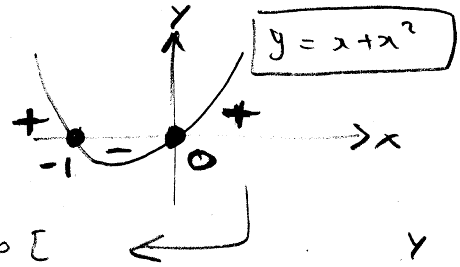
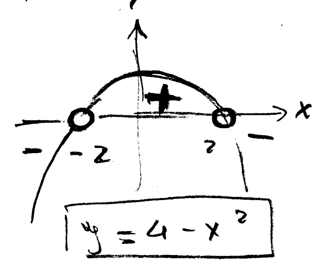
A

① b) $D_f = \{x : x + x^2 \geq 0 \wedge 4 - x^2 > 0 \wedge \ln(4 - x^2) \neq 0\}$



- $\boxed{x + x^2 \geq 0} \Leftrightarrow x(x+1) \geq 0$

$\left(\begin{array}{l} x(x+1) = 0 \\ \Leftrightarrow x = 0 \vee x = -1 \end{array} \right) \Leftrightarrow x \in]-\infty, -1] \cup [0, +\infty[$



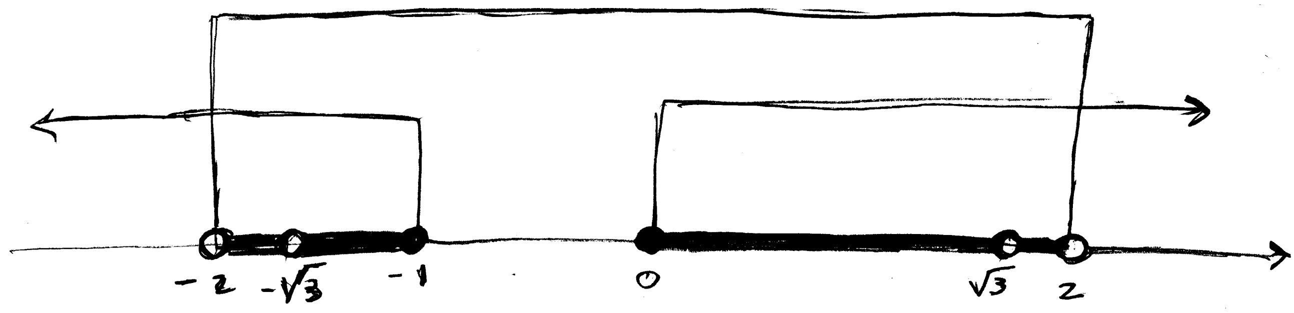
- $\boxed{4 - x^2 > 0} \Leftrightarrow x \in]-2, 2[$

$(x^2 = 4 \Leftrightarrow x = \pm 2)$

- $\boxed{\ln(4 - x^2) \neq 0} \Leftrightarrow 4 - x^2 \neq e^0 \Leftrightarrow 4 - x^2 \neq 1$

$\Leftrightarrow x^2 \neq 3 \Leftrightarrow x \neq \pm\sqrt{3}$

$\sqrt{3} \approx 1.7\ldots$



$D_f =]-2, -\sqrt{3}[\cup]-\sqrt{3}, -1] \cup [0, \sqrt{3}[\cup]\sqrt{3}, 2[$ UFF!

② $D_f = \{x \in \mathbb{R} : \ln(4 - x) \geq 0 \wedge 4 - x > 0\}$

- $\boxed{\ln(4 - x) \geq 0} \Leftrightarrow 4 - x \geq e^0 = 1 \Leftrightarrow -x \geq -3$

$\Leftrightarrow \boxed{x \leq 3}$

- $\boxed{4 - x > 0} \Leftrightarrow -x > -4 \Leftrightarrow \boxed{x < 4}$

$D_f = \{x \in \mathbb{R} : x \leq 3 \wedge x < 4\} =]-\infty, 3]$ //

B

b) $(g \circ f)(x) = g(f(x)) = \operatorname{arctg}(2 f(x)) + \frac{\pi}{2}$

$= \operatorname{arctg}\left(2 \sqrt{\ln(4-x)}\right) + \frac{\pi}{2}$ //

$(f \circ g)(x) = f(g(x)) = \ldots$ (ACABAREM)!

c) $g'(x) = \frac{(2x)'}{1+(2x)^2} + \underbrace{\left(\frac{\pi}{2}\right)'}_{= 0} = \frac{2}{1+4x^2} > 0$

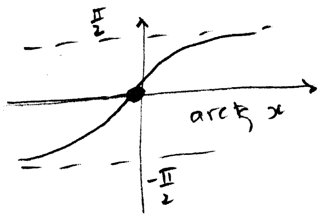
$\Longrightarrow$ $g(x)$ estritamente crescente em $\mathbb{R} = D_g$

$\Longrightarrow$ g invertível em $\mathbb{R}$.

