Whole stand models for even-aged stands and diameter distribution models

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Summary

- Whole stand models for even-aged stands
  - ➔ State variables
  - ➔ Control variables
    - Stand density and stocking
    - Stand density measures:
      - Stand density index (SDI)
      - Crown competition factor (CCF)
      - Relative spacing (Wilson factor)
      - Spacing factor (Sf)
      - Crown cover (CC)
  - ➔ Growth and calculus modules
    - Site productivity
    - Silvicultural treatments and thinning
- Whole stand models – diameter distribution
  - ➔ Modelling diameter distributions
  - ➔ PDF functions (Weibull and Johnson's SB)
  - ➔ The PBRAVO Model

Whole stand models
for
even-aged stands

Whole stand models - state variables

- In whole stand models the state variables are all defined at stand level:
  - → Dominant height (hdom)
  - → Number of trees per ha (N)
  - → Basal area (G)
  - → Volume (V) and merchantable volumes (Vdi or Vhi)
  - → Biomass (W) and biomass per tree component (Wr, Ww, Wb, Wbr, Wl)

Principal variables

Derived variables

- hdom, N and G are almost always principal variables, volume may be derived or not

Whole stand models - control variables

- The most important control variables are

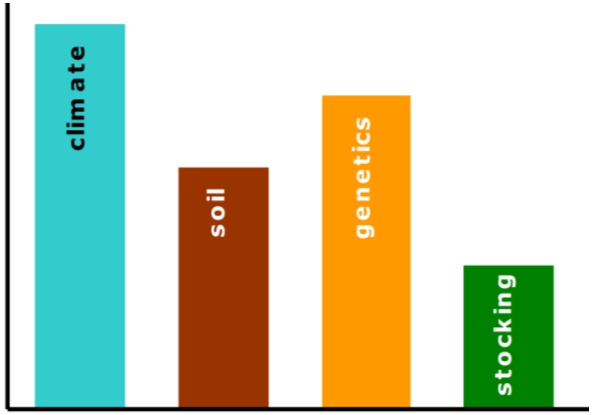
  → Site productivity (climate and soil), very often expressed as site index

  → Genetics

  → Application of **fertilizers**

  → Stocking control, either initial **stand density** and **thinnings**

  → **Other silvicultural techniques** (weeding, pruning, irrigation, etc)



- Selection of **quantitative measures of stand density** is therefore an important step in forest models development and/or application

Stocking and stand density

- Although stocking and stand density are terms that are often applied interchangeably in forestry use, the two terms are not synonymous
  - ➔ **Stand density** denotes a quantitative measurement of the stand
  - ➔ **Stocking**:
    - Stocking refers to the adequacy of a given stand density to meet some management objective (Bickford et al. 1957)
    - Stands may be referred to as "**understocked**", "**fully-stocked**", or overstocked
    - A stand that is "**overstocked**" for one management objective could be "understocked" for another

Quantifying stand density

- Stand density is a quantitative term describing the degree of stem crowding within a stocked area and it can be expressed in:

  → **Absolute** measures of density are determined directly from a given stand without reference to any other stand

  → Basal area

  → Number of trees per ha

  → **Relative** density is based on a selected standard density, usually the "fully-stocked" stand or the open-grown trees (the extremes)

  → Stand density índex (SDI)

  → Crown competition factor (CCF)

  → Other stand density measures

  - Relative spacing (FW)
  - Spacing factor (SF)
  - Percent crown cover (CC)

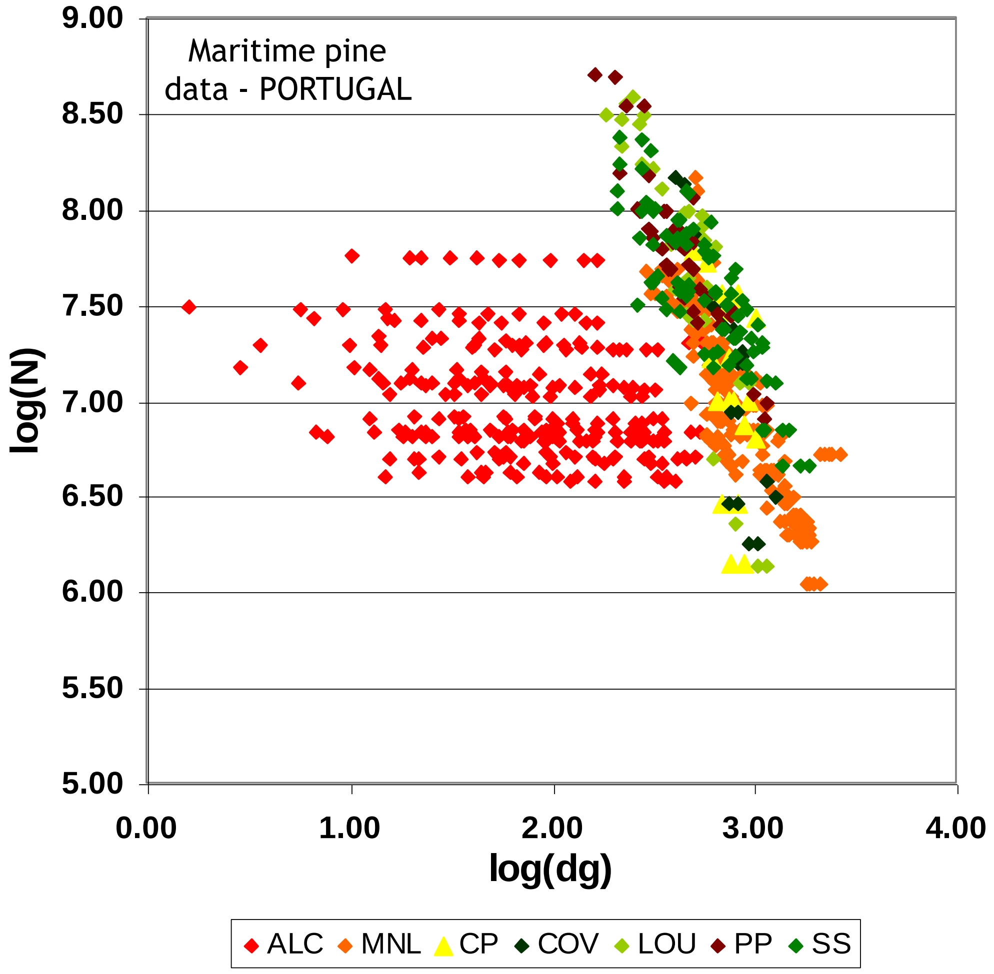
Quantifying stand density

- **Stand density index (SDI)** - Relative stand density measures

  - ➔SDI evaluates stand density by comparing it with the maximum density for a stand with the same quadratic mean dbh (*dg*) – limiting situation or self-thinning line

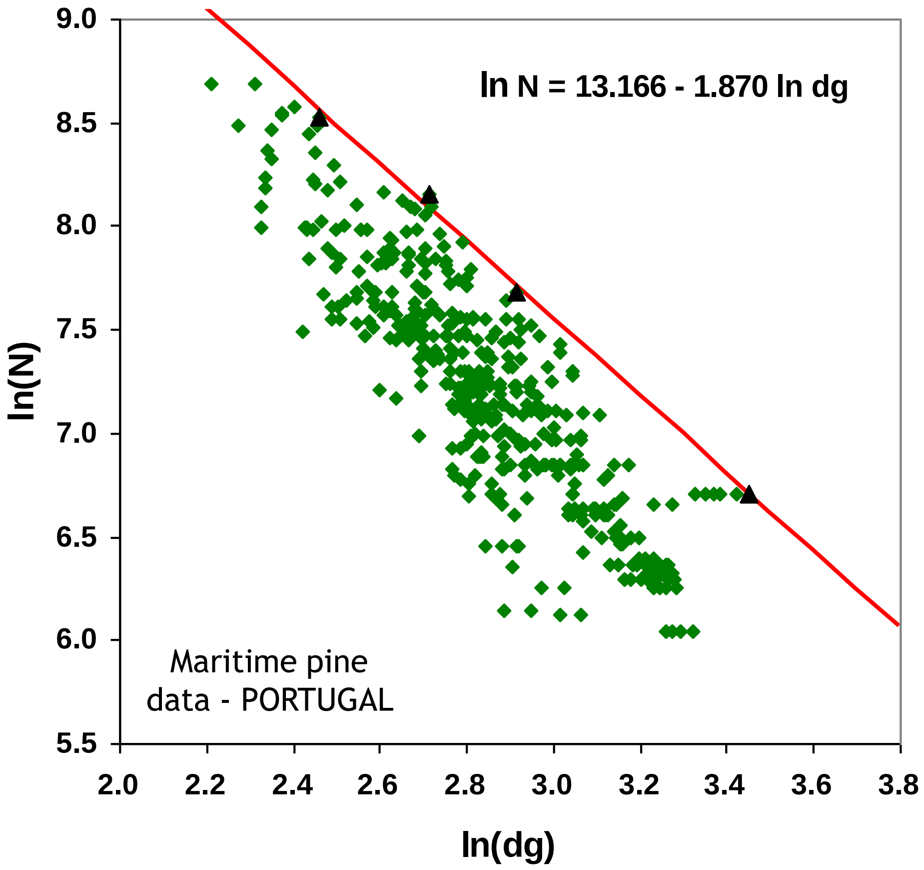
  - ➔For any given *dg* there is a limit to the number of trees per unit that can be carried

  - ➔Reineke (1933) noted that for a variety of species the slope of the limiting line was approximately -1.6 on the log-log scale



Quantifying stand density

- **Stand density index (SDI)** - Relative stand density measures

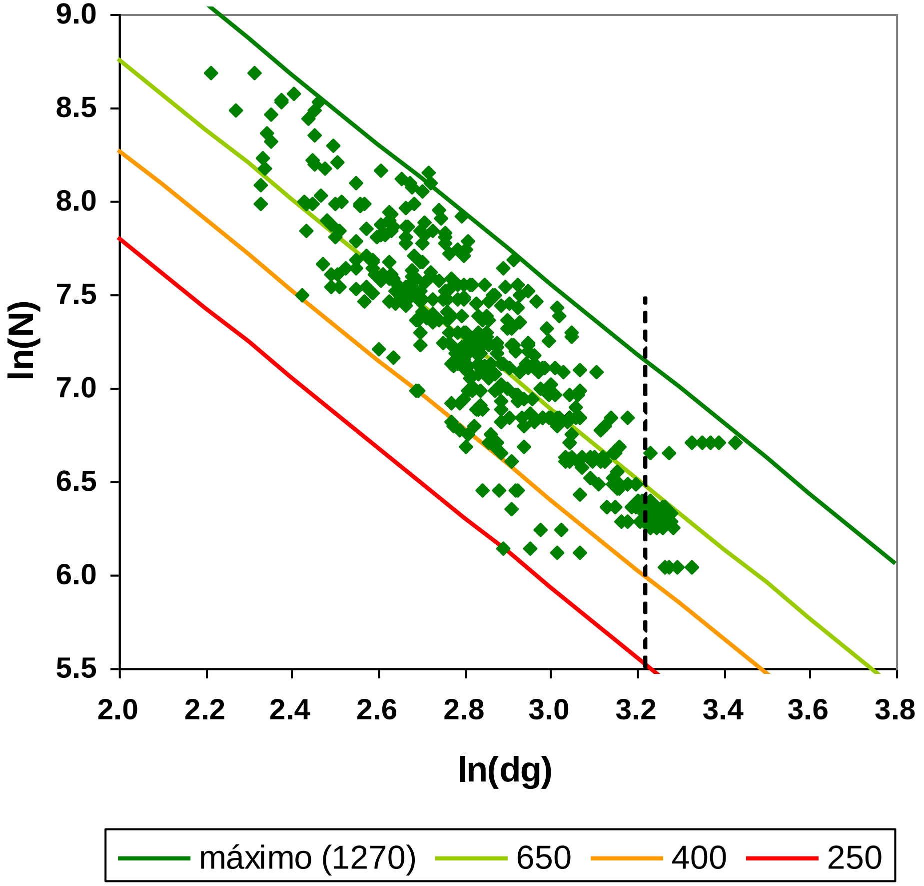


- SDI is based on the evaluation of the difference between the number of trees in the stand and the maximum number of trees it could sustain according to the self-thinning line

- SDI assumes that an understocked stand is located in a logN-logdg line parallel to the self-thinning line but with a smaller intercept

