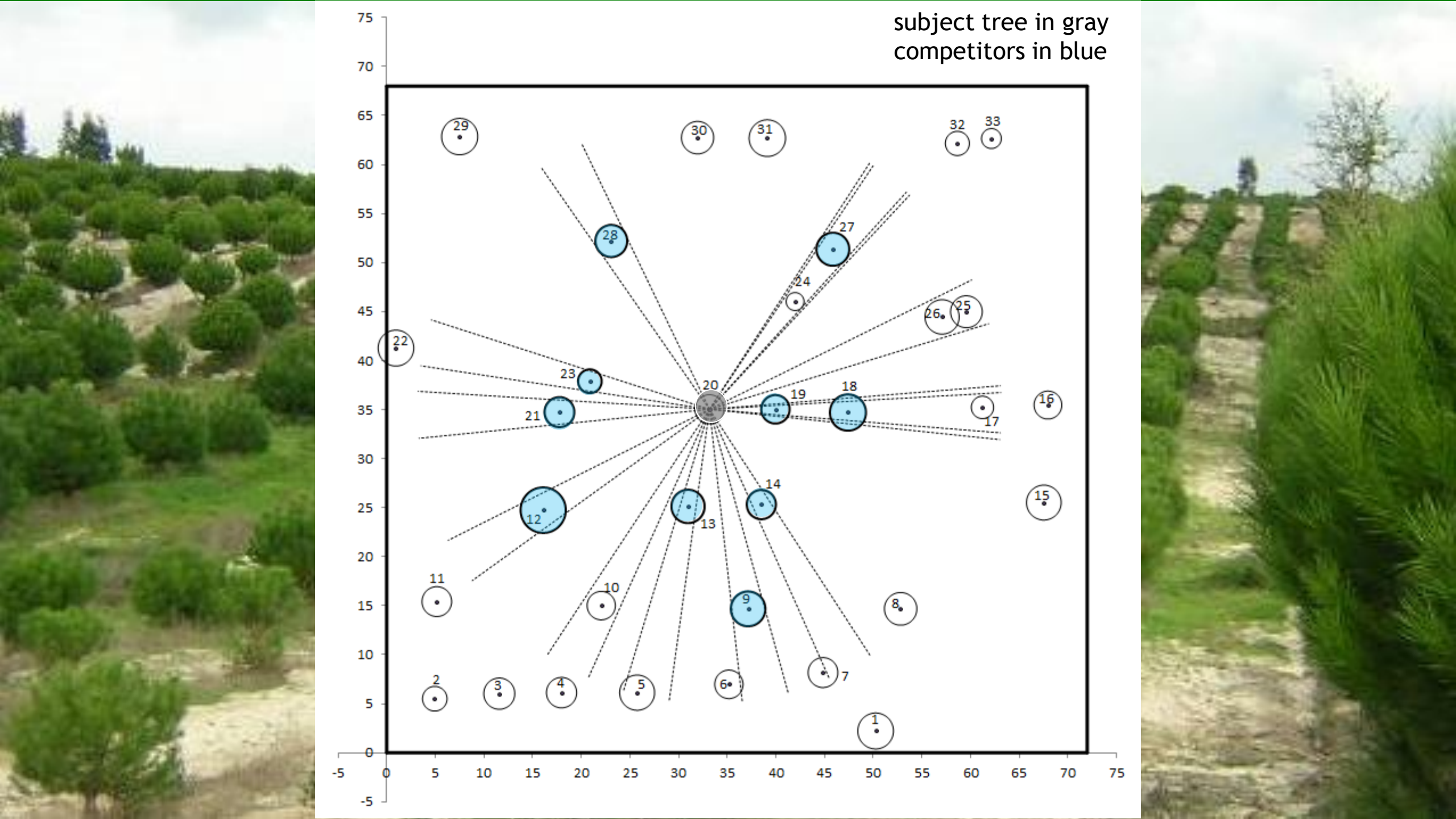


Rule to select competitors

$$dist_{ij} < dist_{lim} = k\,d_j$$

d_j = diameter of *j* tree (m)

$$2500\left(\frac{l}{r}\right)^2 = K \;\; m^2 ha^{-1}$$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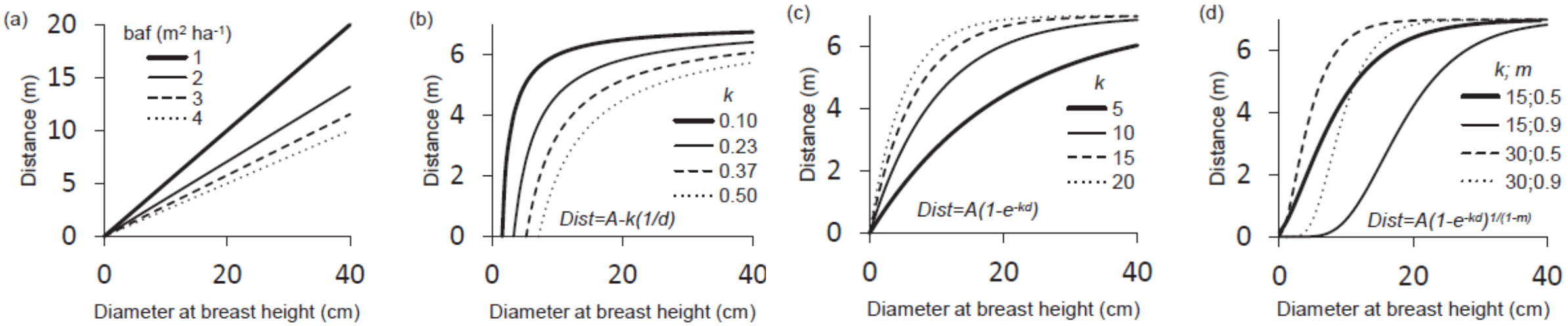
II - Distance dependent competition indices

II.1 selection of competitors

II.1.2 Other methods based on a limiting competition distance, funcion of tree size