$y = \operatorname{arctg}(2x) + \frac{\pi}{2} \iff \operatorname{arctg}(2x) = y - \frac{\pi}{2}$

$\iff 2x = \operatorname{tg}\left(y - \frac{\pi}{2}\right) \iff x = \frac{\operatorname{tg}\left(y-\frac{\pi}{2}\right)}{2}$

$$\boxed{\therefore\ g^{-1}(x) = \frac{\operatorname{tg}\left(x - \frac{\pi}{2}\right)}{2}}$$



$-\frac{\pi}{2} < \operatorname{arctg}(2x) < \frac{\pi}{2}$

$0 < \underbrace{\operatorname{arctg}(2x) + \frac{\pi}{2}}_{g(x)} < \pi \Longrightarrow D_{g^{-1}} = D'_g = \left]0, \pi\right[$

$$\mathbb{R} \underset{g^{-1}(x)}{\overset{g(x)}{\rightleftarrows}} \left]0, \pi\right[$$

(d) $\lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{\sqrt{\ln(4-x)}}{x} \overset{\frac{\infty}{\infty}}{\underset{RC}{=}}$ $\boxed{c}$

$\overset{\boxed{CA}}{=} \lim_{x \to -\infty} \frac{\frac{-1}{2(4-x)\sqrt{\ln(4-x)}}}{1} = 0$ //

$\boxed{CA}$ $\left(\sqrt{\ln(4-x)}\right)' = \left(\left[\ln(4-x)\right]^{\frac{1}{2}}\right)' \underset{1)}{=}$

$\boxed{1)\ (f^{\alpha})' = \alpha f' f^{\alpha-1}}$

$\boxed{2)\ \ln(f)' = \frac{f'}{f}}$

$= \frac{1}{2}\left(\ln(4-x)\right)'\left(\ln(4-x)\right)^{\frac{1}{2}-1}$

$\underset{2)}{=} \frac{1}{2} \frac{-1}{(4-x)} \cdot \frac{1}{\sqrt{\ln(4-x)}} = \frac{-1}{2(4-x)\sqrt{\ln(4-x)}}$

3. (a) $(\sin^3 x)' = \left[(\sin x)^3\right]' = 3(\sin x)'(\sin x)^{3-1}$

$= 3 \cos x \sin^2 x$ //

$(\cos^3 x)' = \dots$

Regra do produto (Leibniz)

$\left[(\sin^3 x)(\cos^3 x)\right]' = (\sin^3 x)' \cos^3 x + (\sin^3 x)(\cos^3 x)' = \dots$

(b) $\left(\sin(\underset{f}{\underbrace{\cos x}})\right)' = \cos(\cos x) \cdot (\cos x)' = \cos(\cos x)(-\sin x)$

$= -\cos(\cos x) \cdot \sin x$ //

$\boxed{(\sin f)' = \cos(f) \cdot f'}$

(e) • $\left(\sqrt{e^{x^2-1}}\right)' = \left[\left(e^{x^2-1}\right)^{1/2}\right]' = \frac{1}{2}\left(e^{x^2-1}\right)'\left(e^{x^2-1}\right)^{-1/2}$ $\boxed{D}$

$\boxed{(e^f)' = f'e^f}$

$\stackrel{\downarrow}{=} \frac{1}{2}\, e^{x^2-1}\,(\cancel{2}x) \cdot \frac{1}{\sqrt{e^{x^2-1}}} = \frac{x\, e^{x^2-1}}{\sqrt{e^{x^2-1}}}$ //

• $\left(\sqrt{(e^x)^2-1}\right)' = \left(\left[(e^x)^2-1\right]^{1/2}\right)' = \frac{1}{2}\left((e^x)^2-1\right)'\left((e^x)^2-1\right)^{-1/2}$

$= \frac{1}{2}\,\frac{2(e^x)(e^x)}{\sqrt{(e^x)^2-1}} = \frac{e^{2x}}{\sqrt{e^{2x}-1}}$ //

(f) $\left[\ln(\ln x)\right]' \stackrel{\uparrow}{=} \frac{(\ln x)'}{\ln x} = \frac{\frac{1}{x}}{\ln x} = \frac{1}{x \ln x}$ //

$\boxed{(\ln(f))' = \frac{f'}{f}}$

(g) $\left(e^{\cos^2 x}\right)' = (\cos^2 x)'\, e^{(\cos^2 x)} \stackrel{\uparrow}{=} 2(-\sin x)\cos x \;\; e^{\cos^2 x}$ //

$\boxed{(e^f)' = fe^f}$ $\quad$ $\boxed{(f^\alpha)' = \alpha f' f^{\alpha-1}}$

(h) $\left[\operatorname{arctg}(e^x)\right]' \stackrel{\nearrow}{=} \frac{(e^x)'}{1+(e^x)^2} = \frac{e^x}{1+e^{2x}}$ //

$\boxed{[\operatorname{arctg} f]' = \frac{f'}{1+f^2}}$

4 (a) $\lim\limits_{x\to 0^+} \dfrac{1-\cos x}{\arcsin x} \overset{\frac{0}{0}}{\underset{RC}{=}} \lim\limits_{x\to 0^+} \dfrac{\sin x}{\dfrac{1}{\sqrt{1-x^2}}} = 0$

E

(d) $\lim\limits_{x\to 0} x e^{\frac{1}{x}}$

$\lim\limits_{x\to 0^+} x e^{\frac{1}{x}} \overset{0\times\infty}{=} \lim\limits_{x\to 0^+} \dfrac{e^{\frac{1}{x}}}{\frac{1}{x}} \overset{\frac{\infty}{\infty}}{\underset{RC}{=}}$

$= \lim\limits_{x\to 0^+} \dfrac{\left(\frac{1}{x}\right)' e^{\frac{1}{x}}}{\left(\frac{1}{x}\right)'} = +\infty$

$\lim\limits_{x\to 0^-} x e^{\frac{1}{x}} = 0$

$\neq s$

$\therefore$ NÃO EXISTE $\lim\limits_{x\to 0} x e^{\frac{1}{x}}$

---

e) $\lim\limits_{x\to \frac{\pi}{2}^-} \dfrac{\operatorname{tg} x}{\ln(\pi-2x)} \overset{\frac{\infty}{\infty}}{\underset{RC}{=}} \lim\limits_{x\to\frac{\pi}{2}^-} \dfrac{\frac{1}{\cos^2 x}}{\frac{-2}{\pi-2x}} = \lim\limits_{x\to\frac{\pi}{2}^-} \dfrac{\pi-2x}{(-2)\cos^2 x} =$