Quantifying stand density

- **Stand density index (SDI)** - Relative stand density measures



- The intercept for a stand can be obtained as

$$\ln N = k - 1.870 \ln dg$$

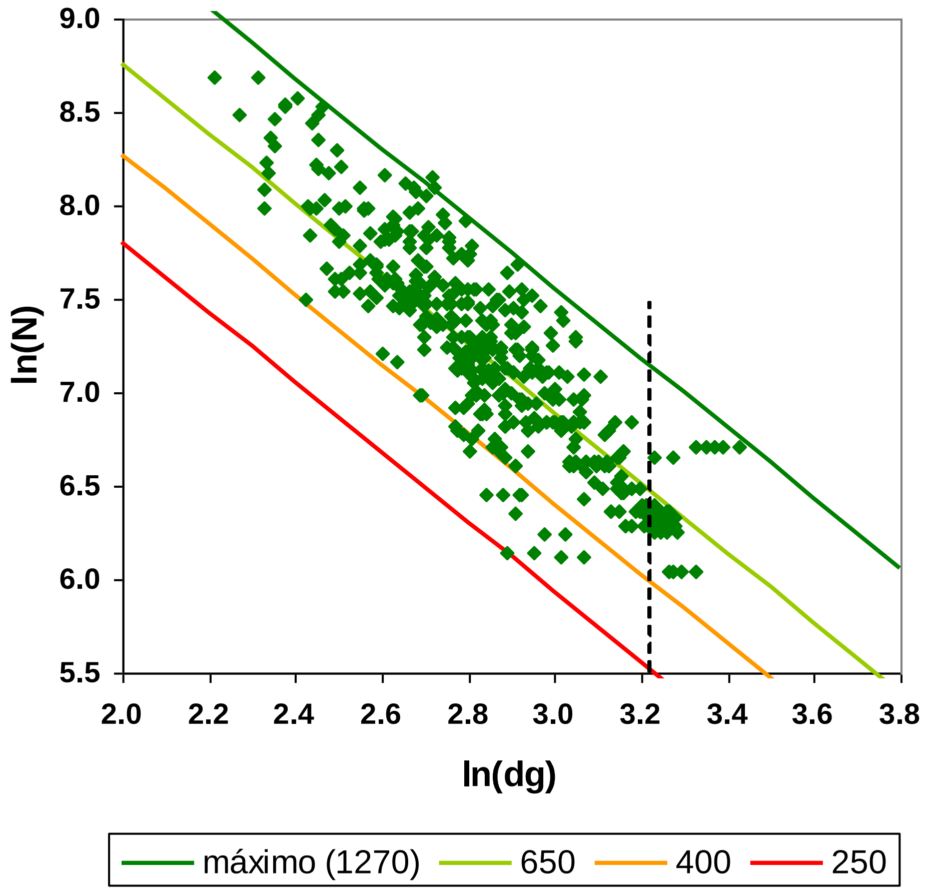
$$k = \ln N + 1.870 \ln dg$$

- The index is “normalized” by using the dg=25 as a basis for comparison

$$\ln SDI = k - 1.870 \ln 25$$

Quantifying stand density

- **Stand density index (SDI)** - Relative stand density measures



The expression for SDI in a particular stand is then obtained:

$$\log SDI = -1.870 \ \log \ 25 + k$$

$$+$$

$$k = \ln \ N + 1.870 \ \ln \ dg$$

$$\ln \ SDI = -1.870 \ \ln \ 25 + \ln \ N + 1.870 \ \ln \ dg$$

$$SDI = N \left(\frac{dg}{25} \right)^{1.870}$$

Quantifying stand density

- **Crown competition factor (CCF)** - Relative stand density measures

➔ CCF reflects the relationship between the **area available for the average tree of the stand** and **the maximum area that the tree could use if it was growing in open space** (open-grown tree)

➔ The computation of CCF requires the study of the relationship between **crown width of an open-grown tree** (cw_{og}) and its **dbh** (d_{og}), usually linear:

$$cw_{og} = b_0 + b_1 d_{og}$$

➔ The crown of an open-grown tree ocupies the area ca_{og}:

$$ca_{og} = \pi \frac{cw_{og}^2}{4} = \pi \frac{\left(b_0 + b_1 d_{og}\right)^2}{4}$$

➔ CCF is then computed as the sum of the ca_{og} values for all the trees in the stand, expressed as a percentage of the plot area:

$$CCF = \frac{100}{A_p} \sum_{i=1}^{N} ca_{og_i}$$

Quantifying stand density

- **Relative spacing (Rs)**

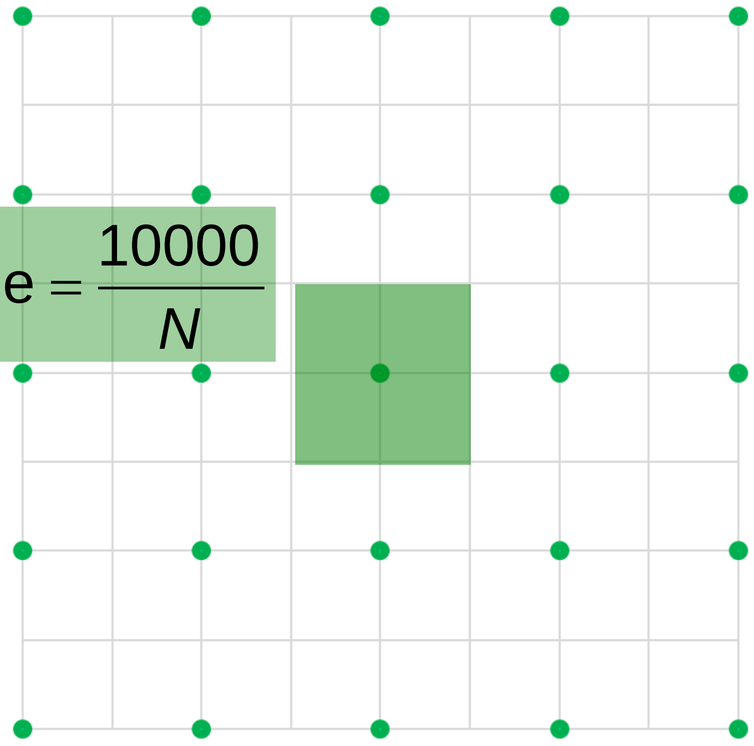
  ➔ RS is a stand density measure that relates the mean distance between trees with the dominant height

  ➔ It is based on the assumption that the stand density must decrease as the stand develops (the dominant height increases)

$$Rs = \frac{\text{average distance between trees}}{hdom}$$

➔ Assuming that the trees are regularly spaced, the area available per tree is:

$$\text{Area per tree} = \frac{10000}{N}$$

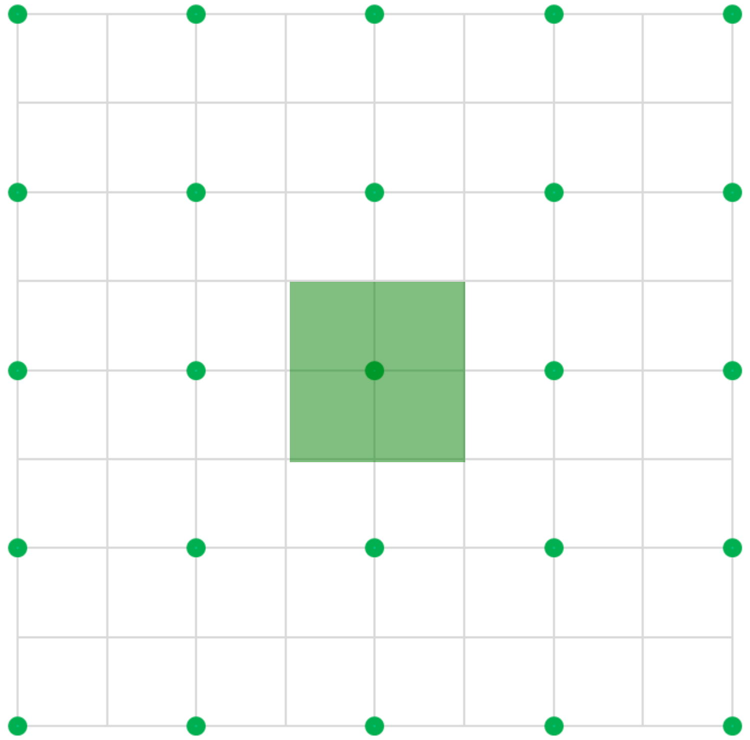


Quantifying stand density

- **Relative spacing (Rs), Wilson factor (Fw)**

→ Assuming that the trees are regularly spaced, the area available per tree is:

$$\text{Area per tree} = \frac{10000}{N} \Rightarrow dist_{mean} = \sqrt{\frac{10000}{N}}$$



→ The relative spacing can be written in the form usually known as wilson factor

$$Fw = \frac{\sqrt{1000/N}}{hdom} = \frac{100}{hdom\ \sqrt{N}}$$

Quantifying stand density

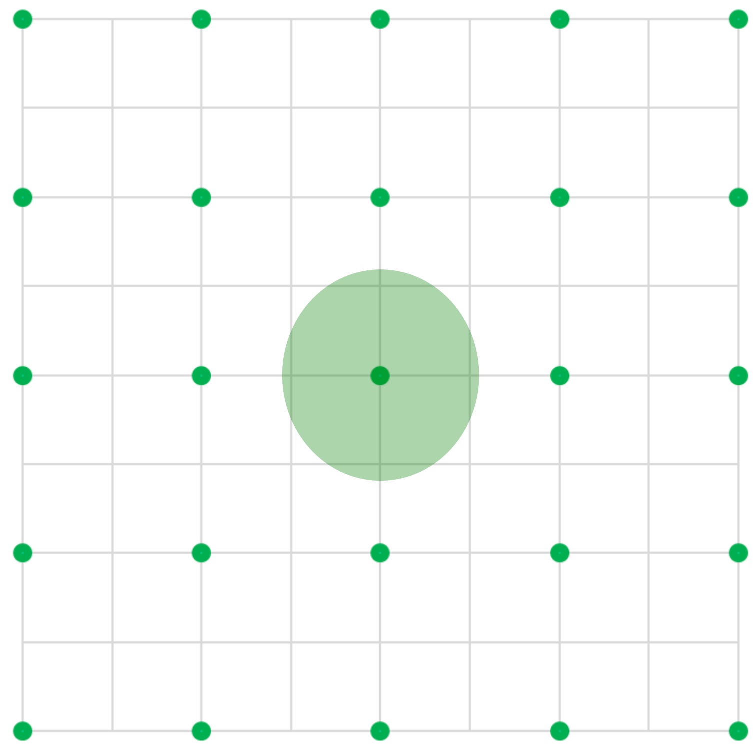
- **Spacing factor (Sf)**

  → Sf is a stand density measure that relates the average distance between trees to the crown width of the average tree:

$$Sf = \frac{\text{average distance between trees}}{cw_{mean}}$$

  → If a regularly spaced stand is assumed, Sf comes as:

$$Sf = \frac{100}{cw_{mean}\sqrt{N}}$$

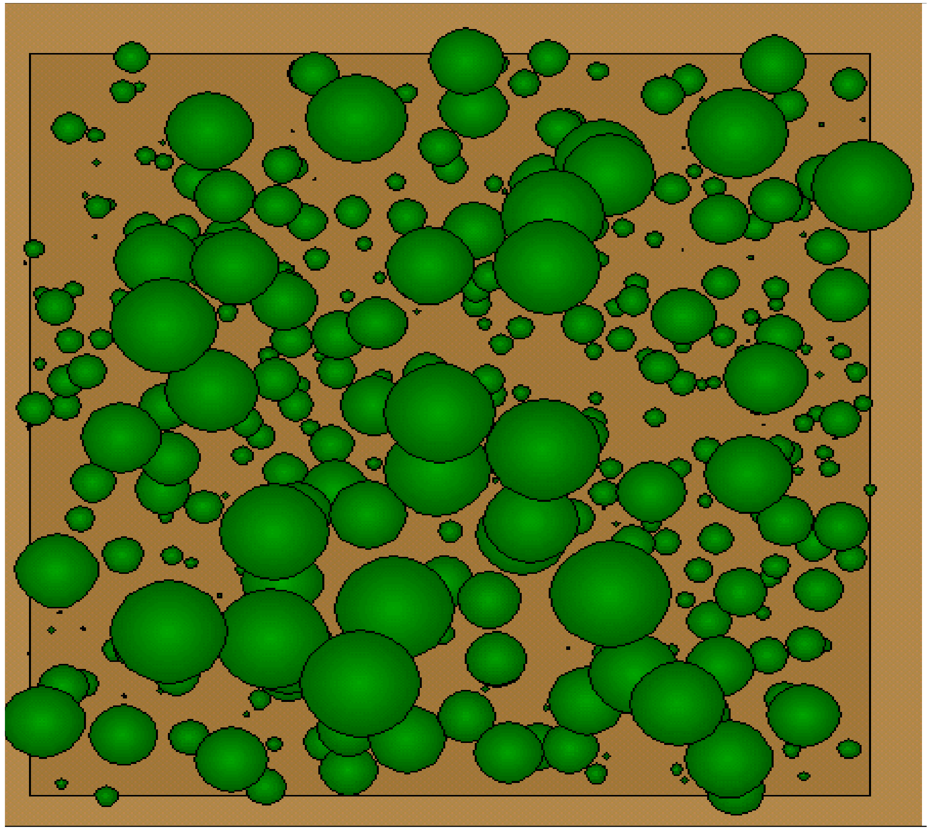


Quantifying stand density

- **Crown cover(*Cc*)**

  ➔ Crown cover (*Cc*) is a stand density measure that computes the percentage of area covered with crowns :

$$Cc = \frac{\sum_{\text{all trees}} \text{crown area}}{\text{Plot area}} \; 100$$



