INSTITUTO SUPERIOR DE AGRONOMIA

Applied Operations Research - *Linear Programming* **- 2018/19**

Exam 1st Call

Consider the linear programming problem P written in the standard form

$$\max Z = 13x_1 + 39x_2$$
$$\begin{cases} x_1 + 3x_2 + x_3 = 18 \\ x_1 + x_2 + x_4 = 10 \\ 3x_1 + x_2 + x_5 = 24 \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{cases}$$

where x_3, x_4 and x_5 are the slack variables.

1. Write P considering only variables x_1 and x_2.

2. Define basic feasible solution to this problem. Give an example of one basic feasible solution with $x_3 = 0$ and illustrate the previous definition by this example.

3. Consider that the Simplex algorithm is applied to P.

a) Suppose that the algorithm starts at the basic feasible solution where x_3, x_4 and x_5 are the basic variables. Let $\begin{cases} x_3 = 18 - x_1 - 3x_2 \\ x_4 = 10 - x_1 - x_2 \\ x_5 = 24 - 3x_1 - x_2 \end{cases}$ be the system of constraints in the standard form expressed in terms of the non-basic variables.

i) Indicate the vertex where the Simplex algorithm starts.

ii) Find which vertex the algorithm goes next and indicate the corresponding objective function value.

b) Suppose that the algorithm goes to the basic feasible solution where x_2, x_4 and x_5 are the basic variables. Let $\begin{cases} x_2 = 6 - \frac{1}{3}x_1 - \frac{1}{3}x_3 \\ x_4 = 4 - \frac{2}{3}x_1 + \frac{1}{3}x_3 \\ x_5 = 18 - \frac{8}{3}x_1 + \frac{1}{3}x_3 \end{cases}$ be the system of constraints in the standard form and $Z = 234 - 39x_3$ the objective function, both expressed in terms of the non-basic variables.

i) Indicate the current vertex.

ii) Is this vertex optimal? Justify the answer.