$\overset{\frac{0}{0}}{\underset{RC}{=}} \lim\limits_{x\to\frac{\pi}{2}^-} \dfrac{-2}{-4(-\sin x)\cos x} = -\infty$

-1 $\quad$ 0^+

$(\cos x > 0)$
$x < \frac{\pi}{2}$

CA $(\operatorname{tg} x)' = \sec^2 x = \dfrac{1}{\cos^2 x}$

F

g) $$\lim_{x\to+\infty} \underbrace{x}_{\downarrow\, +\infty} \ \underbrace{\ln\Big(\underbrace{\frac{x}{1+x}}_{\Downarrow\, 1}\Big)}_{\downarrow\, 0} \overset{0\times\infty}{=} \lim_{x\to+\infty} \frac{\ln\left(\frac{x}{1+x}\right)}{\frac{1}{x}} =$$

$$\underset{RC}{\overset{\frac{0}{0}}{=}} \lim_{x\to+\infty} \frac{\frac{1}{(1+x)^2}}{-\frac{1}{x^2}} = \lim_{x\to+\infty} -\frac{x^2}{(1+x)^2} = \lim_{x\to+\infty} -\Big(\underbrace{\frac{x}{1+x}}_{\downarrow\, 1}\Big)^2$$

$$= -1$$

$\boxed{CA}$

$$\left(\frac{x}{1+x}\right)' = \frac{(x)'(1+x) - x(1+x)'}{(1+x)^2} = \frac{1+x-x}{(1+x)^2} = \frac{1}{(1+x)^2}$$

$$\left(\frac{1}{x}\right)' = (x^{-1})' = (-1)\,x^{-2} = \frac{-1}{x^2}$$

---

h) $$\lim_{x\to+\infty} x\left(1-\cos\frac{1}{x}\right) = \lim_{x\to+\infty} \frac{1-\cos\overset{\to 0}{\frac{1}{x}}}{\frac{1}{x}} \underset{RC}{\overset{\frac{0}{0}}{=}}$$

$$= \lim_{x\to+\infty} \frac{\cancel{\left(\frac{1}{x}\right)'}\sin\frac{1}{x}}{\cancel{\left(\frac{1}{x}\right)'}} = 0 \qquad \left(CA: \ (\cos f)' = -\sin(f)\cdot f'\right)$$

---

i) $$\lim_{x\to+\infty}\left(\sqrt{x^2+1} - \sqrt{x}\right) = \lim_{x\to+\infty} \sqrt{x^2+1}\left(1 - \frac{\sqrt{x}}{\sqrt{x^2+1}}\right)$$

$$= \lim_{x\to+\infty} \sqrt{x^2+1}\left(1 - \sqrt{\underbrace{\frac{x}{x^2+1}}_{\downarrow\, 0}}\right) = +\infty$$

$\boxed{CA}$ $$\lim_{x\to+\infty} \frac{x}{x^2+1} \underset{RC}{\overset{\frac{\infty}{\infty}}{=}}$$

$$\lim_{x\to+\infty} \frac{1}{2x} = 0$$

$\boxed{G}$

j) $\lim_{x\to 0} \dfrac{e^x - 1 - x}{\ln(1+x) - x} \underset{RC}{\overset{\frac{0}{0}}{=}} \lim_{x\to 0} \dfrac{e^x - 1}{\frac{1}{1+x} - 1} \underset{RC}{\overset{\frac{0}{0}}{=}} \lim_{x\to 0} \dfrac{e^x}{-\frac{1}{(x+1)^2}}$

$= \lim_{x\to 0} \left[-e^x (x+1)^2\right] = 0$ //

$\boxed{CA}$ $\left(\frac{1}{1+x}\right)' = \left[(1+x)^{-1}\right]' =$

$= (-1)(1+x)'(1+x)^{-1-1} = \dfrac{-1}{(1+x)^2}$ //

l) $\lim_{x\to 0^+} (\underset{\downarrow\, 0}{\sin x})^{\frac{1}{x}} \overset{0^{\infty}}{=} \lim_{x\to 0^+} e^{\ln(\sin x)^{\frac{1}{x}}} =$

$= \lim_{x\to 0^+} e^{\frac{1}{x}\ln(\sin x)} = 0$ //

$\frac{1}{x} \to +\infty$, $\sin x \to 0^+$, $\ln(\sin x) \to -\infty$, $\frac{1}{x}\ln(\sin x) \to -\infty$

$\boxed{0^{\infty} = 0 \text{ não é indeterm.!!}}$

n) $\lim_{x\to 0^+} (1+5x)^{\frac{1}{x}} = \lim_{x\to 0^+} e^{\ln(1+5x)^{\frac{1}{x}}} =$

$= \lim_{x\to 0^+} e^{\frac{1}{x}\ln(1+5x)} = e^{\lim_{x\to 0^+} \frac{1}{x}\ln(1+5x)} \underset{\boxed{CA}}{=} e^5$ //

$\boxed{CA}$ $\lim_{x\to 0^+} \frac{1}{x}\ln(1+5x) = \lim_{x\to 0^+} \dfrac{\ln(1+5x)}{x} \underset{RC}{\overset{\frac{0}{0}}{=}}$

$= \lim_{x\to 0^+} \dfrac{\frac{5}{1+5x}}{1} = 5$ //

$\boxed{H}$

p) $\lim\limits_{x\to 0^+} (e^x-1)^x \overset{0^0}{=\!=} \lim\limits_{x\to 0^+} e^{\ln(e^x-1)^x} =$

$= \lim\limits_{x\to 0^+} e^{x\ln(e^x-1)} = e^{\lim\limits_{x\to 0^+} x\ln(e^x-1)} \underset{\boxed{CA}}{=} e^0 = 1$ //