Whole stand models for even-aged stands (WSM-eas)

- Site productivity
  - ➔A system of **site index curves** is the most common way to express site productivity in WSM-eas
  - ➔In species in which age is difficult to determine:
    - Site index may be assessed with a **site prediction equation**
    - Site productivity may be included in the several **sub-models through climatic and soil variables**

Whole stand models for even-aged stands (WSM-eas)

Growth modules

→ Growth modules refer to principal variables, the ones whose growth is predicted by the model:

- Direct prediction of growth

  $i_{X1\text{-}2} = f(S, t_1, t_2, SD_1)$

  $X_2 = X_1 + i_{X1\text{-}2}$

- Direct prediction of future value

  $SD2 = f(S, t_1, t_2, SD_1)$

  $X_2 = f(S, t_1, t_2, X_1, SD_1, SD_2,$ *other stand variables)*

→ Notation

- S = site index or site variables (climate and soil)
- t_i = stand age at time t_i
- X_i = principal stand variable X at time t_i
- SD_i = stand density measure at time t_i
- $i_{X1\text{-}2}$ = growth of variable X in the period between t_1 and t_2
- Y_i = derived stand variable Y at time t_i

▪Whole stand models for even-aged stands (GLOBULUS 3.0)

- **Module Growth:** $hdom_2 = f\ (t_1, t_2, hdom_1, Rain)$

SUM | =($B$4+$B$5*$A$48)*(B54/($B$4+$B$5*$A$48))^((A54/A55)^($B$6))

Planted Stand

Number of Days with Rain	Altitude	Number of Days with Frost	Rain	Mean Temperature
114	550	7.00	650.00	15.50

Site Index	Number of Trees at Planting	Rotation	Top Diameter
21.8	1250	0	6.20

Inicialization | Prediction / Growth | Calculus

t	hdom	Nst	N	G	Vu	Vb	Vs	dg	Vdi	Ww	Wl	Wb	Wbr	Wa	Wr
1	2.5	1234	1234	0.6	0.5	0.2	0.0	2.5	0.0	0.2	0.5	0.0	0.2	0.9	0.2
2	=($B$4+$	1217	1217	2.5	5.7	1.6	0.3	5.1	2.3	2.2	1.6	0.4	1.0	5.2	1.3
3	9.4	1201	1201	4.8	16.2	4.1	0.5	7.2	11.5	7.1	2.6	1.1	2.0	12.8	3.2
4	12.1	1185	1185	7.1	30.2	7.3	0.9	8.7	25.0	14.1	3.6	2.0	3.0	22.7	5.6
5	14.3	1170	1170	9.2	46.2	10.7	1.2	10.0	40.6	22.6	4.4	3.1	3.9	34.0	8.5
6	16.2	1154	1154	11.2	63.3	14.3	1.4	11.1	57.4	31.9	5.1	4.3	4.7	46.1	11.5
7	17.9	1139	1139	13.1	80.7	17.8	1.7	12.1	74.5	41.7	5.7	5.5	5.5	58.5	14.6
8	19.3	1124	1124	14.8	98.1	21.3	2.0	12.9	91.8	51.8	6.2	6.8	6.2	71.1	17.7
9	20.6	1109	1109	16.3	115.3	24.6	2.2	13.7	108.8	61.9	6.6	8.1	6.9	83.5	20.8

▪Whole stand models for even-aged stands (GLOBULUS 3.0)

- *Module Growth:* $Nst_2 = f\,(t_1, t_2, Nst_1, NPL, rotation)$

SUM | =C54*EXP(-($B$9+$B$10*$J$48+$B$11*$I$48/1000)*(A55-A54))

Planted Stand

Number of Days with Rain	Altitude	Number of Days with Frost	Rain	Mean Temperature
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Site Index	Number of Trees at Planting	Rotation	Top Diameter
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Inicialization | Prediction / Growth | Calculus

t	hdom	Nst	N	G	Vu	Vb	Vs	dg	Vdi	Ww	Wl	Wb	Wbr	Wa	Wr
1	2.5	1234	1234	0.6	0.5	0.2	0.0	2.5	0.0	0.2	0.5	0.0	0.2	0.9	0.2
2	6.3	=C54*EXP	1217	2.5	5.7	1.6	0.3	5.1	2.3	2.2	1.6	0.4	1.0	5.2	1.3
3	9.4	1201	1201	4.8	16.2	4.1	0.5	7.2	11.5	7.1	2.6	1.1	2.0	12.8	3.2
4	12.1	1185	1185	7.1	30.2	7.3	0.9	8.7	25.0	14.1	3.6	2.0	3.0	22.7	5.6
5	14.3	1170	1170	9.2	46.2	10.7	1.2	10.0	40.6	22.6	4.4	3.1	3.9	34.0	8.5
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7	17.9	1139	1139	13.1	80.7	17.8	1.7	12.1	74.5	41.7	5.7	5.5	5.5	58.5	14.6
8	19.3	1124	1124	14.8	98.1	21.3	2.0	12.9	91.8	51.8	6.2	6.8	6.2	71.1	17.7
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▪Whole stand models for even-aged stands (GLOBULUS 3.0)

- *Module Growth:* $G_2 = f\,(t_1, t_2, Nst_1, Nst_2, G_2, rotation, Rain, altitude)$

SUM

```
=($B$15+$B$16*$A$48)*(E54/($B$15+$B$16*$A$48))^((A54^($E$18*C54/1000))/(A55^($E$18*C55/1000))*((A54/A55)^($E$15+($E$16/(1-($B$48/2000)))+$E$17*$J$48)))
```

Planted Stand

Number of Days with Rain	Altitude	Number of Days with Frost	Rain	Mean Temperature
114	550	7.00	650.00	15.50

Site Index	Number of Trees at Planting	Rotation	Top Diameter
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Inicialization | Prediction / Growth | Calculus

t	hdom	Nst	N	G	Vu	Vb	Vs	dg	Vdi	Ww	Wl	Wb	Wbr	Wa	Wr
1	2.5	1234	1234	0.6	0.5	0.2	0.0	2.5	0.0	0.2	0.5	0.0	0.2	0.9	0.2
2	6.3	1217	1217	\$48)*(E54/($E	5.7	1.6	0.3	5.1	2.3	2.2	1.6	0.4	1.0	5.2	1.3
3	9.4	1201	1201	4.8	16.2	4.1	0.5	7.2	11.5	7.1	2.6	1.1	2.0	12.8	3.2
4	12.1	1185	1185	7.1	30.2	7.3	0.9	8.7	25.0	14.1	3.6	2.0	3.0	22.7	5.6
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10	21.8	1095	1095	17.8	132.1	27.8	2.4	14.4	125.5	71.9	7.0	9.4	7.5	95.7	23.8
11	22.9	1080	1080	19.1	148.5	30.9	2.6	15.0	141.7	81.7	7.3	10.6	8.1	107.7	26.8
12	23.8	1066	1066	20.4	164.3	33.9	2.8	15.6	157.4	91.3	7.5	11.9	8.6	119.3	29.7

▪Whole stand models for even-aged stands (GLOBULUS 3.0)

- ***Module Growth:*** $Vu_2 = f\,(t_1, t_2, hdom_1, hdom_2, G_1, G_2, Vu_1)$

SUM | =F54*((A55/A54)^$B$21)*((B55/B54)^$B$22)*((E55/E54)^$B$23)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
45	**Planted Stand**															
46																
47	Number of Days with Rain	Altitude	Number of Days with Frost	Rain	Mean Temperature			Site Index	Number of Trees at Planting	Rotation	Top Diameter					
48	114	550	7.00	650.00	15.50			21.8	1250	0	6.20					
49																
50																
51	**Inicialization**		**Prediction / Growth**		**Calculus**											
52																
53	**t**	**hdom**	**Nst**	**N**	**G**	**Vu**	**Vb**	**Vs**	**dg**	**Vdi**	**Ww**	**Wl**	**Wb**	**Wbr**	**Wa**	**Wr**
54	1	2.5	1234	1234	0.6	0.5	0.2	0.0	2.5	0.0	0.2	0.5	0.0	0.2	0.9	0.2
55	2	6.3	1217	1217	2.5	=F54*((A5	1.6	0.3	5.1	2.3	2.2	1.6	0.4	1.0	5.2	1.3
56	3	9.4	1201	1201	4.8	16.2	4.1	0.5	7.2	11.5	7.1	2.6	1.1	2.0	12.8	3.2
57	4	12.1	1185	1185	7.1	30.2	7.3	0.9	8.7	25.0	14.1	3.6	2.0	3.0	22.7	5.6
58	5	14.3	1170	1170	9.2	46.2	10.7	1.2	10.0	40.6	22.6	4.4	3.1	3.9	34.0	8.5
59	6	16.2	1154	1154	11.2	63.3	14.3	1.4	11.1	57.4	31.9	5.1	4.3	4.7	46.1	11.5
60	7	17.9	1139	1139	13.1	80.7	17.8	1.7	12.1	74.5	41.7	5.7	5.5	5.5	58.5	14.6
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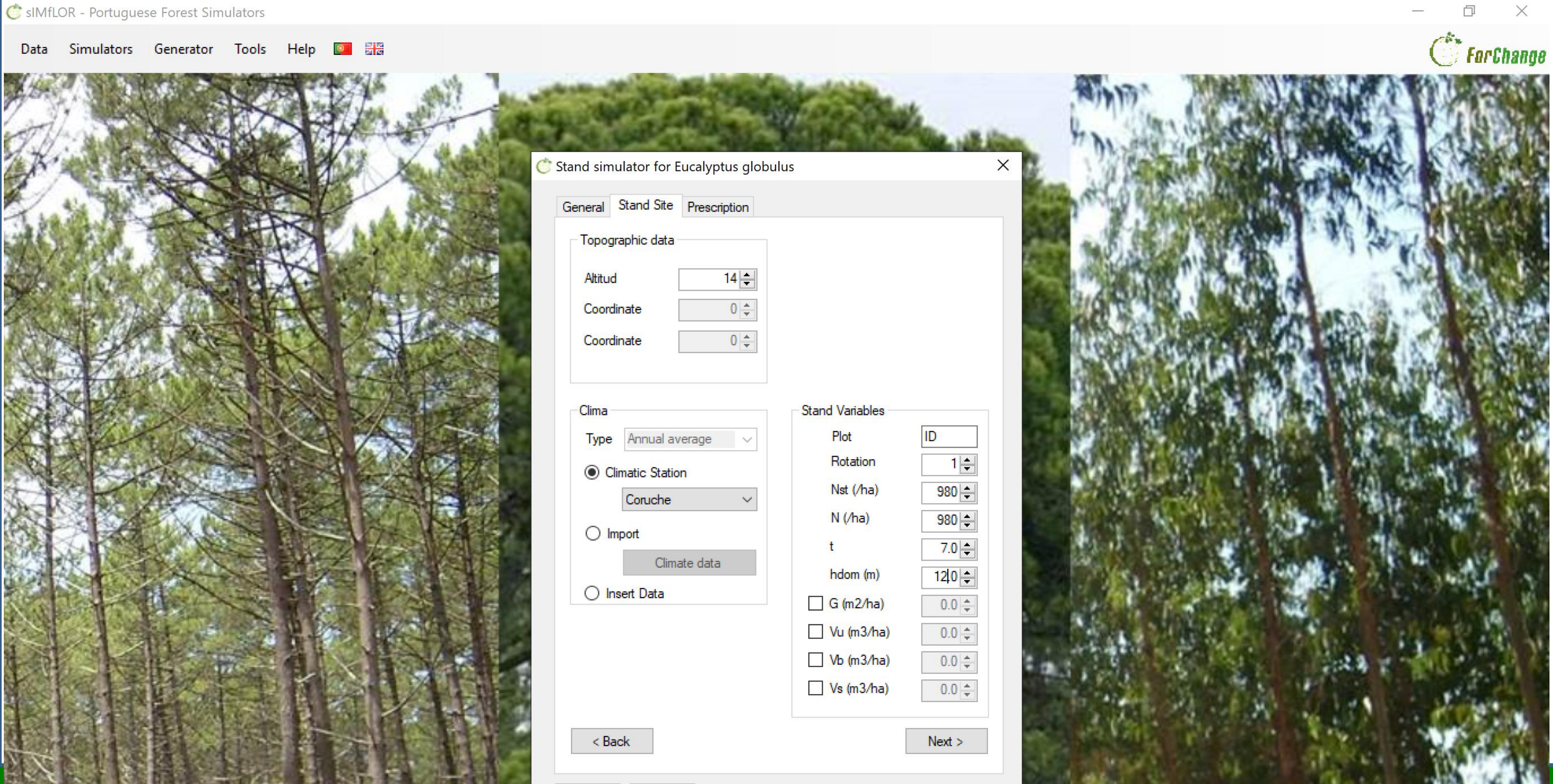

Whole stand models for even-aged stands (WSM-eas)