- rules based on asymptotic functions of the tree dimension

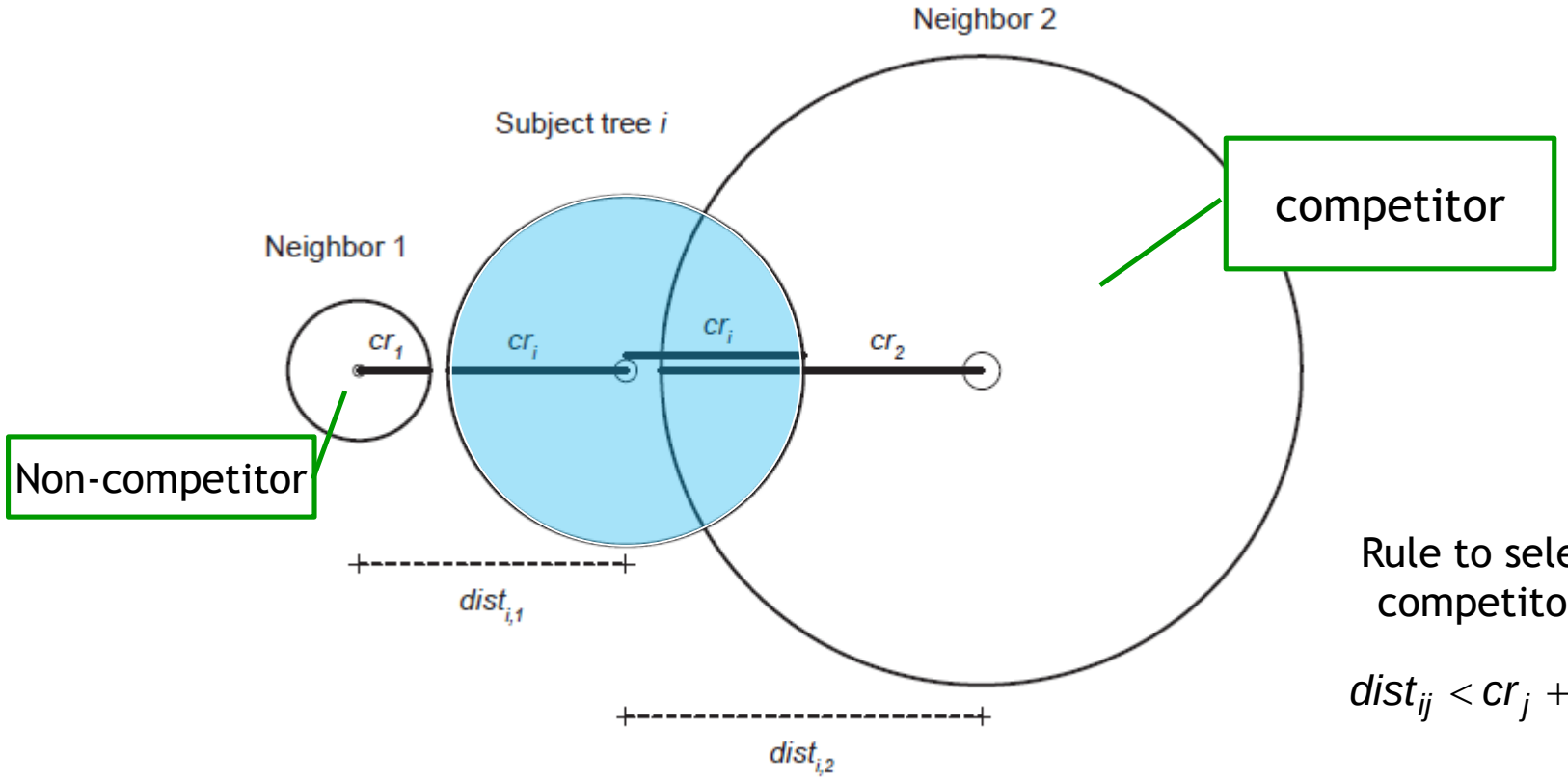
$$Dist=A(1-e^{-kd})^{1/(1-m)}$$



II - Distance dependent competition indices

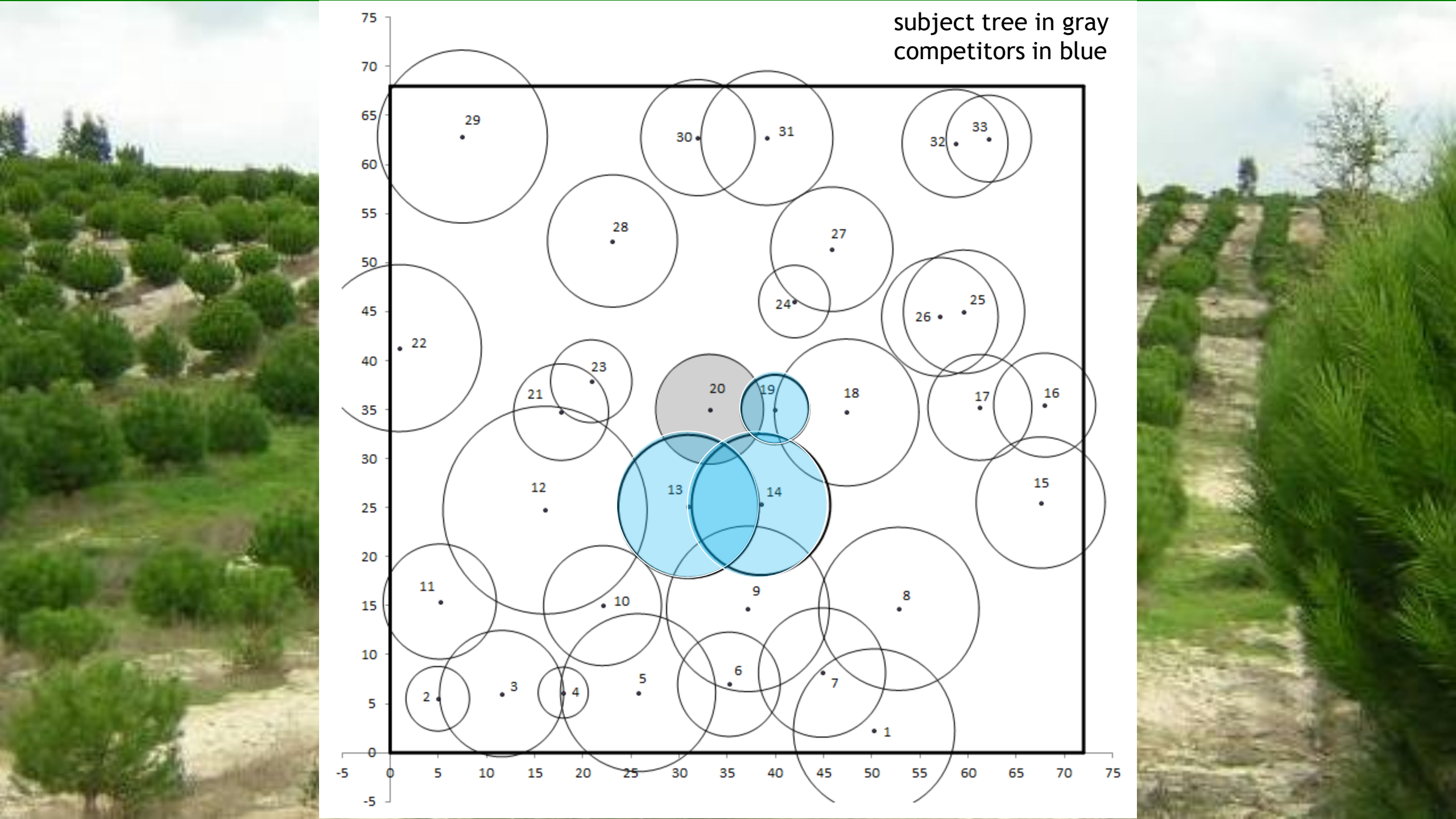
II.1 selection of competitors

II.1.3 Overlap of areas of influence



Rule to select competitors

$$dist_{ij} < cr_j + cr_i$$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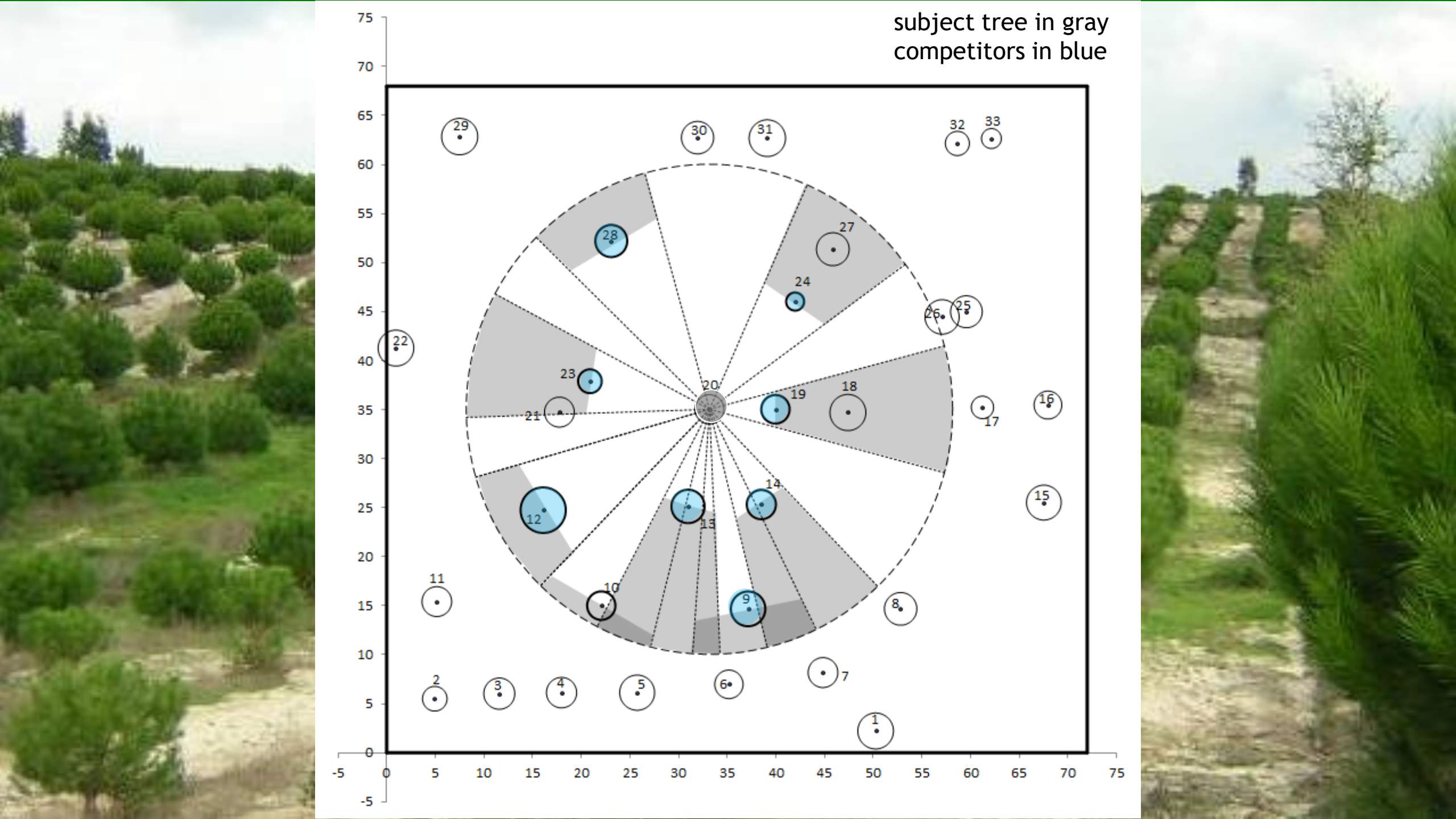
II - Distance dependent competition indices