$\boxed{CA}$

$\lim\limits_{x\to 0^+} \underset{\substack{\downarrow\\ 0}}{x}\ \underbrace{\ln(e^x-1)}_{\substack{\downarrow\\ -\infty}} \overset{0\times\infty}{=\!=} \lim\limits_{x\to 0^+} \dfrac{\ln(e^x-1)}{\frac{1}{x}} \underset{RC}{\overset{\frac{\infty}{\infty}}{=\!=}}$

$= \lim\limits_{x\to 0^+} \dfrac{\frac{e^x}{e^x-1}}{-\frac{1}{x^2}} = \lim\limits_{x\to 0^+}\left[-\dfrac{x^2e^x}{e^x-1}\right] \underset{RC}{\overset{\frac{0}{0}}{=\!=}}$

$= \lim\limits_{x\to 0^+}\left[-\dfrac{2xe^x+x^2e^x}{e^x}\right] = 0$

---

q) $\lim\limits_{x\to 0^+} (\underset{\substack{\downarrow\\ 1}}{\ln^2 x})^x \overset{1^\infty}{=\!=} \lim\limits_{x\to 0^+} e^{\ln\left[(\ln^2 x)^x\right]}$

$= \lim\limits_{x\to 0^+} e^{x\ln(\ln^2 x)} = e^{\lim\limits_{x\to 0^+} x\ln(\ln^2 x)} \underset{\boxed{CA}}{=} e^0 = 1$ //

$\boxed{CA}$ $\lim\limits_{x\to 0^+} \underset{\substack{\downarrow\\ 0}}{x}\ \underbrace{\ln(\underset{\substack{\downarrow\\ 0^+}}{\ln^2 x})}_{\substack{\downarrow\\ -\infty}} \overset{0\times\infty}{=} \lim\limits_{x\to 0^+} \dfrac{\ln(\ln^2 x)}{\frac{1}{x}} \underset{RC}{\overset{\frac{\infty}{\infty}}{=\!=}}$

$\boxed{(\ln f)' = \frac{f'}{f}}$

$$\lim_{x\to 0^+} \frac{\left(\ln^2 x\right)'}{\left(-\frac{1}{x^2}\right)'} \quad \boxed{(f^2)' = 2 f f^{2-1}} \quad = \lim_{x\to 0^+} \frac{2\,\frac{1}{x}\ln x \,/\, \ln^2 x}{-\frac{1}{x^2}} =$$

$$= \lim_{x\to 0^+} \frac{\frac{2 \cancel{\ln} x}{\cancel{x} \ln^{\cancel{2}} x}}{-\frac{1}{x^{\cancel{2}}}} = \lim_{x\to 0^+} -\frac{2x}{\ln x} = 0$$

$\searrow \frac{0}{\infty}$

(5.) $$\lim_{x\to 1} \frac{f(x)-f(1)}{x^2-1} = \lim_{x\to 1} \frac{f(x)-f(1)}{(x-1)(x+1)} =$$

$$= \lim_{x\to 1} \frac{f(x)-f(1)}{x-1} \cdot \frac{1}{x+1} = 2$$

$\frac{f(x)-f(1)}{x-1} \xrightarrow{(*)} f'(1) = 4$

$\frac{1}{x+1} \to \frac{1}{2}$

(*) Def. de Derivada : $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

(6) $$f(x) = \begin{cases} \frac{\pi}{4} x^2, & x > 1 \\ \operatorname{arctg}(x), & x \leq 1 \end{cases}$$

(A função é cont. p/ $x=1$)

Existe $f'(1) \iff$ existem $f'_d(1)$, $f'_e(1)$ $\wedge$ $f'_d(1) = f'_e(1)$

$\boxed{J}$

$$f'_d(1) = \lim_{h \to 0^+} \frac{f(\overset{>1}{1+h}) - f(1)}{h} =$$

$$\lim_{h \to 0^+} \frac{\frac{\pi}{4}(1+h)^2 - \overset{\frac{\pi}{4}}{\text{arctg}(1)}}{h} = \lim_{h \to 0^+} \frac{\frac{\pi}{4}(1+2h+h^2) - \frac{\pi}{4}}{h}$$

$$= \lim_{h \to 0^+} \frac{\frac{\pi}{4}(2\cancel{h}+h^2)}{\cancel{h}} = \lim_{h \to 0^+} \frac{\pi}{4}(2+h) = \frac{\pi}{2} \,/\!/$$

$$f'_e(1) = (\text{arctg}\, x)' \Big|_{x=1} = \frac{1}{1+x^2}\Big|_{x=1} = \frac{1}{2} \,/\!/$$

Uma vez q̄ $f(x) = \text{arctg}(x)$ p/ $x \leq 1$ podemos calcular $f'_e(1)$ recorrendo às regras usuais de derivação!

Como $f'_d(1) = \frac{\pi}{2} \neq f'_e(1) = \frac{1}{2}$

não existe $f'(1)$!

