INSTITUTO SUPERIOR DE AGRONOMIA

Applied Operations Research - *Linear Programming* - 2018/19

Exam 2$^{\text{nd}}$ Call

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1. (20 val.) Consider the following LP problem:

$$\min Z = x_1 + x_2$$
$$\begin{cases} -x_1 & + & 3x_2 & \leq & 3 & (1) \\ 2x_1 & + & 3x_2 & \geq & 12 & (2) \\ x_1 & & & \leq & 5 & (3) \\ x_1, & & x_2 & \geq & 0 & (4) \end{cases}$$

a) Graphically display the feasible region.

b) Determine an optimal solution and the corresponding optimal value. Explain the process you follow to obtain such a solution.

c) Rewrite constraint (3) as $x_1 \leq \alpha$, $\alpha \in \mathbb{R}$.

   i) Determine the values of α for which the above LP problem has feasible solutions.

   ii) For the α values determined in i), does the solution calculated in b) remains optimal? Justify your answer.

d) Write the problem in the standard form.

e) Define basic feasible solution to this problem. Indicate all the basic feasible solutions of the problem and illustrate the previous definition by one of these solutions.

f) Replace the objective function by min $Z = 2x_1 + 3x_2$, and let s_1, s_2 and s_3 be the slack variables of the problem in the standard form with respect to constraints (1), (2) and (3), respectively. The Simplex algorithm is applied to the problem. It starts from a basic feasible solution with non-basic variables s_2 and s_3.

   Let $\begin{cases} x_1 & = & 5 - s_3 \\ x_2 & = & \frac{2}{3} + \frac{1}{3}s_2 + \frac{2}{3}s_3 \\ s_1 & = & 6 - s_2 - 3s_3 \end{cases}$ be the system of constraints in the standard form and $Z = 12 + s_2$ the objective function, both expressed in terms of the non-basic variables.

   i) Indicate the basic feasible solution where the Simplex algorithm starts.

   ii) From the starting basic feasible solution, how much s_2 can increase, without changing the value of s_3, so that the algorithm goes to another basic feasible solution? How much the objective function value changes with this movement?

   iii) From the starting basic feasible solution, how much s_3 can increase, without changing the value of s_2, so that the algorithm goes to another basic feasible solution? Does the objective function value change with this movement?