II.1 selection of competitors

II.1.4 Competition elimination angle

- The method is based on a fixed search radius to select the neighbors of a given subject tree
- Each neighbor may be an active or a passive competitor, based on a competition elimination sector defined by a specific elimination angle fixed *a priori*
- The nearest neighbor is first selected as a competitor and all the trees located within the angle with the vertex at the subject tree and centered at the neighbor are considered as passive competitors and discarded from the selection procedure
- The nearest neighbor outside the elimination angle is then selected and the respective passive competitors identified
- The procedure ends when all the active competitors have been identified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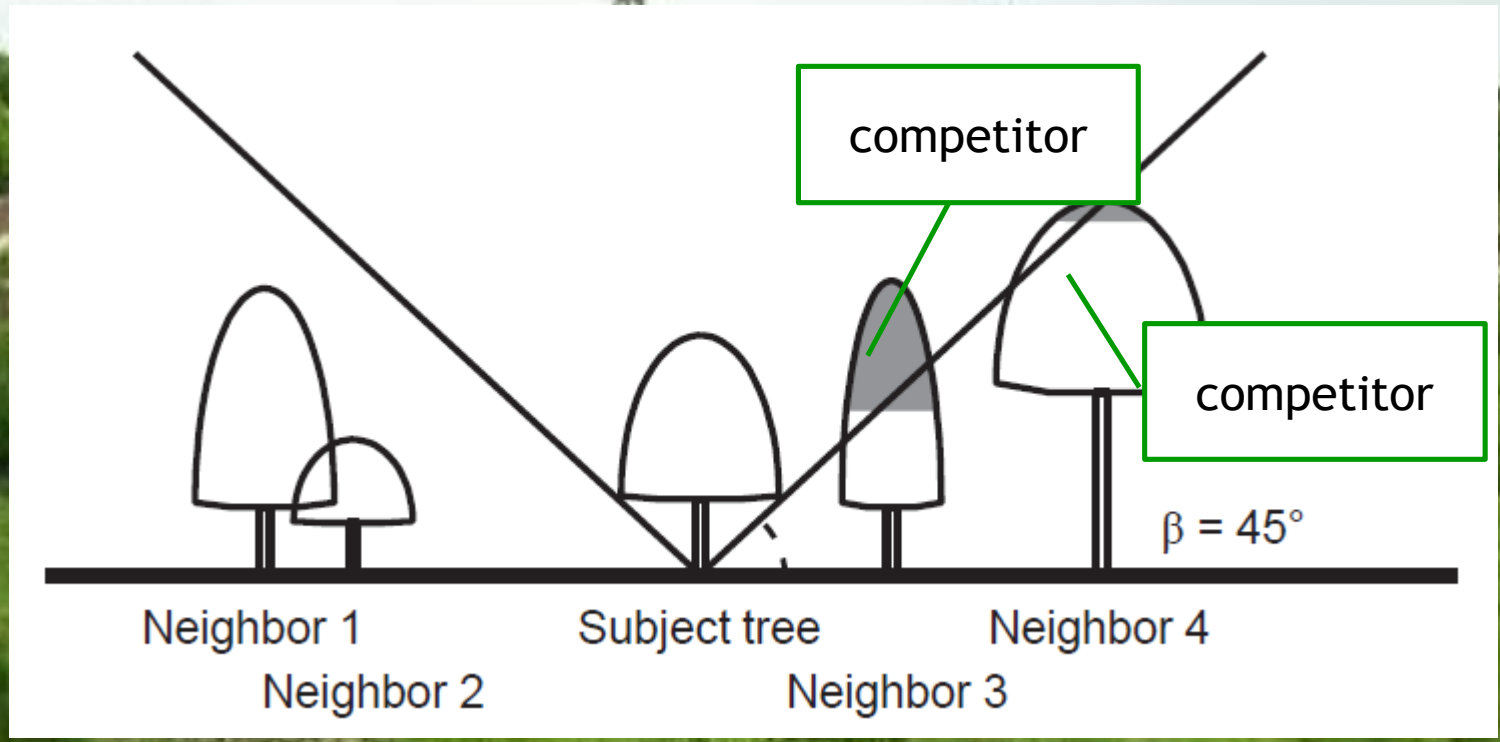
II - Distance dependent competition indices

II.1 selection of competitors

II.1.5 Selection using a vertical search cone

- An upside-down search cone is set up at a certain height of the subject tree (stem base, base of the crown or some point within the crown)
- All the trees whose crowns overlap the search cone are considered as competitors



Rule to select competitors

$$h_j > h_i + dist_{ij} \tan(90 - \beta/2)$$

II - Distance dependent competition indices

II.2 Formulation of the competition index

Formulation of the competition index

- Distance-dependent competition indices include, directly or indirectly, the size of the neighbors and their distance to the subject tree
- The competitive influence of a neighboring tree should be
  - a decreasing function of the **distance** between the neighbor and the subject tree
  - an increasing function of the neighbor's **size**

II - Distance dependent competition indices
II.2 Formulation of the competition index

- Formulation of the competition index
  - Point density indices
  - Area overlap indices
  - Indices based on the size and distance of the neighbors within a search radius
  - Indices based on horizontal or vertical angles centered at the subject tree
  - Growing space and area potentially available
  - Indices based on ecological field theory and field of neighborhood
  - Indices based on the estimation of shading or light interception

II - Distance dependent competition indices

II.2 Formulation of the competition index

- Point density indices
  - Adaptation of Bitterlich method for basal area determination using the centre of the subject tree as a sampling point
  - There are two options for this index, excluding and including the subject tree

$$CI_{PD1i} = \frac{2500}{n}\left[\sum_{j=1}^{n}(j-0.5)\left(\frac{d_j}{dist_{ij}}\right)\right]$$

$$CI_{PD2i} = \frac{2500}{n}\left[\sum_{j=1}^{n}(j+0.5)\left(\frac{d_j}{dist_{ij}}\right)\right]$$

n – number of competitors
d_j – diameter of tree j
$dist_{ij}$ – distance between trees

II - Distance dependent competition indices

II.2 Formulation of the competition index

- Area overlap indices
  - The most common definitions of area of influence **involve open-grown tree crown sizes** or **linear functions of tree diameters or crown radius**