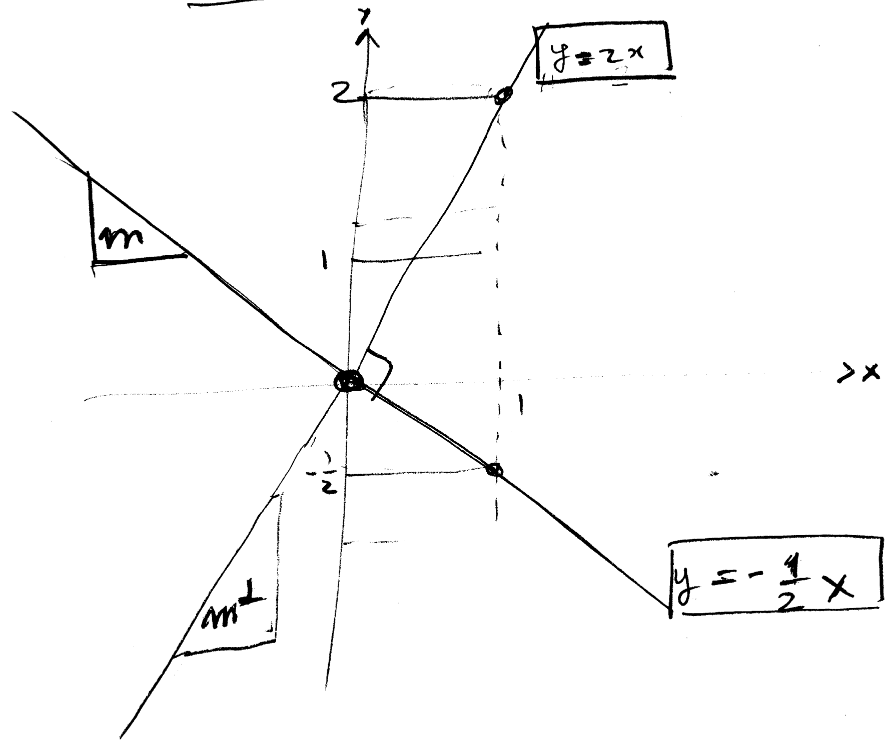
K

9 a) $f(x) = x^3 - 3x^2 + 2x$

declive

$x + 2y = 0 \Leftrightarrow y = -\frac{1}{2}x \Rightarrow \boxed{m = -\frac{1}{2}}$

declive da reta perpend: $m^{\perp} = -\frac{1}{m} = -\frac{1}{-\frac{1}{2}} = 2$ //



? x : $f'(x) = m^{\perp} = 2 \Leftrightarrow 3x^2 - 6x + 2 = 2$

$\Leftrightarrow 3x^2 - 6x = 0$

$\Leftrightarrow 3x(x-2) = 0$

$x = 0 \lor x = 2$

Equação da reta tangente ao gráfico no ponto de abcissa $x = a$ $\longrightarrow$ $\boxed{y = f(a) + f'(a)(x-a)}$

- $a = 0$: $y = \underbrace{f(0)}_{0} + \underbrace{f'(0)}_{2} x \Leftrightarrow \boxed{y = 2x}$
- $a = 2$: $y = \underbrace{f(2)}_{0} + \underbrace{f'(2)}_{2}(x-2) \Leftrightarrow \boxed{y = 2(x-2)}$

b) a, b? tal que f, g têm a mesma reta tangente

$\boxed{L}$

em $x=1$ $\iff$ $\begin{cases} f(1) = g(1) \\ f'(1) = g'(1) \end{cases}$

$f(1) = 1 - 3 + 2 = 0$

$g(1) = (1-a)^2 + b = 0 \implies \boxed{b = -(1-a)^2}$

$f'(1) = 3 - 6 + 2 = -1$

$g'(x) = 2(x-a)$

$\implies g'(1) = 2(1-a) = -1 \implies 1 - a = -\frac{1}{2}$

$a = \frac{3}{2}$ //

$\boxed{\therefore\ a = \frac{3}{2},\ b = -\frac{1}{4}}$

$b = -(1-a)^2 =$

$= -\left(-\frac{1}{2}\right)^2 = -\frac{1}{4}$ //

aproximação linear
em torno de $x = a$

(10) $f(x) = \ln(x) \approx L(x) = f(a) + f'(a)(x-a)$

para x perto de a, com a pt auxiliar onde conhecemos $f(a) = \ln(a)$ e $f'(a) = \frac{1}{a}$

$\ln(0.9) = f(0.9) \approx L(0.9) = \underset{\substack{\parallel \\ \ln(1) \\ \parallel \\ 0}}{f(1)} + \underset{\substack{\parallel \\ \frac{1}{1}}}{f'(1)}(0.9 - 1) = -0.1$

podemos escolher $a = 1$

$\therefore \ln(0.9) \approx -0.1$

(11) $\sqrt{1-x} = \sqrt{0,99} \Rightarrow 1-x = 0.99$ **M**

$\Rightarrow x = 1-0,99 = 0,01$

? a perto 0.01

tal que: $f(a)$ conhecido

$f'(a)$ "

$\left(f'(x) = \dfrac{-1}{2\sqrt{1-x}} \text{ (verifique!)} \right)$

a = 0 serve: $\begin{cases} f(0) = \sqrt{1-0} = 1 \\ f'(0) = -\frac{1}{2} \end{cases}$

$\Rightarrow L(x) = f(0) + f'(0)(x-0) = 1 - \frac{1}{2}x \approx f(x)$ para x perto de $a=0$

$\Rightarrow \sqrt{0,99} = f(0,01) \approx L(0.01) = 1 - \frac{1}{2} \times 0.01$

$= 1 - 0,005$

$= 0.995$ //

$\therefore \sqrt{0,99} \approx 0,995$

0,9949874...

(14) b) $f(x) = e^{-x^2}$

① D_f + A.V
② A. oblíquas
③ Inters. c/ eixos coord.
④ Monot & Extremos
⑤ Concav. & P.I.
⑥ Esboço do gráfico + D'_f

N

① Dom + A.V

f cont. em $\mathbb{R}$ $\Longrightarrow$ não tem A.V.

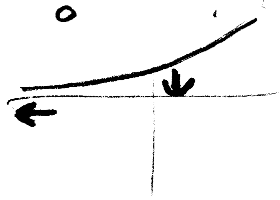
② Assint. não verticais $\quad y = mx + b$

Podemos calcular logo à direita $(+\infty)$ e à esq $(-\infty)$ porque $f(x) = f(-x)$

$$m = \lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{e^{-x^2}}{x} =$$

$$= \lim_{x \to \pm\infty} \frac{1}{\underbrace{x}_{\infty}\,\underbrace{e^{x^2}}_{\infty}} = 0$$

$$b = \lim_{x \to \pm\infty} \left[f(x) - \underbrace{m}_{0}\, x \right] = \lim_{x \to \pm\infty} \left(\underbrace{e^{-x^2}}_{0} - 0 \right) = 0$$



$\therefore$ $y = 0$ A. H. à direita e à esquerda.