- Calculus module

→ Calculus modules refer to derived variables, the ones that are computed from other variables at the same point in time:

Computed variable:

$Y_2 = f(S,\ t_2,\ SD_2,\ \text{other stand variables})$

→ Notation

- S = site index or site variables (climate and soil)
- t_i = stand age at time t_i
- X_i = principal stand variable X at time t_i
- SD_i = stand density measure at time t_i
- $i_{X1\text{-}2}$ = growth of variable X in the period between t_1 and t_2
- Y_i = derived stand variable Y at time t_i

▪Whole stand models for even-aged stands (GLOBULUS 3.0)

- **Module calculus: Vdi = (Vu, Vs, dg, Altitude, S, NPL, top_diameter)**

SUM

```
=(F59-H59)*EXP((($K$48/I59)^($D$27+$D$28*$J$48))*($B$27+$B$28*$J$48+$B$29*$I$48/1000+$B$30*(100/($H$48*SQRT($I$48)))+$B$31*(1/(1-($B$48/2000))))
```

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1	2.5	1234	1234	0.6	0.5	0.2	0.0	2.5	0.0	0.2	0.5	0.0	0.2	0.9	0.2
2	6.3	1217	1217	2.5	5.7	1.6	0.3	5.1	2.3	2.2	1.6	0.4	1.0	5.2	1.3
3	9.4	1201	1201	4.8	16.2	4.1	0.5	7.2	11.5	7.1	2.6	1.1	2.0	12.8	3.2
4	12.1	1185	1185	7.1	30.2	7.3	0.9	8.7	25.0	14.1	3.6	2.0	3.0	22.7	5.6
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6	16.2	1154	1154	11.2	63.3	14.3	1.4	11.1	=(F59-H5	31.9	5.1	4.3	4.7	46.1	11.5
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9	20.6	1109	1109	16.3	115.3	24.6	2.2	13.7	108.8	61.9	6.6	8.1	6.9	83.5	20.8
10	21.8	1095	1095	17.8	132.1	27.8	2.4	14.4	125.5	71.9	7.0	9.4	7.5	95.7	23.8
11	22.9	1080	1080	19.1	148.5	30.9	2.6	15.0	141.7	81.7	7.3	10.6	8.1	107.7	26.8
12	23.8	1066	1066	20.4	164.3	33.9	2.8	15.6	157.4	91.3	7.5	11.9	8.6	119.3	29.7

▪Whole stand models for even-aged stands (GLOBULUS 3.0)

▪ **Module calculus: Vdi = (Vu, Vs, dg, Altitude, S, NPL, top_diameter)**

SUM

```
=(F59-H59)*EXP((($K$48/I59)^($D$27+$D$28*$J$48))*($B$27+$B$28*$J$48+$B$29*$I$48/1000+$B$30*(100/($H$48*SQRT($I$48)))+$B$31*(1/(1-($B$48/2000)))))
```

Planted Stand

Number of Days with Rain	Altitude	Number of Days with Frost	Rain	Mean Temperature
114	550	7.00	650.00	15.50

Site Index	Number of Trees at Planting	Rotation	Top Diameter
21.8	1250	0	6.20

Inicialization | Prediction / Growth | Calculus

t	hdom	Nst	N	G	Vu	Vb	Vs	dg	Vdi	Ww	Wl	Wb	Wbr	Wa	Wr
1	2.5	1234	1234	0.6	0.5	0.2	0.0	2.5	0.0	0.2	0.5	0.0	0.2	0.9	0.2
2	6.3	1217	1217	2.5	5.7	1.6	0.3	5.1	2.3	2.2	1.6	0.4	1.0	5.2	1.3
3	9.4	1201	1201	4.8	16.2	4.1	0.5	7.2	11.5	7.1	2.6	1.1	2.0	12.8	3.2
4	12.1	1185	1185	7.1	30.2	7.3	0.9	8.7	25.0	14.1	3.6	2.0	3.0	22.7	5.6
5	14.3	1170	1170	9.2	46.2	10.7	1.2	10.0	40.6	22.6	4.4	3.1	3.9	34.0	8.5
6	16.2	1154	1154	11.2	63.3	14.3	1.4	11.1	=(F59-H59	31.9	5.1	4.3	4.7	46.1	11.5
7	17.9	1139	1139	13.1	80.7	17.8	1.7	12.1	74.5	41.7	5.7	5.5	5.5	58.5	14.6
8	19.3	1124	1124	14.8	98.1	21.3	2.0	12.9	91.8	51.8	6.2	6.8	6.2	71.1	17.7
9	20.6	1109	1109	16.3	115.3	24.6	2.2	13.7	108.8	61.9	6.6	8.1	6.9	83.5	20.8
10	21.8	1095	1095	17.8	132.1	27.8	2.4	14.4	125.5	71.9	7.0	9.4	7.5	95.7	23.8
11	22.9	1080	1080	19.1	148.5	30.9	2.6	15.0	141.7	81.7	7.3	10.6	8.1	107.7	26.8
12	23.8	1066	1066	20.4	164.3	33.9	2.8	15.6	157.4	91.3	7.5	11.9	8.6	119.3	29.7

▪Whole stand models for even-aged stands (GLOBULUS 3.0)

- **Module calculus: Ww = (t, hdom, G, Nst, S, rotation)**

SUM

```
=B$34*$E59^(B$35+B$36*$J$48+B$37*($C59/1000)+B$38*($H$48/1000)+B$39*($A59/1000))*$B59^B$40
```

Planted Stand

Number of Days with Rain	Altitude	Number of Days with Frost	Rain	Mean Temperature
114	550	7.00	650.00	15.50

Site Index	Number of Trees at Planting	Rotation	Top Diameter
21.8	1250	0	6.20

Inicialization | Prediction / Growth | Calculus

t	hdom	Nst	N	G	Vu	Vb	Vs	dg	Vdi	Ww	Wl	Wb	Wbr	Wa	Wr
1	2.5	1234	1234	0.6	0.5	0.2	0.0	2.5	0.0	0.2	0.5	0.0	0.2	0.9	0.2
2	6.3	1217	1217	2.5	5.7	1.6	0.3	5.1	2.3	2.2	1.6	0.4	1.0	5.2	1.3
3	9.4	1201	1201	4.8	16.2	4.1	0.5	7.2	11.5	7.1	2.6	1.1	2.0	12.8	3.2
4	12.1	1185	1185	7.1	30.2	7.3	0.9	8.7	25.0	14.1	3.6	2.0	3.0	22.7	5.6
5	14.3	1170	1170	9.2	46.2	10.7	1.2	10.0	40.6	22.6	4.4	3.1	3.9	34.0	8.5
6	16.2	1154	1154	11.2	63.3	14.3	1.4	11.1	57.4	=B$34*$E	5.1	4.3	4.7	46.1	11.5
7	17.9	1139	1139	13.1	80.7	17.8	1.7	12.1	74.5	41.7	5.7	5.5	5.5	58.5	14.6
8	19.3	1124	1124	14.8	98.1	21.3	2.0	12.9	91.8	51.8	6.2	6.8	6.2	71.1	17.7

▪Whole stand models for even-aged stands (GLOBULUS 3.0)

- **Module inicialization: hdom = f(t, Rain, S)**

SUM | =($B$4+$B$5*$A$48)*($H$48/($B$4+$B$5*$A$48))^((10/A54)^$B$6)

Planted Stand

Number of Days with Rain	Altitude	Number of Days with Frost	Rain	Mean Temperature
114	550	7.00	650.00	15.50

Site Index	Number of Trees at Planting	Rotation	Top Diameter
21.8	1250	0	6.20

Inicialization | Prediction / Growth | Calculus

t	hdom	Nst	N	G	Vu	Vb	Vs	dg	Vdi	Ww	Wl	Wb	Wbr	Wa	Wr
1	=($B$4+$	1234	1234	0.6	0.5	0.2	0.0	2.5	0.0	0.2	0.5	0.0	0.2	0.9	0.2
2	6.3	1217	1217	2.5	5.7	1.6	0.3	5.1	2.3	2.2	1.6	0.4	1.0	5.2	1.3
3	9.4	1201	1201	4.8	16.2	4.1	0.5	7.2	11.5	7.1	2.6	1.1	2.0	12.8	3.2
4	12.1	1185	1185	7.1	30.2	7.3	0.9	8.7	25.0	14.1	3.6	2.0	3.0	22.7	5.6
5	14.3	1170	1170	9.2	46.2	10.7	1.2	10.0	40.6	22.6	4.4	3.1	3.9	34.0	8.5
6	16.2	1154	1154	11.2	63.3	14.3	1.4	11.1	57.4	31.9	5.1	4.3	4.7	46.1	11.5
7	17.9	1139	1139	13.1	80.7	17.8	1.7	12.1	74.5	41.7	5.7	5.5	5.5	58.5	14.6
8	19.3	1124	1124	14.8	98.1	21.3	2.0	12.9	91.8	51.8	6.2	6.8	6.2	71.1	17.7
9	20.6	1109	1109	16.3	115.3	24.6	2.2	13.7	108.8	61.9	6.6	8.1	6.9	83.5	20.8

▪Whole stand models for even-aged stands (GLOBULUS 3.0)

- Module inicialization: G = f(t, Nst, Rain, Altitude, S, NPL, rotation)

SUM

```
=($B$15+$B$16*$A$48)*EXP(-($H$15+$H$16*$H$48+$H$17*100/($H$48*SQRT($I$48))+$H$18*$J$48)*(1/A54)^($E$15+$E$16/(1-($B$48/2000))+$E$17*$J$48+$E$
C54/1000))
```

Planted Stand

Number of Days with Rain	Altitude	Number of Days with Frost	Rain	Mean Temperature
114	550	7.00	650.00	15.50

Site Index	Number of Trees at Planting	Rotation	Top Diameter
21.8	1250	0	6.20

Inicialization | Prediction / Growth | Calculus

t	hdom	Nst	N	G	Vu	Vb	Vs	dg	Vdi	Ww	Wl	Wb	Wbr	Wa	Wr
1	2.5	1234	1234	=($B$15+B	0.5	0.2	0.0	2.5	0.0	0.2	0.5	0.0	0.2	0.9	0.2
2	6.3	1217	1217	2.5	5.7	1.6	0.3	5.1	2.3	2.2	1.6	0.4	1.0	5.2	1.3
3	9.4	1201	1201	4.8	16.2	4.1	0.5	7.2	11.5	7.1	2.6	1.1	2.0	12.8	3.2
4	12.1	1185	1185	7.1	30.2	7.3	0.9	8.7	25.0	14.1	3.6	2.0	3.0	22.7	5.6
5	14.3	1170	1170	9.2	46.2	10.7	1.2	10.0	40.6	22.6	4.4	3.1	3.9	34.0	8.5
6	16.2	1154	1154	11.2	63.3	14.3	1.4	11.1	57.4	31.9	5.1	4.3	4.7	46.1	11.5
7	17.9	1139	1139	13.1	80.7	17.8	1.7	12.1	74.5	41.7	5.7	5.5	5.5	58.5	14.6
8	19.3	1124	1124	14.8	98.1	21.3	2.0	12.9	91.8	51.8	6.2	6.8	6.2	71.1	17.7
9	20.6	1109	1109	16.3	115.3	24.6	2.2	13.7	108.8	61.9	6.6	8.1	6.9	83.5	20.8
10	21.8	1095	1095	17.8	132.1	27.8	2.4	14.4	125.5	71.9	7.0	9.4	7.5	95.7	23.8
11	22.9	1080	1080	19.1	148.5	30.9	2.6	15.0	141.7	81.7	7.3	10.6	8.1	107.7	26.8
12	23.8	1066	1066	20.4	164.3	33.9	2.8	15.6	157.4	91.3	7.5	11.9	8.6	119.3	29.7

Whole stand models for even-aged stands (WSM-eas)

- Stand response to silvicultural treatments

  ➔ Including stand response to silvicultural treatments into the forest models is crucial for the selection of the most efficient management

  ➔ In spite of this importance, there is no established theory and the study of such models is usually made through examples

  ➔ Some examples from Burkhart and Tomé (2012) are presented here as an illustration

Whole stand models for even-aged stands (WSM-eas)

- Stand response to thinning

  → Pienaar and Shiver (1986)

$$\ln G = b_0 + b_1 \frac{1}{t} + b_2 \ln N + b_3 \ln h_{dom} + b_4 \frac{\ln N}{t} + b_5 \frac{\ln h_{dom}}{t} + b_6 \frac{N_t t_t}{N_{at} t}$$

- t_t = plantation age at last thinning
- N = present number of trees per unit area
- N_t = number of trees removed in last thinning
- N_{at} = number of trees remaining after last thinning
- G = basal area per unit area
- t = plantation age
- hdom = dominant height

Whole stand models for even-aged stands (WSM-eas)

- Stand response to thinning

  ➔ Pienaar and Shiver (1986)

  - The term $(N_t t_t / N_{at} t)$ modifies the basal area of unthinned plantations of given age, stems per unit area, and average dominant height to predict the basal area for comparable thinned plantations
  - In the non-logarithmic form of the prediction equation, it is a multiplicative modifier theoretically between 0 and 1
  - For any given age, t, the earlier a thinning of given intensity (N_t / N_{at}) occurs, the larger (closer to 1) the modifier will be
  - If thinnings of different intensities occur the same time ago, so that (t_t / t) and N_{at} are the same, then the modifier will be larger for the less intensive thinning.