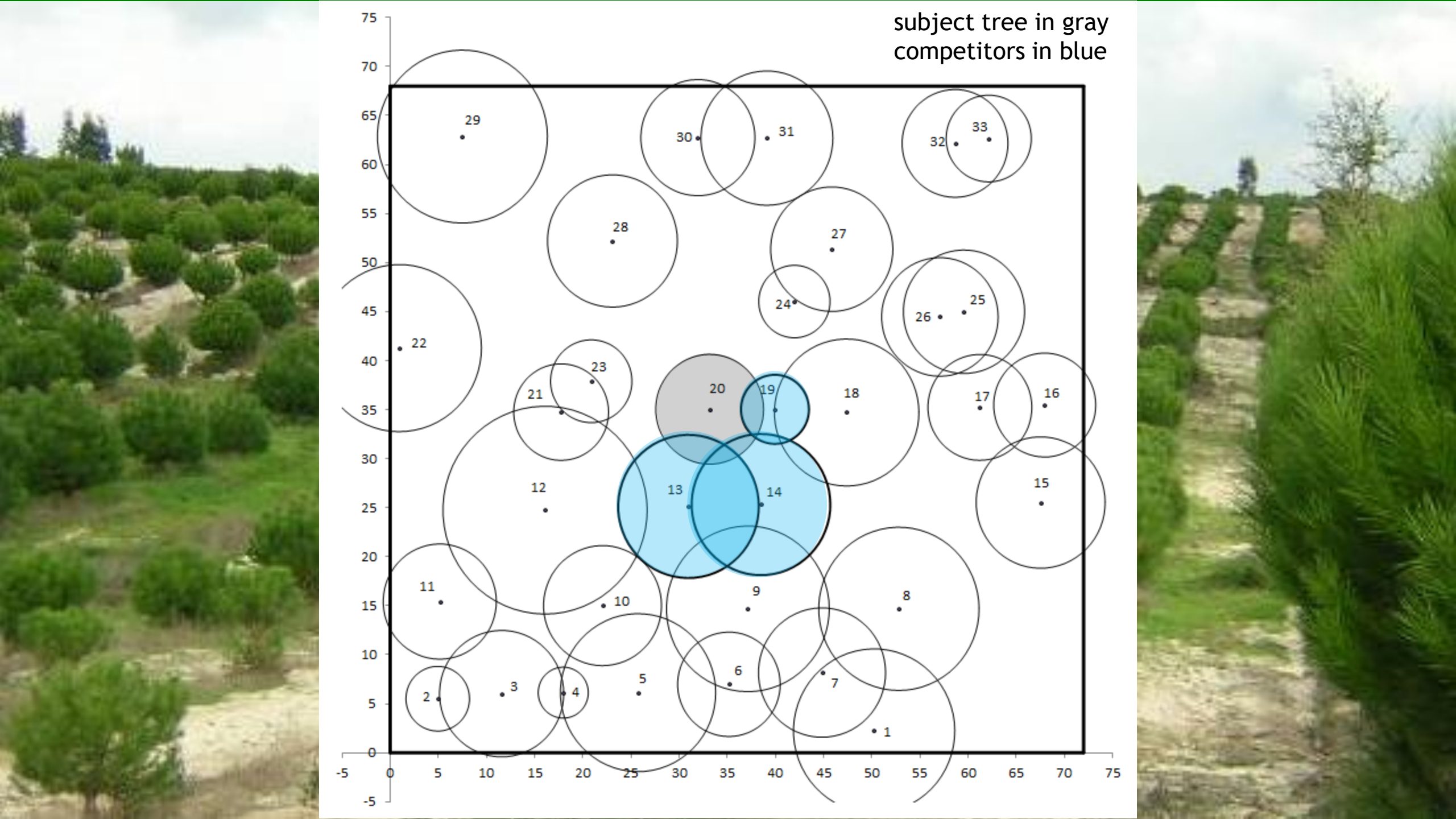
$$CI_{AO_i} = \sum_{j=1}^{n} \frac{ao_{ij}}{AI_i} \left(R_{ji}\right)^m$$

n – number of competitors

AI_i – area of influence of subject tree i *(area over which the tree obtains/competes for resources)*

ao_{ij} – area overlap between trees i and j

R_{ji} – ratio between dimensions of tree j and subject tree i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II - Distance dependent competition indices

II.2 Formulation of the competition index

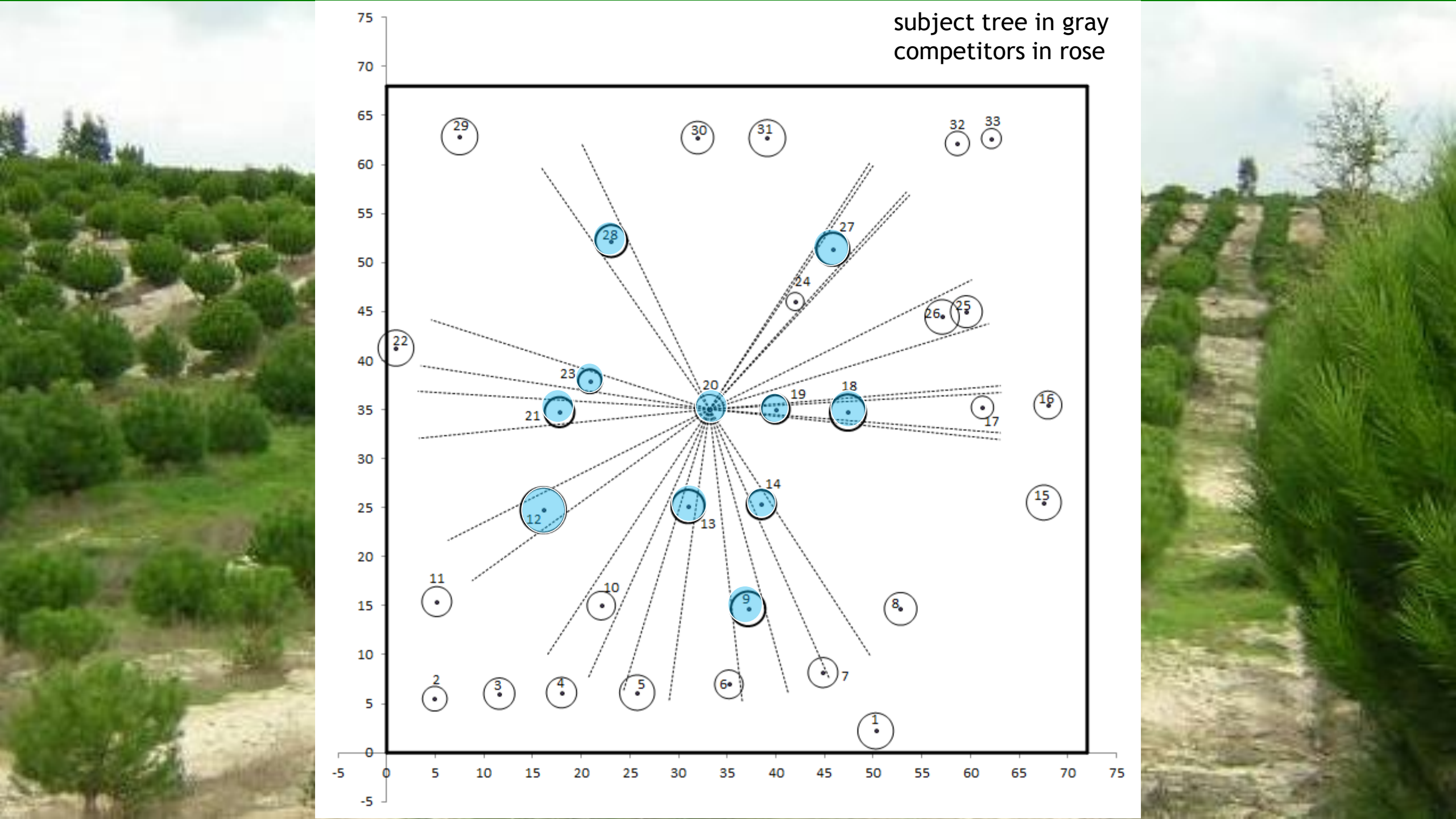
- Indices based on the size and distance of the neighbors within a search radius
  - The best-known indices of this type fall under the category of distance-weighted size ratio indices, known as **Hegyi indices**

$$CI_{DR\,i} = \sum_{j=1}^{n} R_{ji}\ f\left(dist_{ij}\right)$$

n – number of competitors

$f\ (dist_{ij})$ - *function of the* distance between trees i and j

R_{ji} – ratio between dimensions of tree j and subject tree i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II - Distance dependent competition indices