③ Intersecção c/ os eixos coordenados

- Inters. c/ yy : $f(0) = 1 \Rightarrow$ intersecta no pt $(0, 1)$
- Inters. c/ xx : $f(x) = 0 \Leftrightarrow e^{-x^2} = 0$ impossível!

$(e^{-x^2} > 0 \quad \forall x)$

não intersecta o eixo dos xx $\qquad f(x) > 0 \ \forall x$

④ Monot. & Extremos

$\boxed{0}$

$$f'(x) = \left(e^{-x^2}\right)' = (-x^2)' e^{-x^2} = -2x \underbrace{e^{-x^2}}_{>0}$$

$$f'(x) = 0 \iff x = 0$$

$$f'(x) > 0 \iff -2x \underbrace{e^{-x^2}}_{>0} > 0 \iff -x > 0 \iff x < 0$$

$$f'(x) < 0 \iff x > 0$$

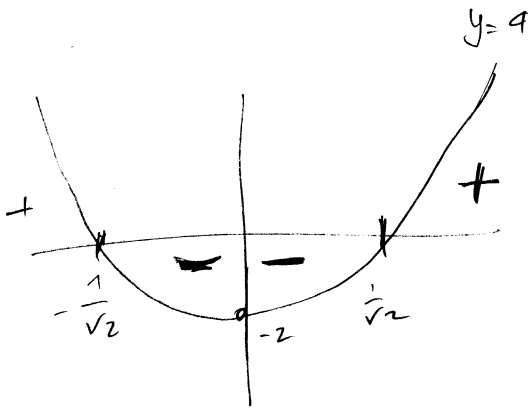
		0	
f'	+	0	−
f	↗	MAX	↘

⑤ Concavidades & P.I

$$f''(x) = \left(-2x e^{-x^2}\right)' = -2e^{-x^2} + (-2x)\left(-2x e^{-x^2}\right)$$

$$= e^{-x^2}\left(4x^2 - 2\right) =$$

$$f''(x) = 0 \iff x^2 = \frac{1}{2} \iff x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$



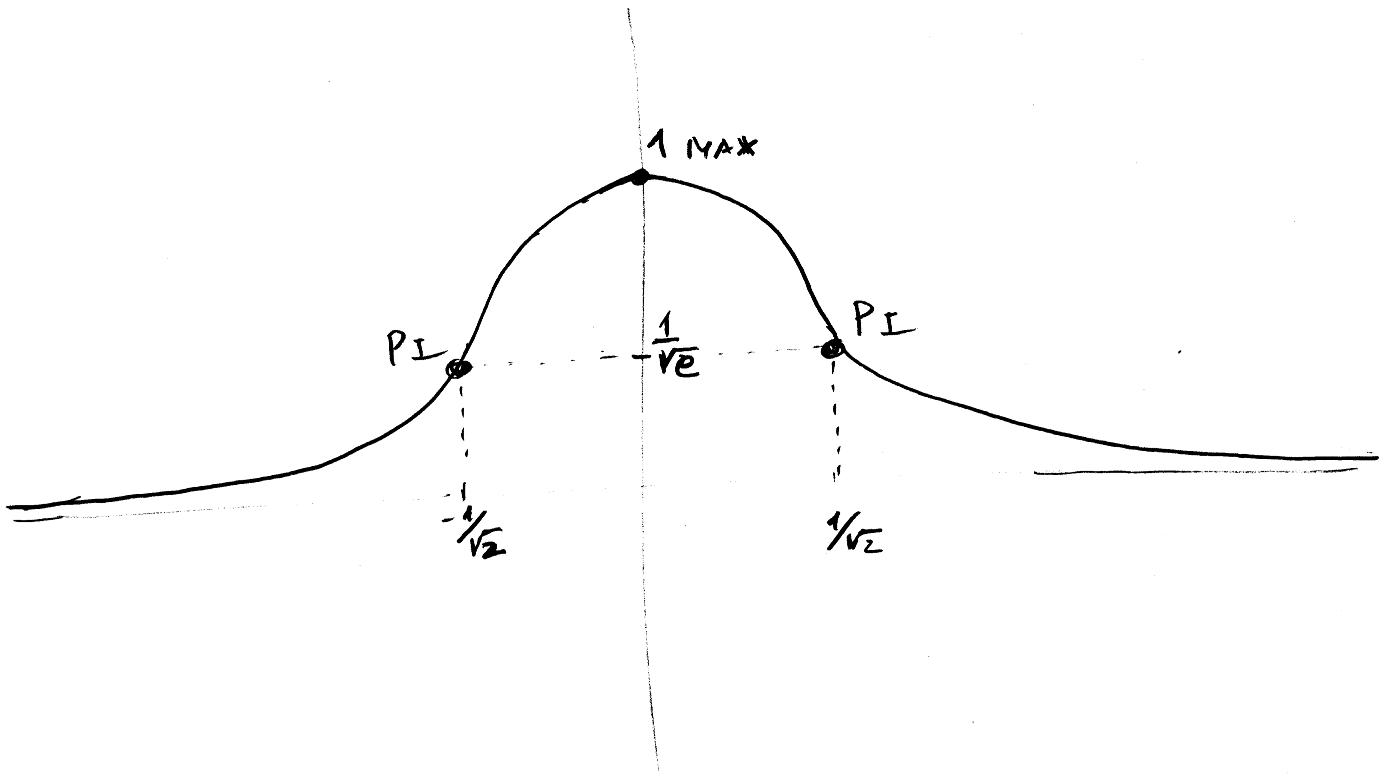
		$-\frac{1}{\sqrt{2}}$		$\frac{1}{\sqrt{2}}$	
f''	+	0	−	0	+
f	∪	PI	∩	PI	∪

$$f\left(\pm\frac{1}{\sqrt{2}}\right) = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