Whole stand models for even-aged stands (WSM-eas)

- Stand response to thinning
  - ➔ Pienaar and Shiver (1986)
    - A basal area projection equation was derived from the prediction equation

$$
\begin{aligned}
ln G_2 = ln G_1 + b_1\left(\frac{1}{t_2} - \frac{1}{t_1}\right) + b_2(t_2 - t_1) + b_3\left(1 - \frac{t_1}{t_2}\right) + b_4\left(\frac{1}{t_2^2} - \frac{1}{t_1 t_2}\right) \\
+ b_5 \, ln N_1\left(\frac{1}{t_2} - \frac{1}{t_1}\right) + b_6 \, ln \, h_{dom1}\left(\frac{1}{t_2} - \frac{1}{t_1}\right) + b_7\left(\frac{N_t t_t}{N_{at} t_2} - \frac{N_t t_t}{N_{at} t_1}\right)
\end{aligned}
$$

Whole stand models
with
diameter distributions

Diameter distribution models

- The idea behind diameter distribution models is:

  ➔To start by simulating the growth of some variables (principal variables):

  - dominant height
  - number of trees per ha
  - stand basal area estimation
  - some variables characterizing the diameter distribution such as the minimum diameter, some percentile of the diameter distribution or the variance of diameters (depending on the pdf used for diameter distribution)

Diameter distribution models

- The idea behind diameter distribution models is:

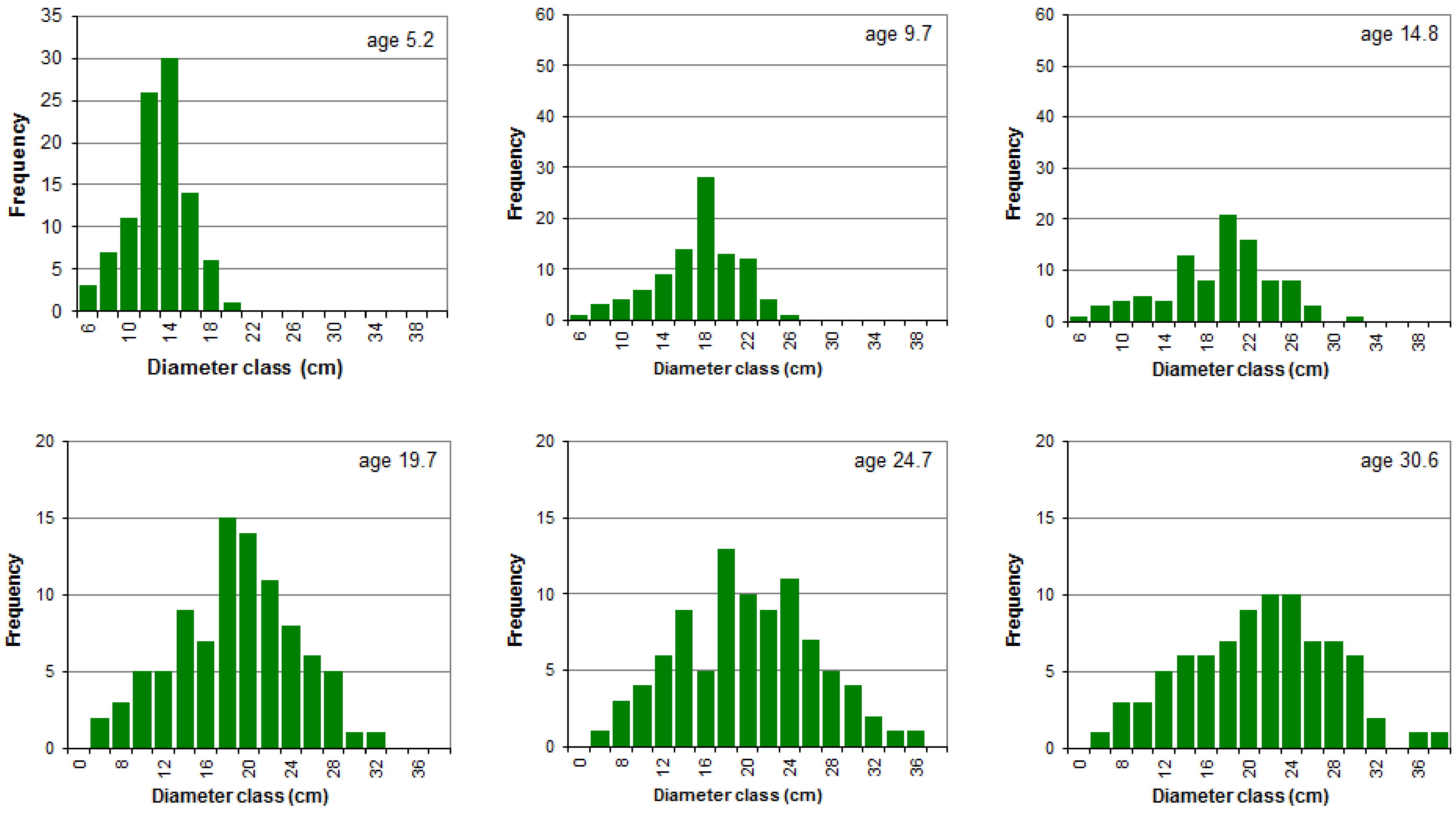
  → to estimate the **distribution of trees by diameter classes** (diameter distribution)

  → Usually the simulation of diameter distribution implies the need to predict other variables, namely **minimum diameter** and some **percentile in the upper part of the distribution**

  → to estimate **stand volume** (total and merchantable) from the **diameter distribution**, by **using tree volume equations**

Diameter distribution models

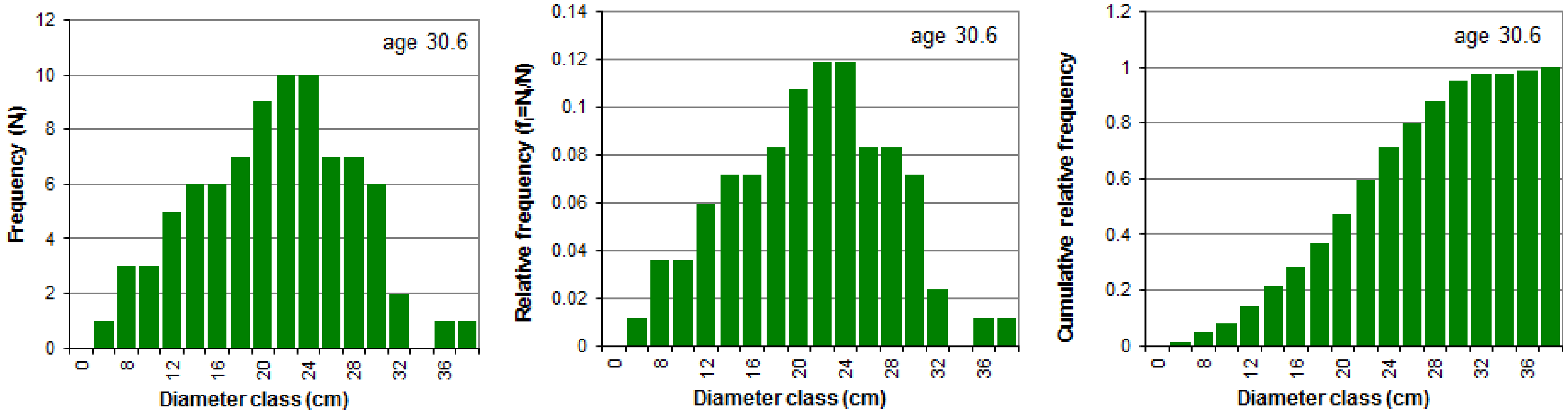
- Diameter distributions of a permanent plot over time



Diameter distribution models

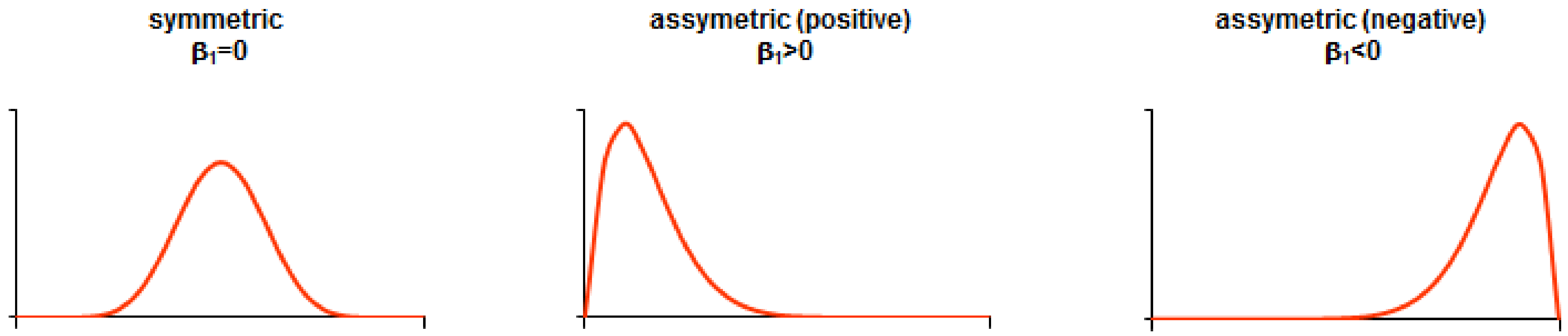
- **Diameter distribution in relative frequencies**
  - ➔ The diameter distributions may be expressed in terms of relative frequencies by expressing the frequency each diameter class (N_i) relative to the total number of trees per ha (N)
- **Diameter distribution in cumulative relative frequencies**
  - ➔ The cumulative relative frequency of a diameter distribution leads to the empirical distribution function

$\Sigma f_i=1$



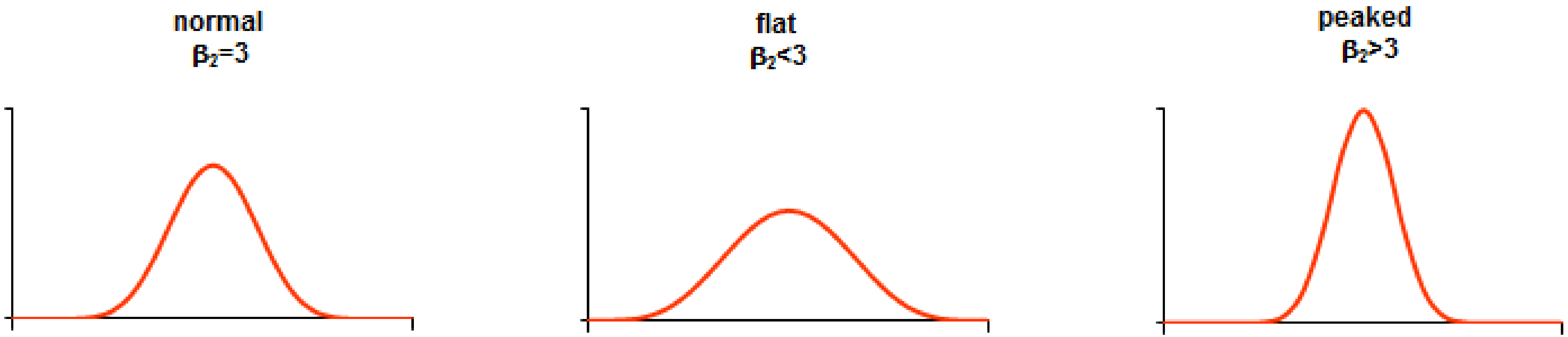
Modelling diameter distributions

- A typical diameter distribution for pure, even-aged stands is **unimodal and slightly skewed**
- Skewness coefficient (β_1) is used to measure the symmetry of a distribution



Modelling diameter distributions

- Diameter distributions for pure, even-aged stands can also be **more or less flat**
- Kurtosis coefficient (β_2) is used to measure flatness or peakdness of a distribution



Modelling diameter distributions

- Diameter distributions can be modelled by a variety of mathematical functions from the probability density functions (pdfs) type
- **Probability density functions** express the relative likelihood for a random variable to take on a given value
- The **probability density function** is non-negative everywhere, and its integral over the entire space is equal to one
- The **probability that the random variable** takes a **value<*x*** is equal to the integral of the pdf from the start to *x*

Probability density functions (pdfs)

- A pdf is described by a mathematical expression that contains parameters
- The values of the parameters give a different shape to the pdf
- For instance, a **Normal distribution**, the most well know pdf, has the following expression

$$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

that includes two parameters, **the mean** (μ) and the **standard deviation** (σ), and is designated by $N(\mu,\sigma)$

Probability density functions (pdfs)