II.2 Formulation of the competition index

- Indices based on the size and distance of the neighbors within a search radius
  - Other indices do not utilize distance as a weighting factor, they use the crown volume or crown surface area of the competitor tree above the point where a vertical angle from the base of the subject tree cuts the axis of the stem of the competitor relative to the crown volume or crown surface area of the subject tree

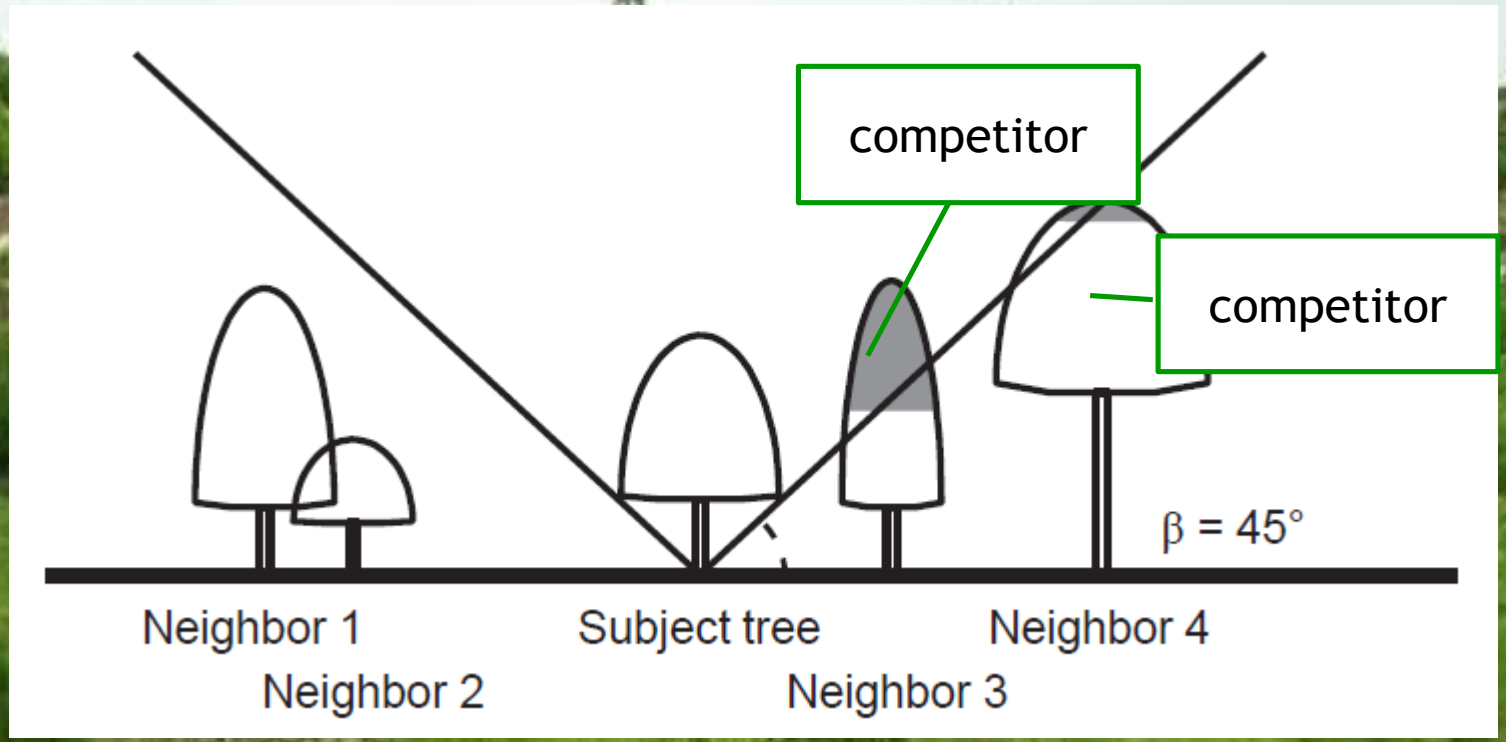
$$CI_{cvha_i} = \sum_{j=1}^{n} \frac{cvha_j}{c_{vi}}$$

n – number of competitors

$cvha_j$ - crown volume of tree j above the height at which the vertical angle cuts its stem axis

c_{vi} – crown volume of subject tree i

An identical index with crown surface can also be defined



Rule to select competitors

$$h_j > h_i + dist_{ij} \tan(90 - \beta/2)$$

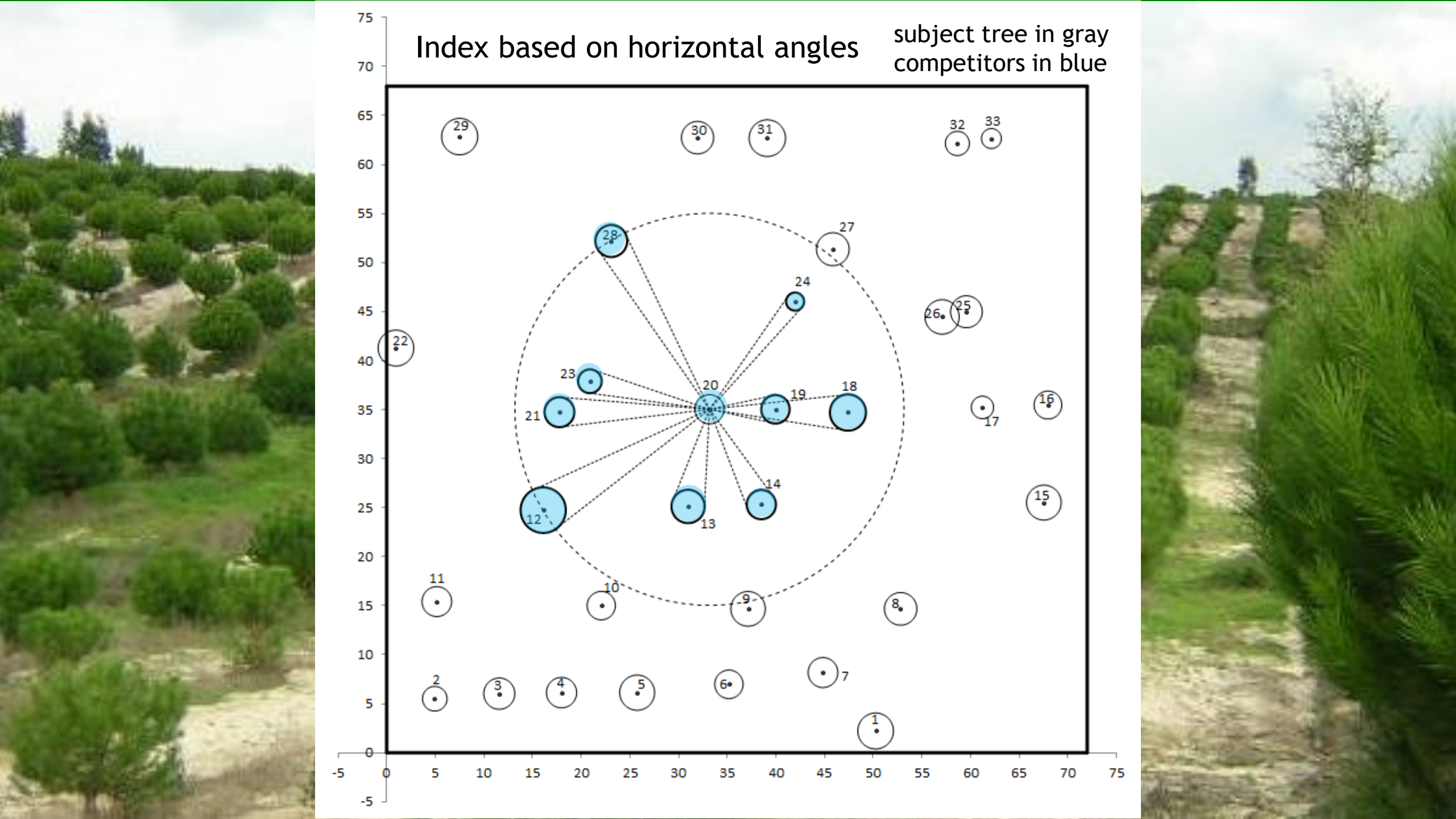
II - Distance dependent competition indices