P

⑥ Esboço do gráfico e D'_f

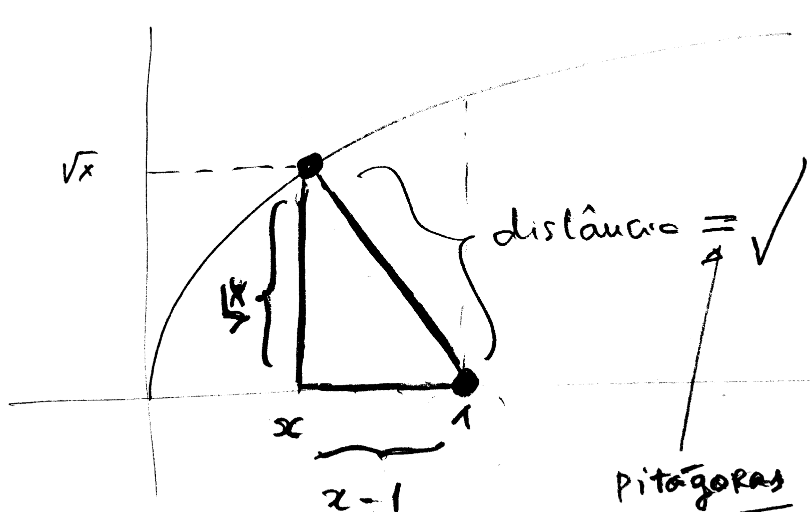
		$-1/\sqrt{2}$		0		$1/\sqrt{2}$	
f'	$+$	$+$	$+$	0	$-$	$-$	$-$
f	↗	↗	↗	MAX	↘	↘	↘
f''	$+$	0	$-$	$-$	$-$	0	$+$
f	∪	PI	∩	∩	∩	PI	∪



$D'_f =]0,1]$

UFF!!

(15)



distância $= \sqrt{(x-1)^2 + (\sqrt{x})^2} = \sqrt{(x-1)^2+x}$

pitágoras

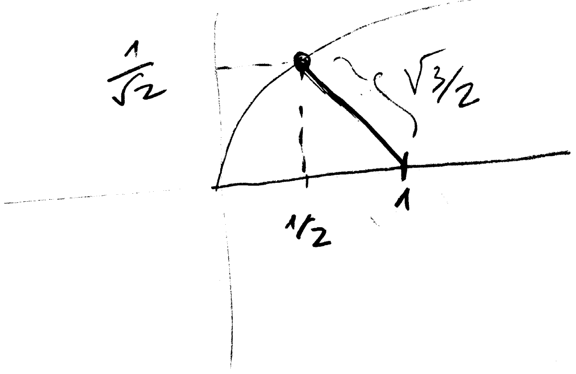
$= \sqrt{x^2-2x+1+x}$

$= \underbrace{\sqrt{x^2-x+1}}_{f(x)}$

minimizar a função $\sqrt{x^2-x+1}$ é o mesmo que minimizar $g(x) = f^2(x) = x^2-x+1$, $x>0$

g atinge um min relativo em $x=a>0$ $\Longrightarrow$ a pt crítico de g i.e $g'(a)=0$

$$g'(x) = 2x-1 = 0 \iff \boxed{x = \frac{1}{2}}$$



∴ o ponto do gráfico $(x,\sqrt{x})$ c/ $x>0$ que se encontra à menor distância de $(1,0)$ é o pt $\left(\frac{1}{2}, \sqrt{\frac{1}{2}}\right)$

cuja distância a $(1,0)$ é:

$$\sqrt{\left(\frac{1}{2}\right)^2 - \frac{1}{2} + 1} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

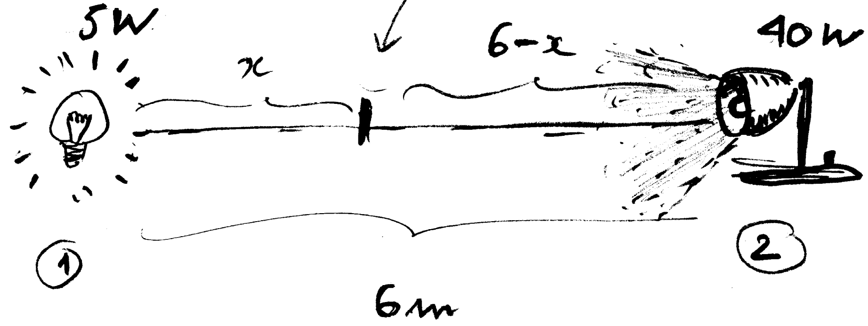
OBS

$dist((1,0),(0,0)) = 1 > \frac{\sqrt{3}}{2}$

17

R

Ponto menos iluminado ?



constante proporcionalidade

$$\text{Intensidade} = \frac{\alpha \text{ potência}}{\text{distância}^2} \quad (W\,m^{-2})$$

Intensidade no pt x pelas fontes 1 e 2:

$$I_1(x) = \frac{5\alpha}{x^2}, \quad I_2(x) = \frac{40\alpha}{(6-x)^2} \quad (0 < x < 6)$$

$$x? \quad \min \underbrace{I_1 + I_2}_{f(x)} = \min \alpha\left(\frac{5}{x^2} + \frac{40}{(6-x)^2}\right)$$

Se f atinge o mínimo em $x = a \in]0,6[$

$\Longrightarrow f'(a) = 0$ (a pt crítico de f)