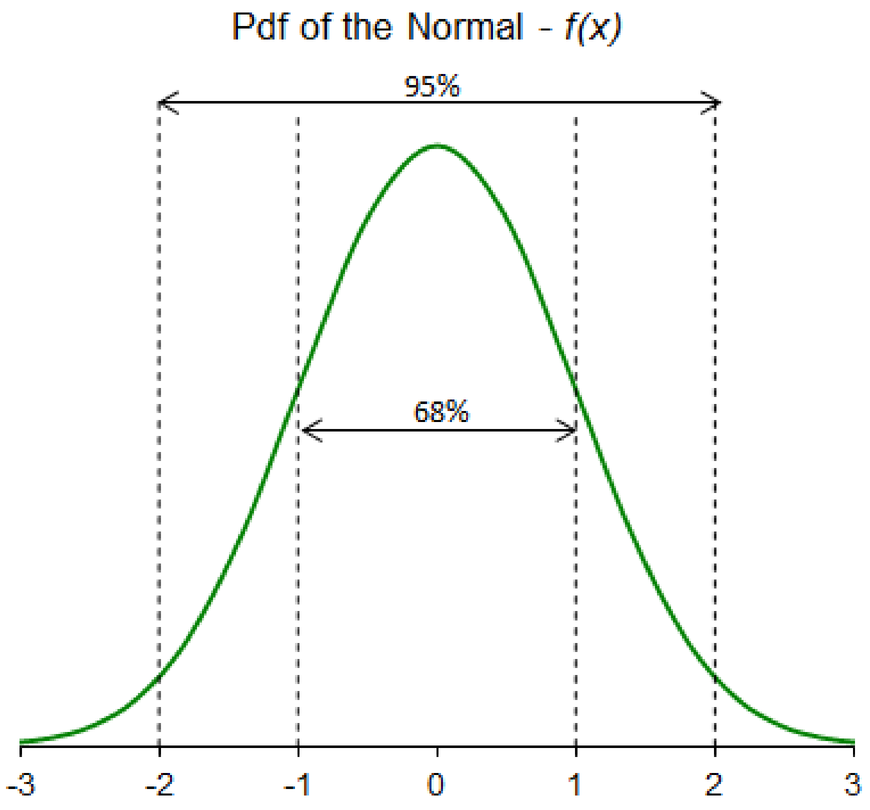
- Integrating the pdf produces the **cumulative distribution function** that, for the Normal distribution is

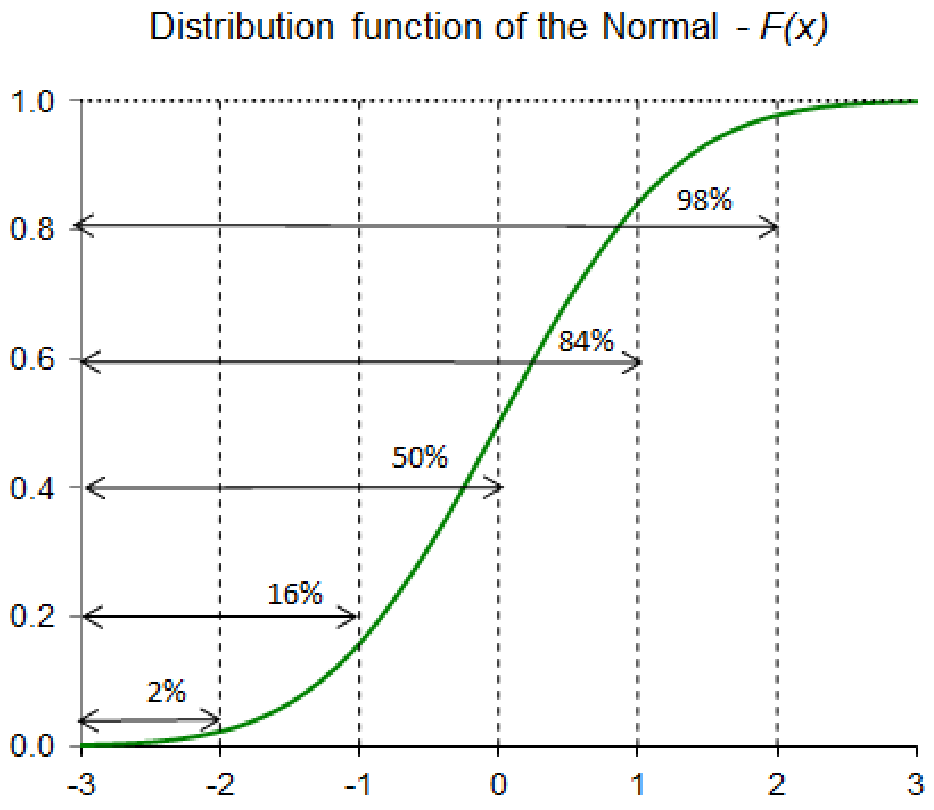
$$F(x;\mu,\sigma)=\text{Prob}(X <= x)=\int_{-\infty}^{x}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{2}}dx$$

- The Normal distribution **is not appropriate to model diameter distributions** because of its symmetry

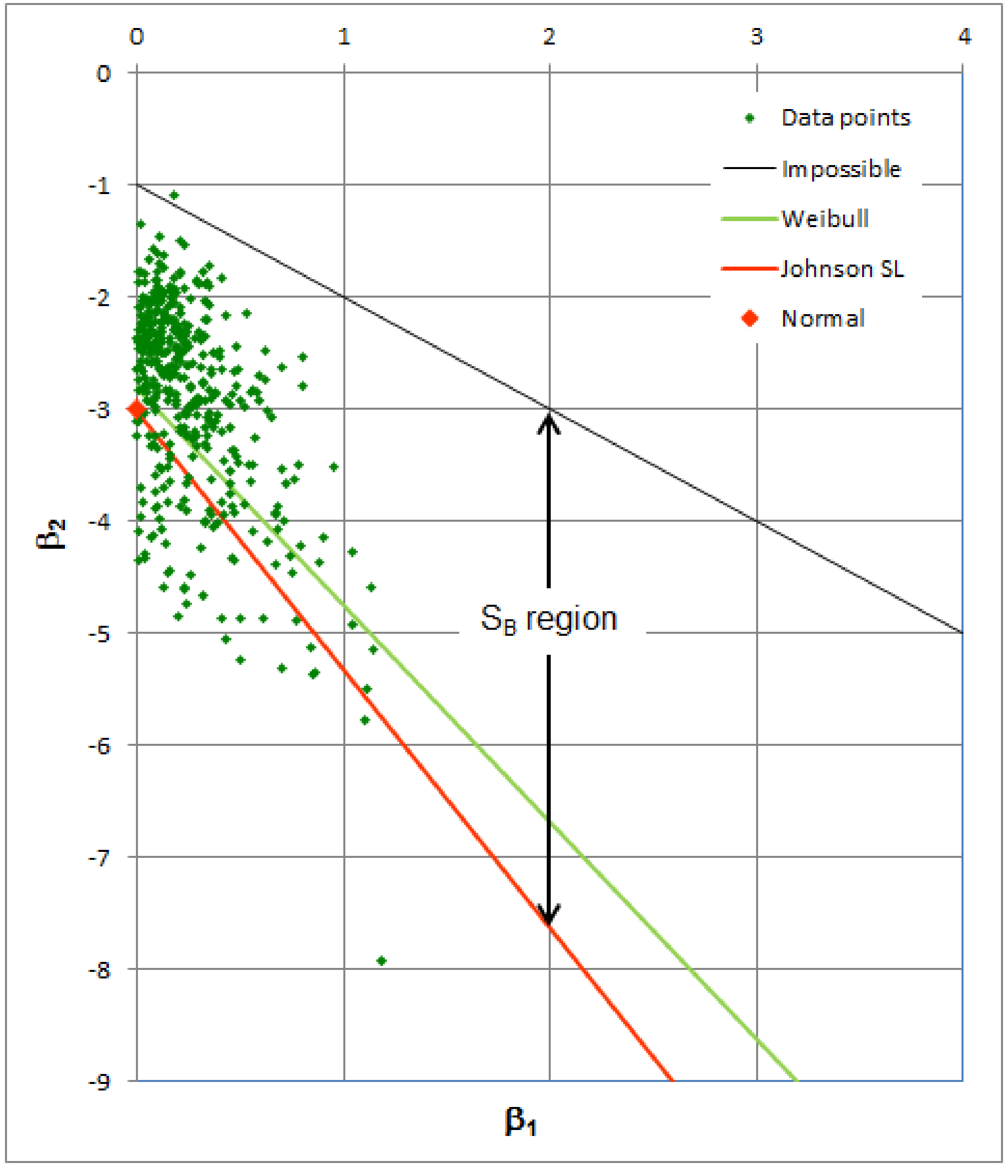
Probability density functions (pdfs)

- The normal distribution is symmetric around the mean (μ)





Probability density functions (pdfs)



- Other pdfs, that can take different values for the pair (β_1,β_2), have been used for diameter distribution modelling

(β_1,β_2) values for the pdfs most used for diameter distribution modeling and for a set of eucalyptus permanent plots

Estimators for β_1 and β_2:

$$\sqrt{b_1} = \sqrt{n}\,\frac{m_3}{m_2^{3/2}}$$

$$b_2 = n\,\frac{m_4}{m_2^2}$$

$$m_j = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^j}{n}, \qquad j = 2,3,4$$

n - number of observations

Weibull pdf

- The Weibull, one of the most used pdfs in diameter distribution modelling, is a three-parameter pdf

$$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] \quad (a <= x < \infty)$$

$$= 0 \quad \text{otherwise}$$

a – location parameter (related to the d_{min})

b – scale parameter (>0)

c – shape parameter (>0; if c>1 implies a inverse J shape; if c=3.6 is close to Normal; c<3.6 is right skewed; if c>3.6 is left skewed)

a+b is close to percentile 63% (P_{63}) of the distribution

Weibull pdf

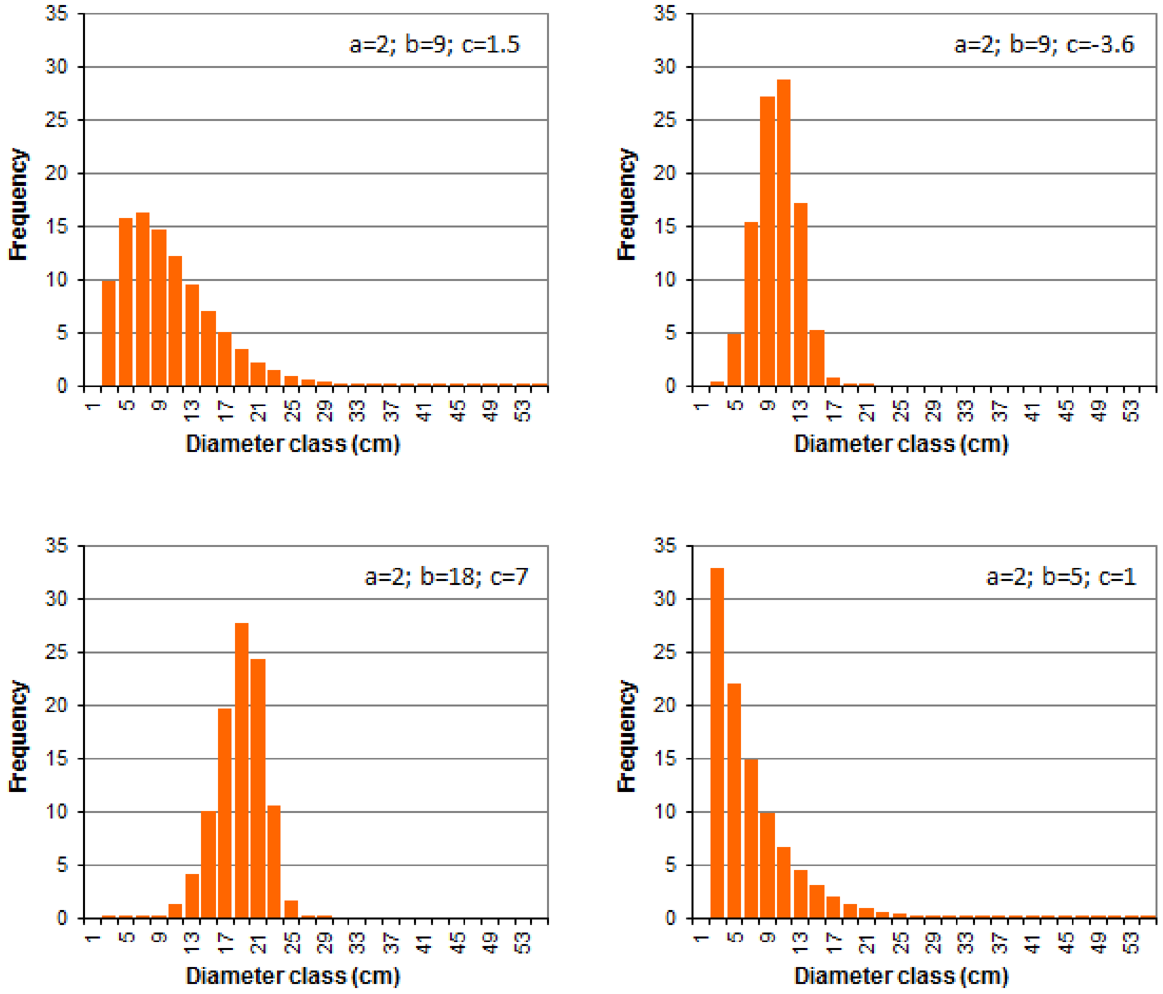
- Integrating the pdf produces the cumulative distribution function for the Weibull distribution

$$F(x) = 1 - exp\left[-\left(\frac{x-a}{b}\right)^c\right] \quad (a <= x < \infty)$$

$$= 0 \quad \text{otherwise}$$

- The Weibull distribution has the advantage of having a closed integral form which makes it very tractable