II.2 Formulation of the competition index

- Indices based on horizontal or vertical angles centered at the subject tree
  - The indices are based on the sum of the horizontal/vertical angles from the subject tree to all the neighbors within a fixed search radius (competitors)

$$CI_{Sang} = \sum_{j=1}^{n} \sum_{k=1}^{na} \alpha_{jk}$$

n - number of competitors
α_{jk} - horizontal/vertical angle subtended by some dimension of the neighbor


Index based on horizontal angles
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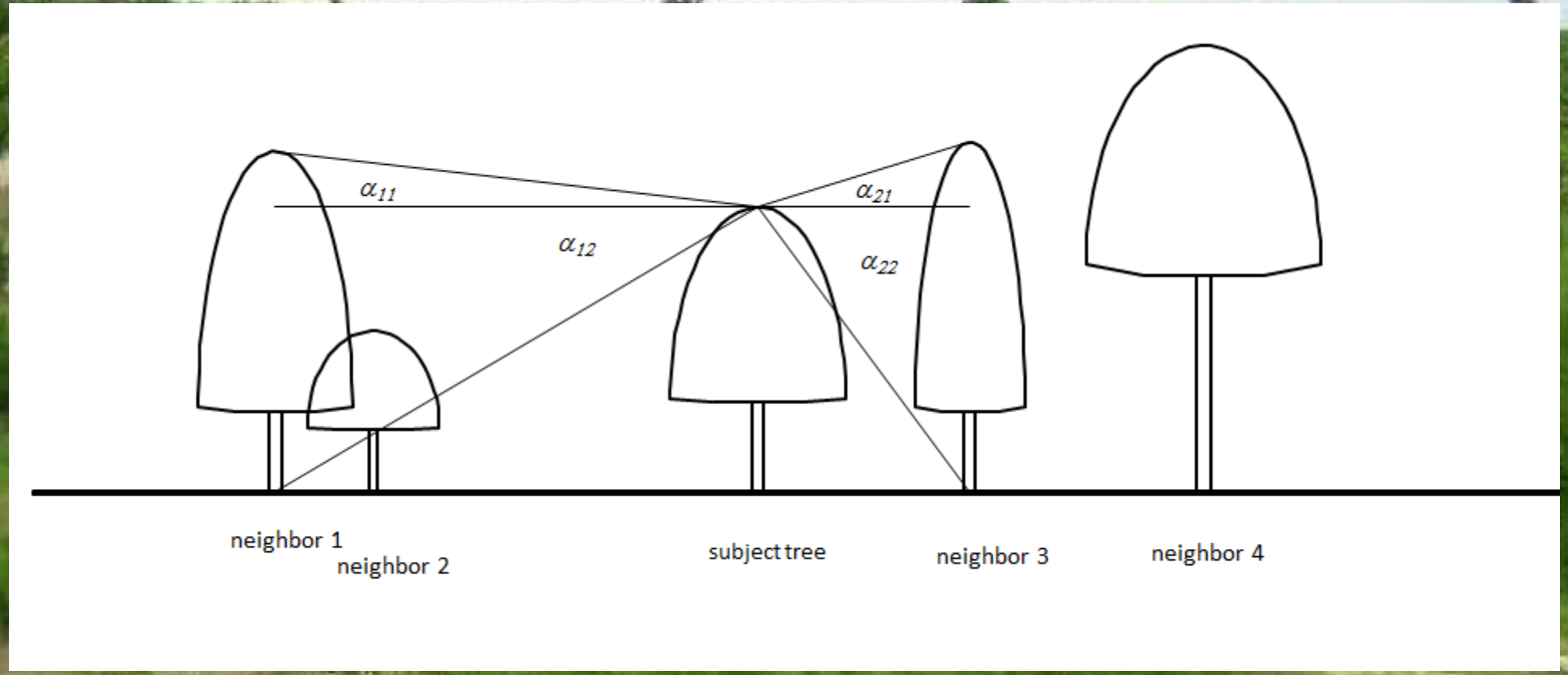
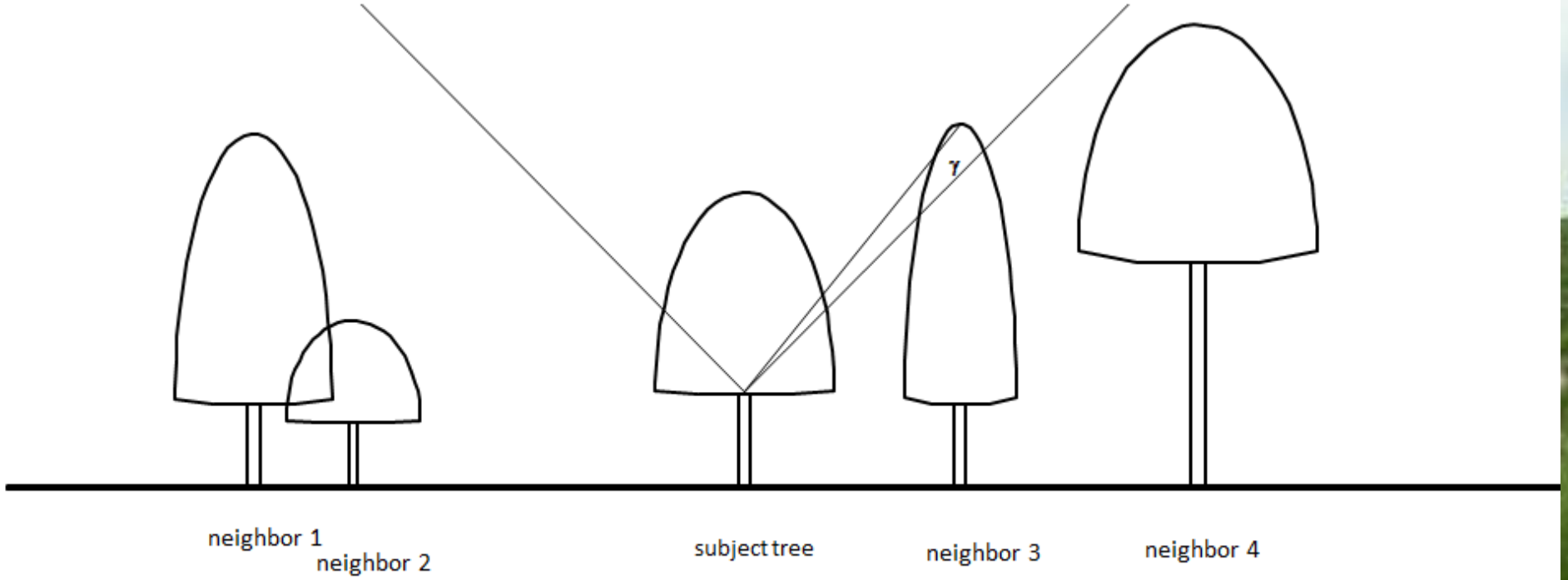
Index based on vertical angles



II - Distance dependent competition indices

II.2 Formulation of the competition index

- Indices based on horizontal or vertical angles centered at the subject tree – vertical search cone
  - The sum of the angles γ between the surface line of the search cone and the line connecting the tip of the competitor tree *j* with the cone apex on the subject tree
  - The closer and taller the competitor compared to the subject tree, the greater the angle and the competitive strength of this neighbor
  - The competition measure may also include the ratio of the competitor and subject tree crown cross-sectional areas at the height of the search cone insertion and a species-specific light transmission coefficient



$$CI_{Sang\,i} = \sum_{j=1}^{n} \gamma_j \frac{ca_{hc\,j}}{ca_{hc\,i}}\, lt_j = \sum_{j=1}^{n} \left[\arctan\left(\frac{h_j - (p/100)h_i}{dist_{ij}} \right) - \beta \right] \frac{ca_{hc\,j}}{ca_{hc\,i}}\, lt_j$$

n – number of competitors

ca_{hc} - crown cross-sectional area at height of cone insertion hc

γ_j - angle defined by the surface line of the search cone and the line between the insertion point of the cone and the top of the competitor tree

lt_j - light transmission coefficient for competitor j (dependent on species)

