INSTITUTO SUPERIOR DE AGRONOMIA

Applied Operations Research - *Integer Programming and Network Models* - **2018/19**

Exam 1st Call

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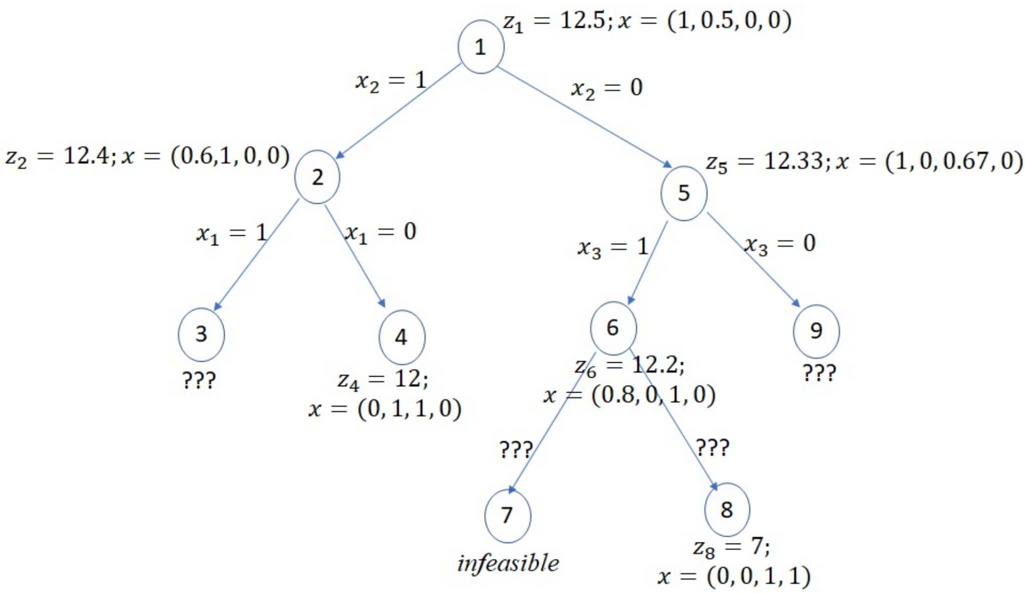
1. Consider the following integer linear programming problem, further denoted by (IP):

$$\max z = 9x_1 + 7x_2 + 5x_3 + 2x_4$$
$$\begin{cases} 5x_1 + 4x_2 + 3x_3 + 2x_4 \leq 7 \\ x_1, \; x_2, \; x_3, \; x_4 \in \{0, 1\} \end{cases}$$

The Figure below shows the branch-and-bound for solving (IP). Information concerning the linear programming relaxation solution of each subproblem is displayed near the corresponding node, where z_i is subproblem i optimal solution value.



a) Display an optimal solution for (IP).

b) Complete Node 3. Justify your answer.

c) Nodes 7 and 8 are obtained by adding which constraint to (IP)? In each case, specify the constraint.

d) Determine a solution to node 9.

e) At the end of node 4, can you conclude that the resulting solution $x_2 = x_3 = 1; x_1 = x_4 = 0$ is optimal for (IP)? Justify your answer.

2. A company wishes to assign three customers, C_1, C_2 and C_3, to two warehouses, W_1 and W_2. The assignment costs, the warehouse capacities and the customer demands are given in the following table.

	W_1	W_2	Demand
C_1	2	8	18
C_2	5	3	15
C_3	7	3	14
Capacity	30	20	

The following integer programming model translates the problem that the company would like to solve.

$$\begin{aligned}
\min \quad & Z = 2x_{11} + 8x_{12} + 5x_{21} + 3x_{22} + 7x_{31} + 3x_{32} & & (1)\\
\text{s.t.} \quad & x_{11} + x_{12} = 1 & & (2)\\
& x_{21} + x_{22} = 1 & & (3)\\
& x_{31} + x_{32} = 1 & & (4)\\
& 18x_{11} + 15x_{21} + 14x_{31} \leq 30 & & (5)\\
& 18x_{12} + 15x_{22} + 14x_{32} \leq 20 & & (6)\\
& x_{11},\ x_{12},\ x_{21},\ x_{22},\ x_{31},\ x_{32} \in \{0,1\} & & (7)
\end{aligned}$$

a) What can be the meaning of decision variables x_{ij} $(i = 1, ..., 3,\ j = 1, 2)$, objective function (1) and constraints (2) to (6)?

b) Find a feasible solution for the problem and give the corresponding assignment cost.