Ora,

$$f'(x) = \alpha\left(\frac{5}{x^2} + \frac{40}{(6-x)^2}\right)' = \alpha\left(5\left(x^{-2}\right)' + 40\left(\left(6-x\right)^{-2}\right)'\right)$$

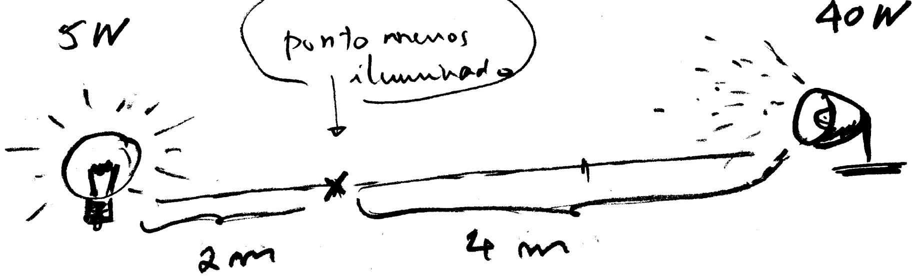
$$= \alpha\left(-10x^{-3} + 80(6-x)^{-3}\right) \quad \text{confirme!}$$

$$f'(x) = 0 \iff -10x^{-3} + 80(6-x)^{-3} = 0 \iff \frac{80}{(6-x)^3} = \frac{10}{x^3}$$

5

$$\Leftrightarrow \frac{(6-x)^3}{x^3} = \frac{80}{10} = 8 \Leftrightarrow \frac{6-x}{x} = \sqrt[3]{8} = 2$$

$$\Leftrightarrow 6-x = 2x \Leftrightarrow 3x = 6 \Leftrightarrow \underline{x = 2\,m}$$



(20.) b) $P \frac{3}{\sqrt[6]{x^5}} = 3 P\, x^{-5/6} = 3 \frac{x^{-\frac{5}{6}+1}}{-\frac{5}{6}+1}$

$= 3 \frac{x^{1/6}}{1/6} = 18 \sqrt[6]{x}$ // $\left(2) \; \underset{(\alpha \neq -1)}{P} x^\alpha = \frac{x^{\alpha+1}}{\alpha+1} \right)$

f) $P \frac{2x-5}{x^3} = P\left(\frac{2x}{x^3} - \frac{5}{x^3} \right)$

$= 2 P x^{-2} - 5 P x^{-3} = 2 \frac{x^{-1}}{-1} - 5 \frac{x^{-2}}{-2} = \frac{-2}{x} + \frac{5}{2x^2}$ //

g) **NÃO FAZER !** (é necessário dividir os polinómios...)

h) $P\left(\frac{x - x^3}{1+x^4} \right) = P\left(\frac{x}{1+x^4} - \frac{x^3}{1+x^4} \right) =$

$= \frac{1}{2} P \frac{2x}{1+(x^2)^2} - \frac{1}{4} P \frac{4x^3}{1+x^4} = \frac{1}{2} \operatorname{arctg}(x^2) - \frac{1}{4} \ln|1+x^4|$ //

$\frac{f'}{1+f^2}$ $\quad \frac{f'}{f}$

$\boxed{T}$

i) $= P \frac{x}{x^2+1} + P \frac{2}{x^2+1} + P \frac{\sin x}{\cos^2 x}$

α $f = \cos x$, $f' = -\sin x$

$= \frac{1}{2} P \frac{2x}{x^2+1} + 2 P \frac{1}{1+x^2} + (-1) P(-\sin x) \cos^{-2} x$

$= \frac{1}{2} \ln(x^2+1) + 2 \operatorname{arctg} x - \frac{(\cos x)^{-2+1}}{-2+1}$

$= \frac{1}{2} \ln(x^2+1) + 2 \operatorname{arctg} x + \frac{1}{\cos x}$ //

j)
k) } NÃO FAZER (primitiva por partes - ppp)

l) $P \frac{e^x + e^{2x}}{1+e^{2x}} = P \frac{e^x}{1+e^{2x}} + \frac{1}{2} P \frac{2e^{2x}}{1+e^{2x}}$

$= \operatorname{arctg}(e^x) + \frac{1}{2} \ln(1+e^{2x})$

m)
o)
p)
q)
r) } NÃO FAZER (ppp)

$\boxed{U}$

s) $P \underbrace{\frac{1}{x}(1+\ln x)^{1/2}}_{f}$ $\underset{f' f^{\alpha}}{=\!=\!=}$ $\frac{(1+\ln x)^{3/2}}{3/2} = \frac{2}{3}\sqrt{(1+\ln x)^3}$ //

t) $P \frac{\frac{1}{x}}{1+\ln x} = \ldots$

u) $P \frac{\frac{1}{x}}{1+\ln^2 x} = \ldots$

v) NÃO FAZER (ppp)

w) $P \frac{e^{\sqrt{x}}}{\sqrt{x(1-e^{2\sqrt{x}})}} = P \frac{e^{\sqrt{x}}}{\sqrt{x}\sqrt{1-e^{2\sqrt{x}}}}$

$= (-1) P \underbrace{\frac{(-1)e^{\sqrt{x}}}{\sqrt{x}}}_{f'} \cdot \underbrace{(1-e^{2\sqrt{x}})}_{f}{}^{-1/2} \overset{\alpha}{} = - \frac{(1-e^{2\sqrt{x}})^{-1/2+1}}{-1/2+1}$

$= -2\sqrt{1-e^{2\sqrt{x}}}$ //

(x) NÃO FAZER (ppp)

$\boxed{V}$

21 $F = \int f \Longleftrightarrow$

$F' = f$

logo temos q ver q

$$\left(x \arccos x - \sqrt{1-x^2} \right)' = \arccos x$$

...

24 NÃO FAZER (MP)