II - Distance dependent competition indices

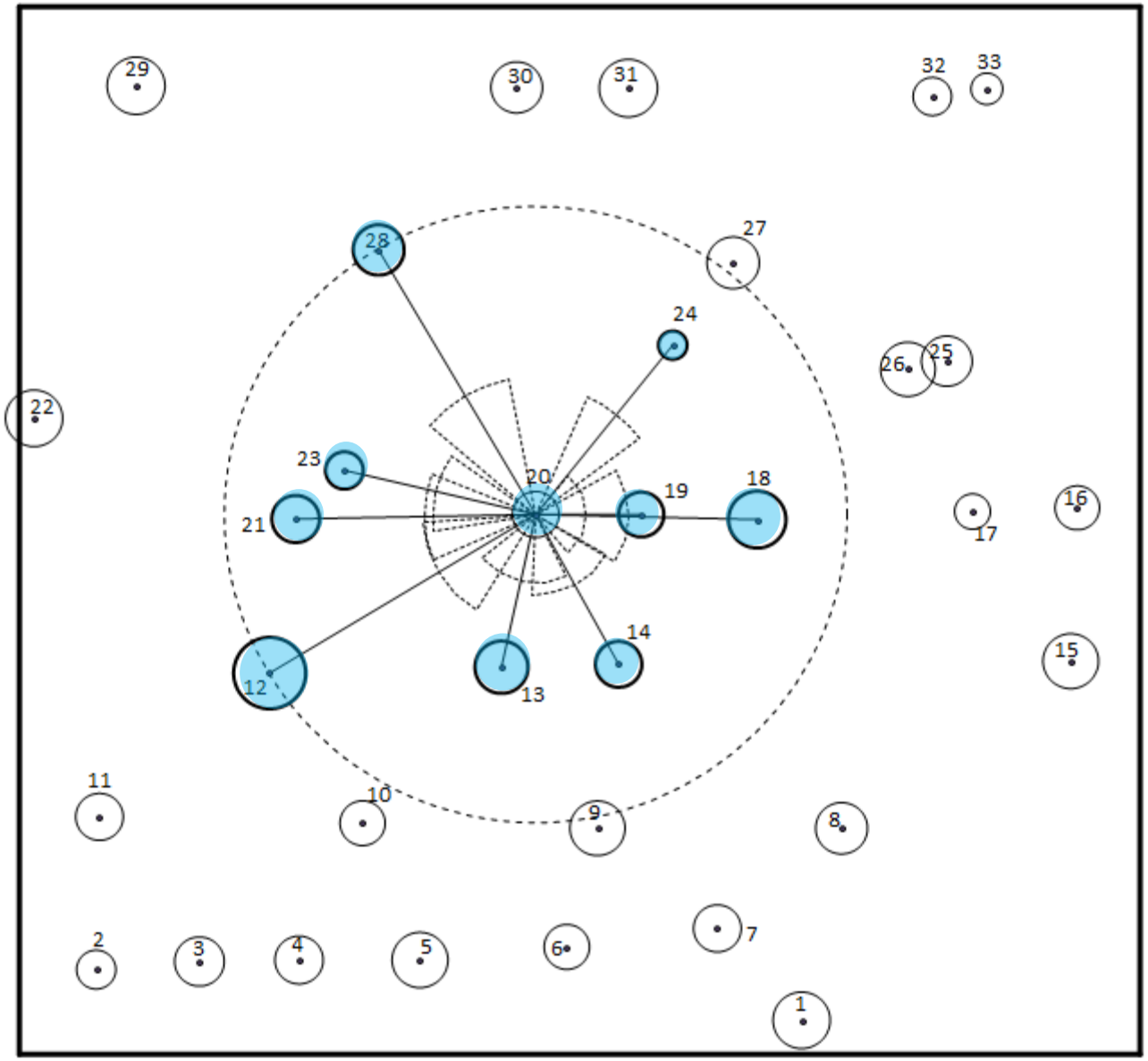
II.2 Formulation of the competition index

Growing space

- The stand surrounding a central tree *i* is divided into as many imaginary circle segments as there are competitors
- Each circle has a radius proportional to the size of the subject tree in relation to the sum of its size and that of the corresponding competitor
- The sum of the area of these *n* segments, assumed to be the area available for tree growth, is used as the competition index

$$CI_{GS} = \sum_{j=1}^{n} \underbrace{\pi \left[dist_{ij} \frac{d_i}{d_i + d_j} \right]^2}_{\text{area of the circle}} \underbrace{\frac{d_j}{dist_{ij}} \Big/ \frac{\sum_{j=1}^{n} d_j}{\sum_{j=1}^{n} dist_{ij}}}_{\text{opening width of the segment}}$$

subject tree in gray
competitors in blue

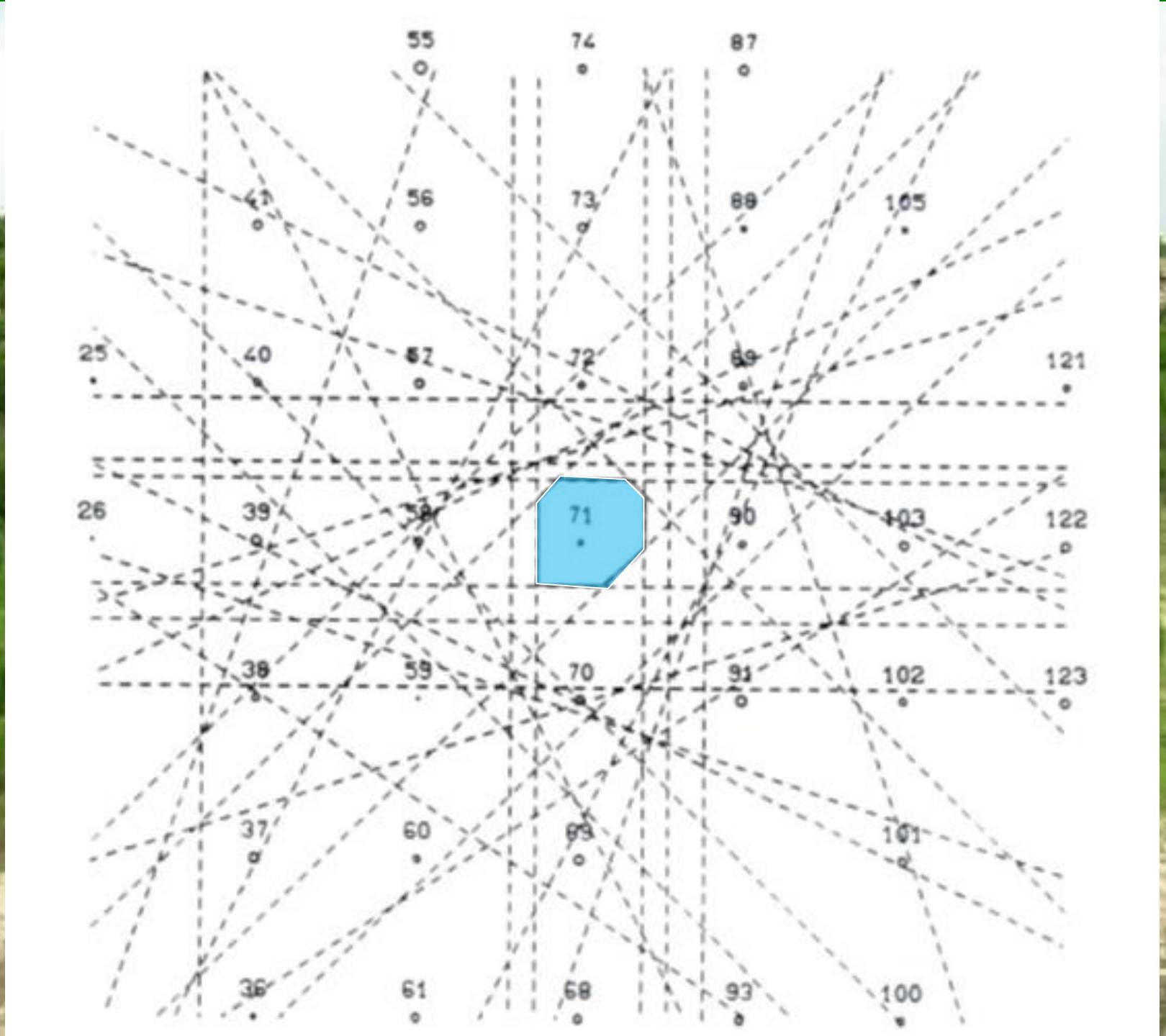


II - Distance dependent competition indices

II.2 Formulation of the competition index

Area potentially available (APA)

