Applied Operations Research

Solving applications of Integer Linear Programming with Excel (One resolution)

- 2018/2019-

1. **Hiring rangers**

(a) **Indicate the districts that a ranger can protect.**

Districts	Districts where the rangers can be placed										
	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	0	0	0	0	0	0	0	0
2	1	1	1	0	1	1	0	0	0	0	0
3	1	1	1	1	0	1	1	0	0	0	1
4	0	0	1	1	0	0	1	1	0	0	0
5	0	1	0	0	1	1	0	0	1	0	0
6	0	1	1	0	1	1	1	0	1	0	1
7	0	0	1	1	0	1	1	1	0	0	1
8	0	0	0	1	0	0	1	1	0	1	1
9	0	0	0	0	1	1	0	0	1	1	1
10	0	0	0	0	0	0	0	1	1	1	1
11	0	0	1	0	0	1	1	1	1	1	1

Table 1: Districts that a ranger can protect.

(b) **Formulate and solve the problem.**

The decision variables are as follows:

$$x_j = \begin{cases} 1 & \text{if a ranger is placed in district } j \\ 0 & \text{otherwise.} \end{cases}$$

The model is the following:

$$\min Z = \sum_{j=1}^{11} x_j \tag{1}$$

subject to

$$
\begin{aligned}
x_1 + x_2 + x_3 &\geq 1\\
x_1 + x_2 + x_3 + x_5 + x_6 &\geq 1\\
x_1 + x_2 + x_3 + x_4 + x_6 + x_7 + x_{11} &\geq 1\\
x_3 + x_4 + x_7 + x_8 &\geq 1\\
x_2 + x_5 + x_6 + x_9 &\geq 1\\
x_2 + x_3 + x_5 + x_6 + x_7 + x_9 + x_{11} &\geq 1\\
x_3 + x_4 + x_6 + x_7 + x_8 + x_{11} &\geq 1\\
x_4 + x_7 + x_8 + x_{10} + x_{11} &\geq 1\\
x_5 + x_6 + x_9 + x_{10} + x_{11} &\geq 1\\
x_8 + x_9 + x_{10} + x_{11} &\geq 1\\
x_3 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} &\geq 1\\
x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11} &\in \{0, 1\}.
\end{aligned}
$$

Expression (1) minimizes the number of rangers hired. All constraints bebore the last ensure that each district is protected. The last constraints state the nature of the variables.

The optimal solution is obtained with three rangers, placed in districts 2, 3 and 11.

2. **Sheet cutting planning**

   Formulate this problem as an IP model and solve the model.

Sheet of paper	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	1	0	2	1	0	3	2	1	0	5	4	3	2	1	0
3	0	0	0	2	2	2	1	1	1	1	0	0	0	0	0	0
4	0	1	3	0	1	3	0	2	3	5	0	1	3	5	6	8

Table 2: Possible cutting patterns on a 48 cm × 96 cm sheet.

The decision variables are as follows:

x_j- number of 48 cm $\times$ 96 cm sheets assigned to cutting pattern $P_j,\ j = 1, ..., 16$.

$$\min\ Z = \sum_{i=1}^{16} x_i$$

$$\begin{array}{llllllllllll}
\textit{subject to} \\
x_1+ & x_2+x_3 & & & & & & & & & & = 800 \\
2x_1+ & x_2+ & 2x_4+x_5+ & & 3x_7+ & 2x_8+x_9+ & & 5x_{11}+ & 4x_{12}+3x_{13}+ & 2x_{14}+x_{15} & & = 1300 \\
 & & 2x_4+2x_5+ & 2x_6+x_7 & & x_8+x_9+ & x_{10} & & & & & = 500 \\
 & x_2+3x_3+ & x_5+ & 3x_6+ & & 2x_8+3x_9+ & 5x_{10}+ & & x_{12}+3x_{13}+ & 5x_{14}+6x_{15}+ & 8x_{16} & = 1500 \\
x_j & & & & & & & & & & & \in \mathbb{N}_0,\ i = 1, ..., 16.
\end{array}$$

The first expression minimizes the number of 48 cm × 96 cm sheets used. All constraints before the last ensure that the characteristics of the order are satisfied. The last constraints state the integer requirements on the variables.

The optimal number of 48 cm × 96 cm sheets used is 1088. The optimal values of the variables are displayed in Table 3.

Number of 48 cm × 96 cm sheets															
P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}
642	4	154	1	0	249	0	0	0	0	1	0	0	1	3	33

Table 3: Optimal solution.

3. **Project selection**

(a) **What is the optimal solution for this problem?**

$x_1 = x_2 = x_3 = x_5 = 1$ and the remaining variables are equal to zero, with the objective function value of 22.

(b) **Formulate constraints and find the optimal solutions for the following conditions:**

i. **Exactly two projects out of projects 2, 3, 4 and 5 must be selected.**

$$x_2 + x_3 + x_4 + x_5 = 2.$$

The optimal solution is $x_2 = x_3 = 1$ and $x_1 = x_4 = x_5 = x_6 = 0$, with the objective function value of 21.

ii. **Project 1 may be selected if and only if project 6 is selected.**

$$x_1 = x_6.$$

$x_2 = x_3 = x_4 = x_5 = 1$ and $x_1 = x_6 = 0$, with the objective function value of 21.

iii. **If project 2 is selected, projects 4 and 5 must both be selected.**

$$x_2 \leq x_4$$

$$x_2 \leq x_5$$

$x_2 = x_3 = x_4 = x_5 = 1$ and $x_1 = x_6 = 0$, with the objective function value of 21.

iv. **If project 1 and 2 are both selected, 6 must be selected.**

$$x_1 + x_2 - 1 \leq x_6$$

$x_2 = x_3 = x_4 = x_5 = 1$ and $x_1 = x_6 = 0$, with the objective function value of 21.