Applied Operations Research

Solving applications of Integer Linear Programming with

Excel

- 2018/2019 -

1. **Hiring rangers**

   The forest service of a country needs to set up sites for district rangers. The forest is made up of a number of districts, as illustrated in the following figure (Figure 1).

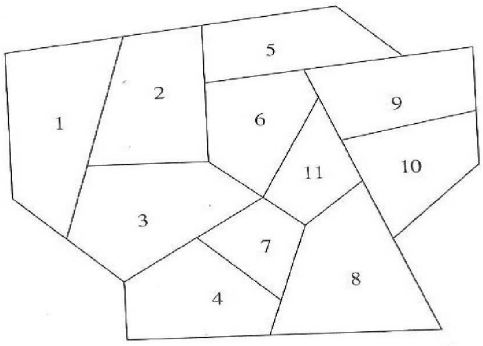

Figure 1: Forest.

   A district ranger can be placed in any district and is able to handle the job of protecting the forest resources for future generations and to protect visitors for both its district and any adjacent districts. Consider that two districts are adjacent if they share one point at least. The objective is to minimize the number of district rangers hired.

   (a) Indicate the districts that a ranger can protect.

   (b) Formulate and solve the problem.

2. **Sheet cutting planning**

   A pulp mill cuts sheets of 48 cm $\times$ 96 cm paper into smaller sheets. This company received an order with the characteristics indicated in Table 1. Table 2 and Figure 2 indicate the possible cutting patterns on a 48 cm $\times$ 96 cm sheet. The goal is to determine the cutting plan in order to minimize the number of 48 cm $\times$ 96 cm sheets used. Formulate this problem as an IP model and solve the model.

Type	Sheet of paper Dimensions cm × cm	Number
1	36 × 50	800
2	24 × 36	1300
3	20 × 60	500
4	18 × 30	1500

Table 1: Characteristics of the order.

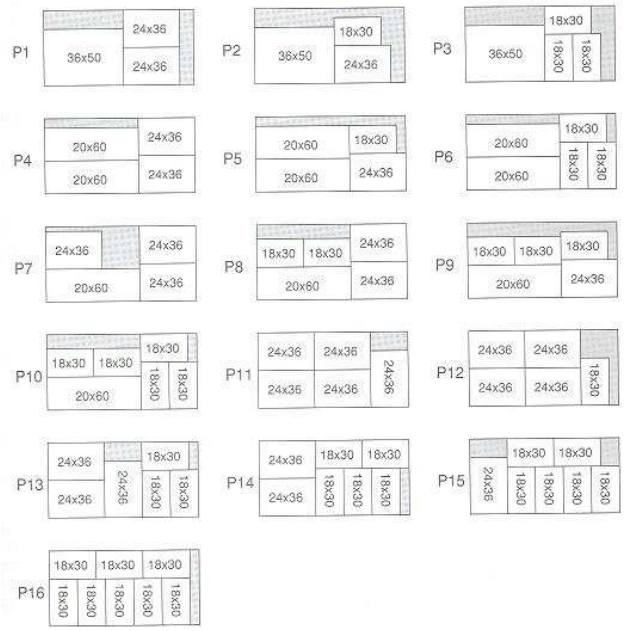

Figure 2: Possible cutting patterns on a 48 cm × 96 cm sheet.

3. **Project selection**

Consider the following integer LP problem to select a combination of projects from projects 1 through 6:

Sheet of paper	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	1	0	2	1	0	3	2	1	0	5	4	3	2	1	0
3	0	0	0	2	2	2	1	1	1	1	0	0	0	0	0	0
4	0	1	3	0	1	3	0	2	3	5	0	1	3	5	6	8

Table 2: Possible cutting patterns on a 48 cm $\times$ 96 cm sheet.

$$\max\ 4x_1 + 8x_2 + 6x_3 + 3x_4 + 4x_5 + 7x_6 \tag{1}$$

s.t.

$$500x_1 + 700x_2 + 550x_3 + 400x_4 + 450x_5 + 750x_6 \leq 2200 \tag{2}$$

$$10x_1 + 7x_2 + 9x_3 + 9x_4 + 8x_5 + 5x_6 \leq 35 \tag{3}$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \in \{0, 1\}. \tag{4}$$

(a) What is the optimal solution for this problem?

(b) Formulate constraints and find the optimal solutions for the following conditions:

i. Exactly two projects out of projects 2, 3, 4 and 5 must be selected.

ii. Project 1 may be selected if and only if project 6 is selected.

iii. If project 2 is selected, projects 4 and 5 must both be selected.

iv. If project 1 and 2 are both selected, 6 must be selected.