- The area available to each tree is calculated as the area of a polygon defined by the bisectors to the inter-tree lines
- The most used index is obtained by bisecting the inter-tree lines proportionally to the subject tree and competitor sizes
- The distance to a polygon side maybe restricted by a function of the radius of an open-grown tree of the same diameter as the subject tree to prevent polygon areas from becoming excessively large.
- This variant of the APA index may be useful in stands with irregular spatial patterns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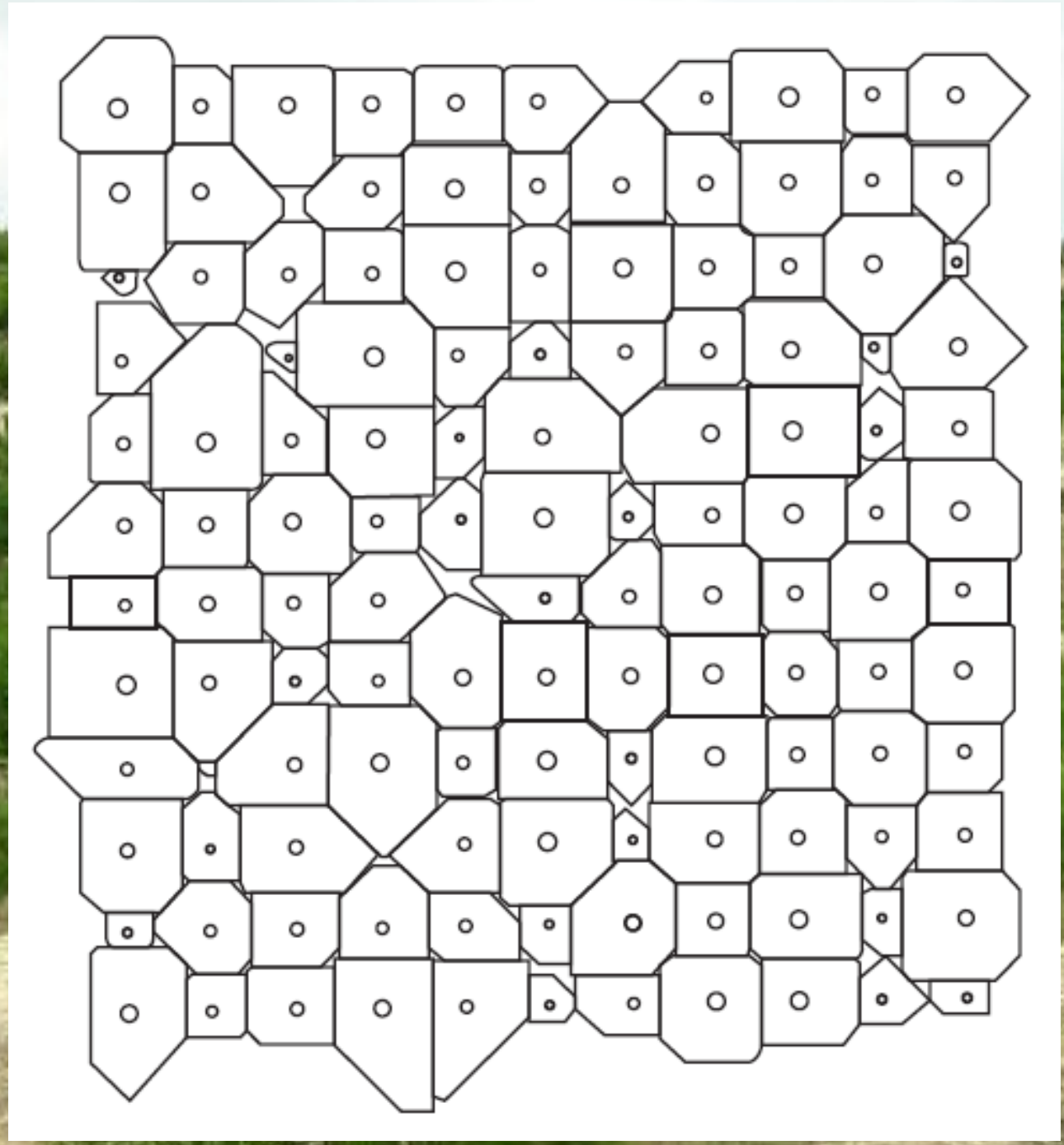

II - Distance dependent competition indices

II.2 Formulation of the competition index

- Indices based on the estimation of shading or light interception
  - Indices that estimate light interception or the degree of shading from neighbors seem useful to model competition for light
  - The simulation of light interception within a forest stand has been a topic of research in ecology with the objective of better understanding stand production and dynamics
  - Some recent studies have used simplified models of light interception as a way to include inter-tree competition in modeling tree growth

Asymmetric/one-sided versions of the competition indices

- Competition processes have been defined according to two basic models: symmetric/asymmetric and one-sided/two-sided competition
- In **two-sided competition**, resources are shared by all the trees while in **one-sided competition** larger trees are not affected by smaller neighbors.
- Resources may be shared:
  - equally,
  - proportionally to size or
  - using some intermediate sharing rule
- When there is perfect sharing relative to size, competition is **symmetric**

Asymmetric/one-sided versions of the competition indices

Two-sided competition
(resources are shared by all trees)

Equally shared
(**Symmetric competition** – eg: competition for water and nutrients)

Shared proportionally to size
(**Asymmetric competition** – eg: competition for light)

One-sided competition
(The extreme situation of **asymmetric competition** – bigger trees are not affected by smaller ones)

Asymmetric/one-sided versions of the competition indices

- Depending on the respective formulation, competition indices implicitly assume an asymmetric or symmetric partitioning of resources among neighboring trees
  - All the **distance-independent competition** indices that are based on the trees larger than the subject tree implicitly assume asymmetric competition
  - In **distance-dependent indices** the distinction between these two models relate also to the selection of competitors

Asymmetric/one-sided versions of the competition indices

- The formulations that reflect asymmetric competition restrict the competitors to those neighbors that are larger than the subject tree
- The competitive status between each tree and its neighbors may be established taking into account their relative dimensions, neighbors larger than the subject tree place it at a competitive disadvantage, whereas those smaller put it at a competitive advantage
- This leads to competition indices that are sums of positive and negative values. Dominant neighbors make a positive contribution to the index while suppressed neighbors subtract from the index

For more details read chapter 9

MODELLING FOREST TREES AND STANDS (BURKHART AND TOMÉ, 2012)

Asymmetric/one-sided versions of the competition indices - Hegyi type indices

$$FDD_{rdi} = \sum_{j=1}^{n_1} f(d_j, d_i)\, f(D_{ij}) \qquad \text{dominant neighbors}$$

$$-\sum_{j=1}^{n_2} f(d_i, d_j)\, f(D_{ij}) \qquad \text{suppressed neighbors}$$

$$-\sum_{j=1}^{m} f(d_i, d_{j0})\, f(D_{ij}) \qquad \text{dead neighbors}$$

Asymmetric/one-sided versions of the competition indices – Area overlap indices

$$SA_{rdi} = \sum_{j=1}^{n_1} \left(\frac{as_{ji}}{AI_i} \right) (R_{ji})^k$$ dominant neighbors ;

$$-\sum_{j=1}^{n_2} \left(\frac{as_{ij}}{AI_j} \right) (R_{ij})^k$$ suppressed neighbors ;

$$-\sum_{j=1}^{m} \left(\frac{as_{ij0}}{AI_{j0}} \right) (R_{ij0})^k$$ dead neighbors