Weibull distribution - examples



The Johnson's SB system of pdfs

- The system of random variables generated by

$$Z = \gamma + \delta \ ln\left(\frac{X - \varepsilon}{\varepsilon + \lambda - X}\right)$$

$$\varepsilon < X < \varepsilon + \lambda$$

$$-\infty < \gamma < \infty$$

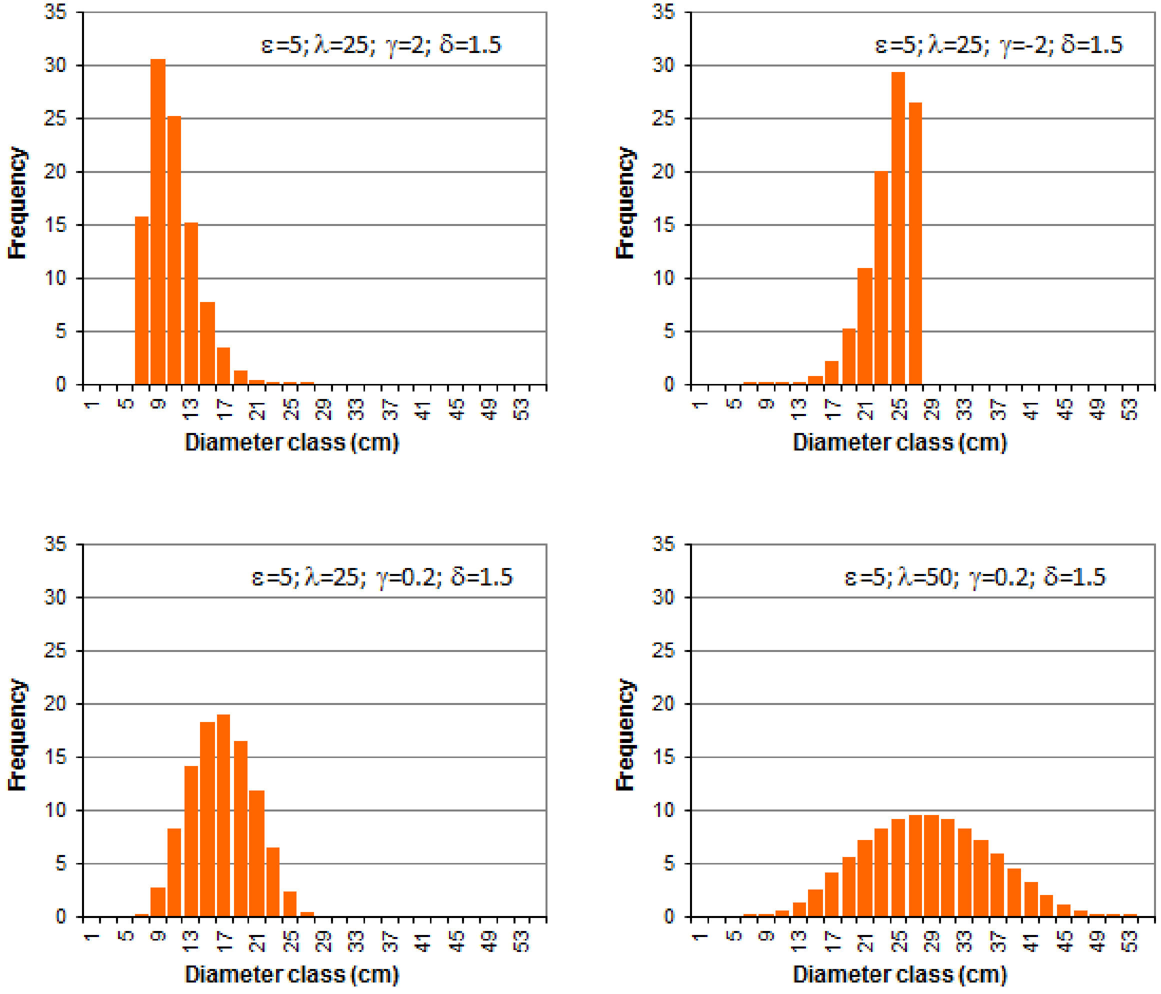
$$-\infty < \varepsilon < \infty$$

$$\lambda > 0$$

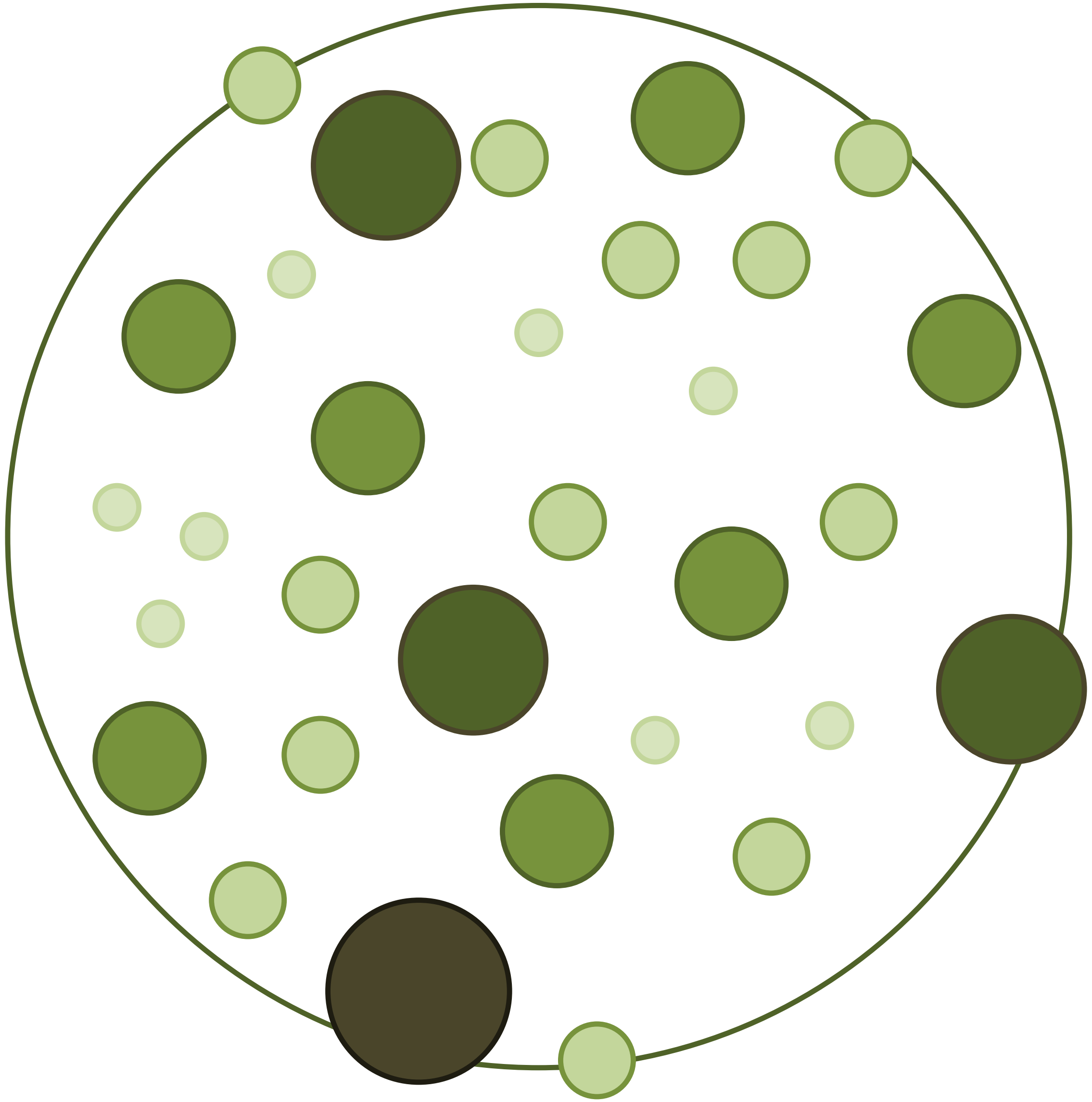
is called the Johnson's SB system of distributions

- It is very flexible and can take several shapes

Johnson's SB system - examples



Diameter distributions

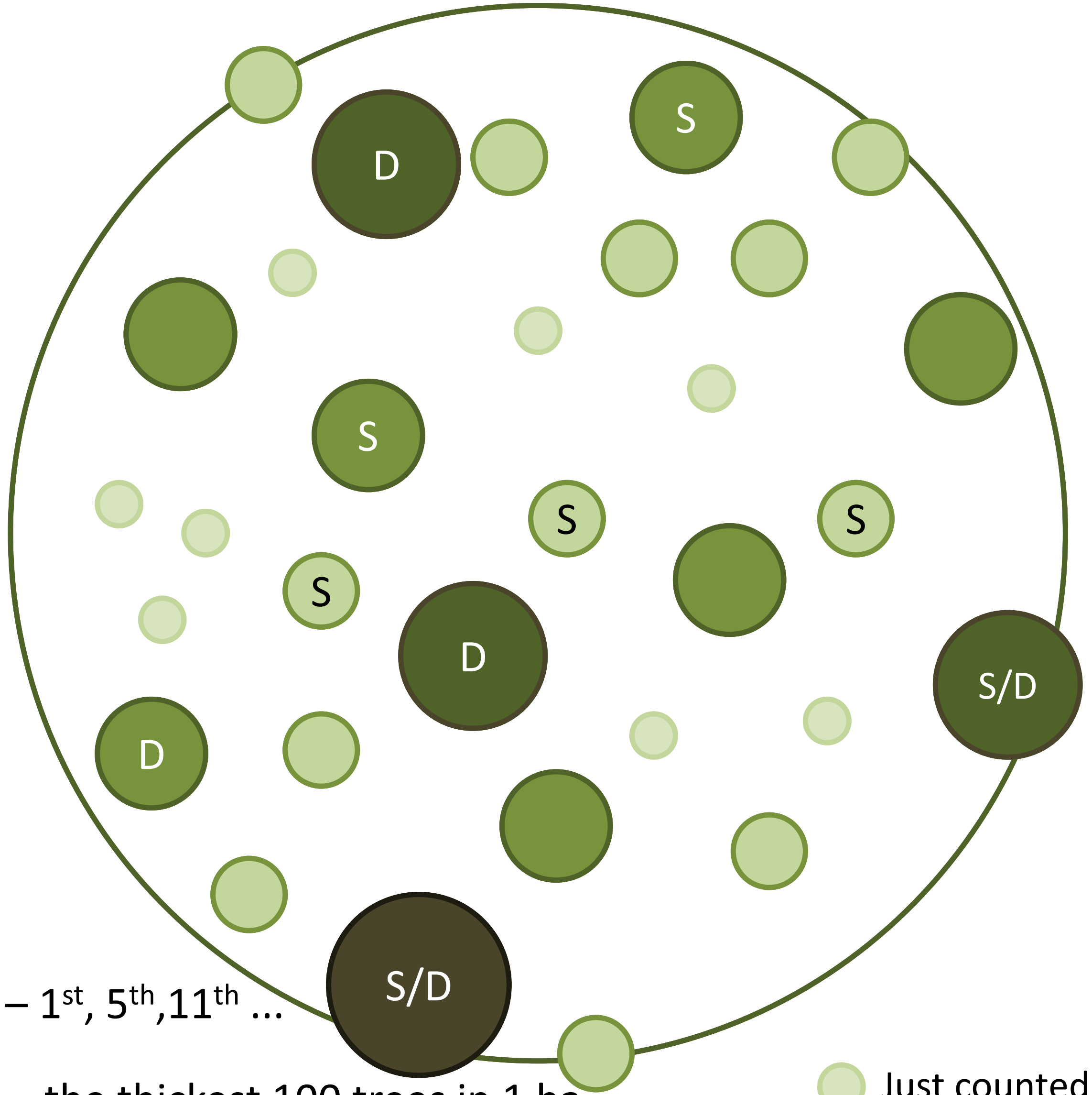


To characterize a plot:

1. Total enumeration

Tree Number	DBH (cm)	g (m2)
1	25.4	0.05067
2	25.4	0.05067
3	26.4	0.05474
4	25.2	0.04988
5	24.1	0.04562
6	21.5	0.03631
7	21.1	0.03497
8	22.1	0.03836
9	19.6	0.03017
10	18.2	0.02602
11	17.1	0.02297
12	14.5	0.01651
13	14.6	0.01674
14	23.5	0.04337
15	24.1	0.04562
16	30.6	0.07354
17	26.0	0.05309
18	23.2	0.04227
19	22.7	0.04047
20	22.7	0.04047
21	25.7	0.05187
22	24.2	0.046
23	11.1	0.00968
	g =	0.92
	G =	18.39999

Diameter distributions



S – 1st, 5th,11th …

D – the thickest 100 trees in 1 ha

Just counted (no sample trees in this class)

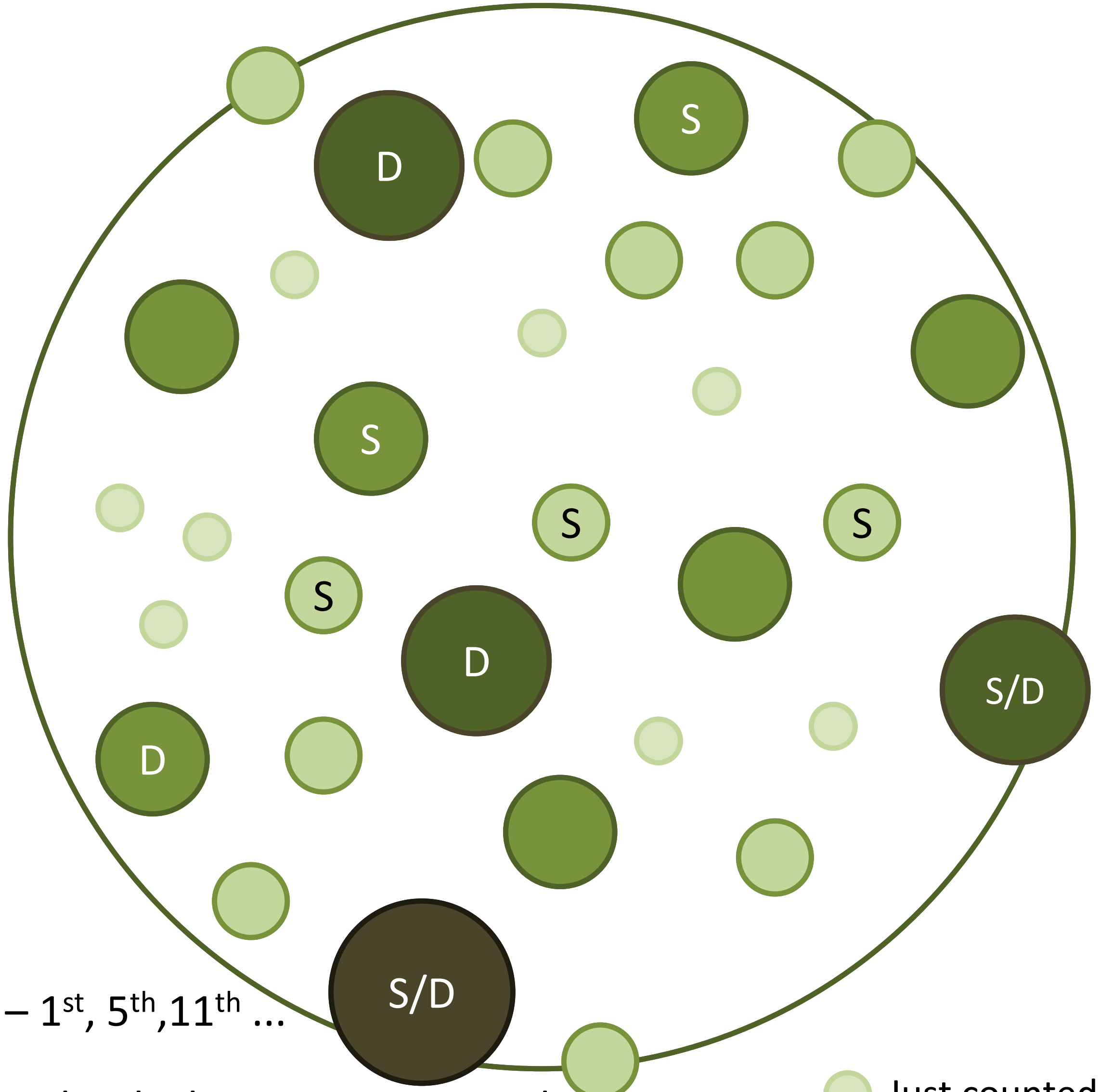
To characterize a plot:

1. Total enumeration
2. Sample trees

Plot: MEDFOR			
d Class	Main Species: Pb		
2.5-7.4	卌	III	
7.5-12.4	卌	卌	II
12.5-17.4	卌	II	
17.5-22.4	III		
22.5-27.4	I		
27.5-32.4			
32.5-37.4			

Tree Number	DBH (cm)	h (m)
23	11.1	9.40
12	14.5	14.5
13	14.6	14.20
11	17.1	
10	18.2	
9	19.6	16.1
7	21.1	
6	21.5	
8	22.1	
19	22.7	
20	22.7	
18	23.2	
14	23.5	
5	24.1	
15	24.1	
22	24.2	
4	25.2	10.5
1	25.4	
2	25.4	14.90
21	25.7	15.2
17	26.0	14.50
3	26.4	17.7
16	30.6	19.2
	hdom =	16.30

Diameter distributions



To characterize a plot:

1. Total enumeration
2. Sample trees

Plot: MEDFOR			
d Class	Main Species: Pb		
2.5-7.4	卌	III	
7.5-12.4	卌	卌	II
12.5-17.4	卌	II	
17.5-22.4	III		
22.5-27.4	I		
27.5-32.4			
32.5-37.4			

d Class	d central	n
7.5-12.4	10	12
12.5-17.4	15	7
17.5-22.4	20	3
22.5-27.4	25	1

S – 1st, 5th, 11th …

D – the thickest 100 trees in 1 ha

Just counted (no sample trees in this class)

Diameter distributions

- ✓ **How are volume and biomass calculated?**
- ✓ Stand volume is estimated from the simulated diameter distribution using a methodology similar to the one used in stand table projection

dcentral (cm)	N (ha^{-1})	h (m)	v (árvore)	V (m^3ha^{-1})
5.0	120	1.4	0.003848	0.5
10.0	539	4.6	0.021739	11.7
15.0	214	6.8	0.060050	12.8
20.0	5	8.4	0.119334	0.6
25.0	0	0.0	0.000000	0.0
30.0	0	0.0	0.000000	0.0
35.0	0	0.0	0.000000	0.0
	878			25.6

$$= (0.021739)\ 539$$

Diameter distributions

- ✓ **How are volume and biomass calculated?**
- ✓ Stand volume is estimated from the simulated diameter distribution using a methodology similar to the one used in stand table projection
- ✓ Predicting the **height** and **volume** of the **average tree** of each d class, it is possible to estimate volume per d class by multiplying tree volume by the number of trees per ha
- ✓ Stand volume is estimated by summing up these values

Diameter distributions

PBRAVO

✓ Estimate accumulated probabilities of trees to occur below each dbh class using the **Weibull parameters** a=2.6, b=9.0, c=3.2 and the stand density of 878

classe d	dcentral (cm)	dsup (cm)	Weibull P(d<=dsup)
[2.5, 7.5]	5	7.5	0.137
]7.5, 12.5]	10	12.5	0.751
]12.5, 17.5]	15	17.5	0.994
]17.5, 22.5]	20	22.5	1.000
]22.5, 27.5]	25	27.5	1.000
]27.5, 32.5]	30	32.5	1.000
]32.5, 37.5]	35	37.5	1.000

$$F(12.5) = 1 - e^{-\left(\frac{12.5 - 2.6}{9.0}\right)^{3.2}}$$

Weibull function to estimates the accumulated frequency of trees per dbh class

Diameter distributions

PBRAVO

- ✓ Multiplying the accumulated probabilities by N and making the differences between consecutive d classes one obtains the diameter distribution and the respective G

dcentral (cm)	Weibull P(d<=dsup)	Nacum (ha^{-1})	N (ha^{-1})	G ($m^2 ha^{-1}$)
5	0.137	120	120	0.2
10	0.751	659	539	4.2
15	0.994	873	214	3.8
20	1.000	878	5	0.2
25	1.000	878	0	0.0
30	1.000	878	0	0.0
35	1.000	878	0	0.0
		Stand	878	8.4

$$= \left(\frac{\pi}{4} (10/100)^2 \right) 539$$

Diameter distributions

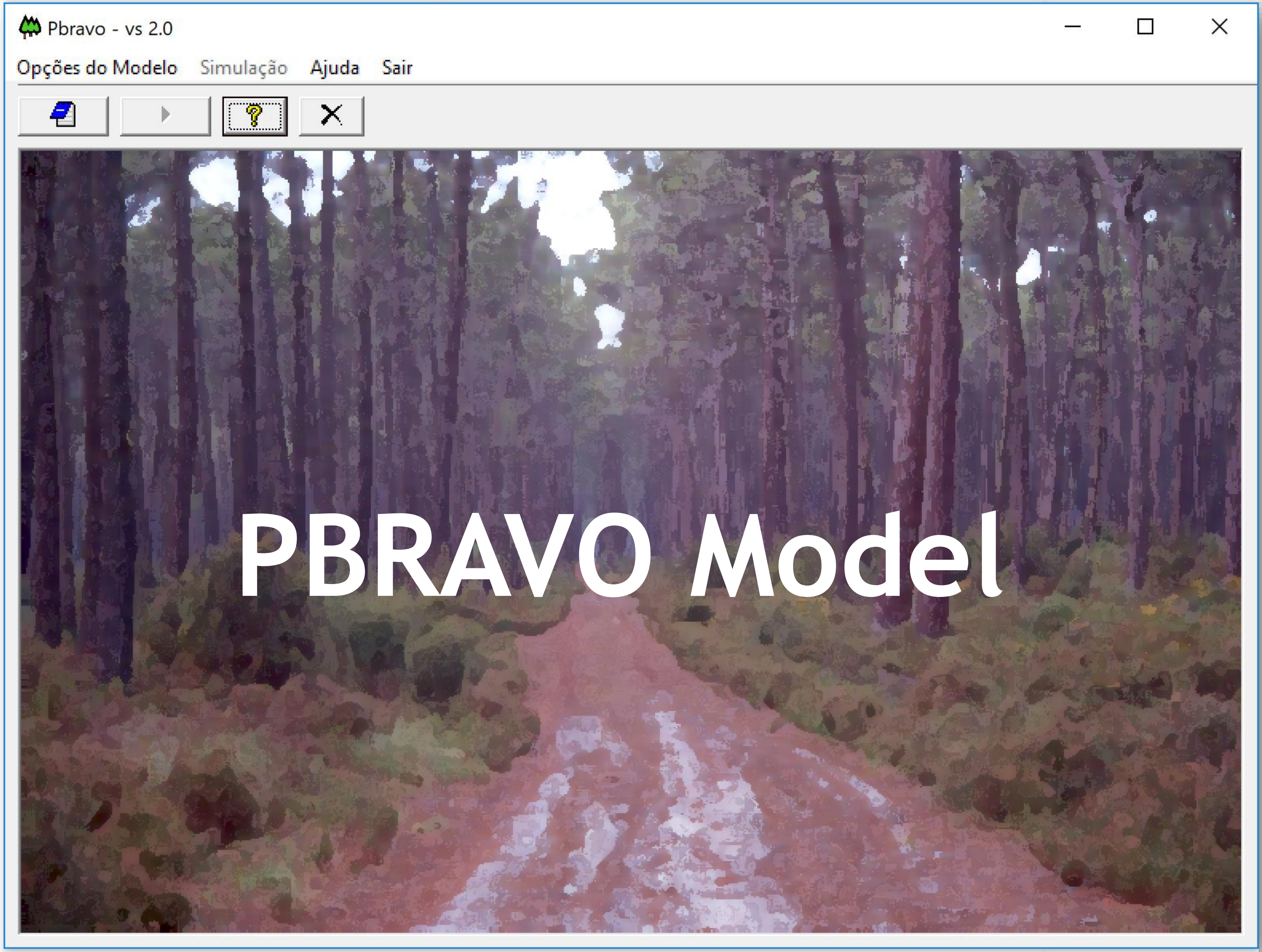
PBRAVO

✓ Predicting the height and volume of the average tree of each d class, we estimate volume per d class by multiplying tree volume by the number of trees per ha.

dcentral (cm)	N (ha^{-1})	h (m)	v (árvore)	V ($m^3 ha^{-1}$)
5.0	120	1.4	0.003848	0.5
10.0	539	4.6	0.021739	11.7
15.0	214	6.8	0.060050	12.8
20.0	5	8.4	0.119334	0.6
25.0	0	0.0	0.000000	0.0
30.0	0	0.0	0.000000	0.0
35.0	0	0.0	0.000000	0.0
	878			25.6

$= (0.021739)\ 539$

Stand volume is estimated by summing up these values


Pbravo - vs 2.0
Opções do Modelo
Simulação
Ajuda
Sair
PBRAVO Model

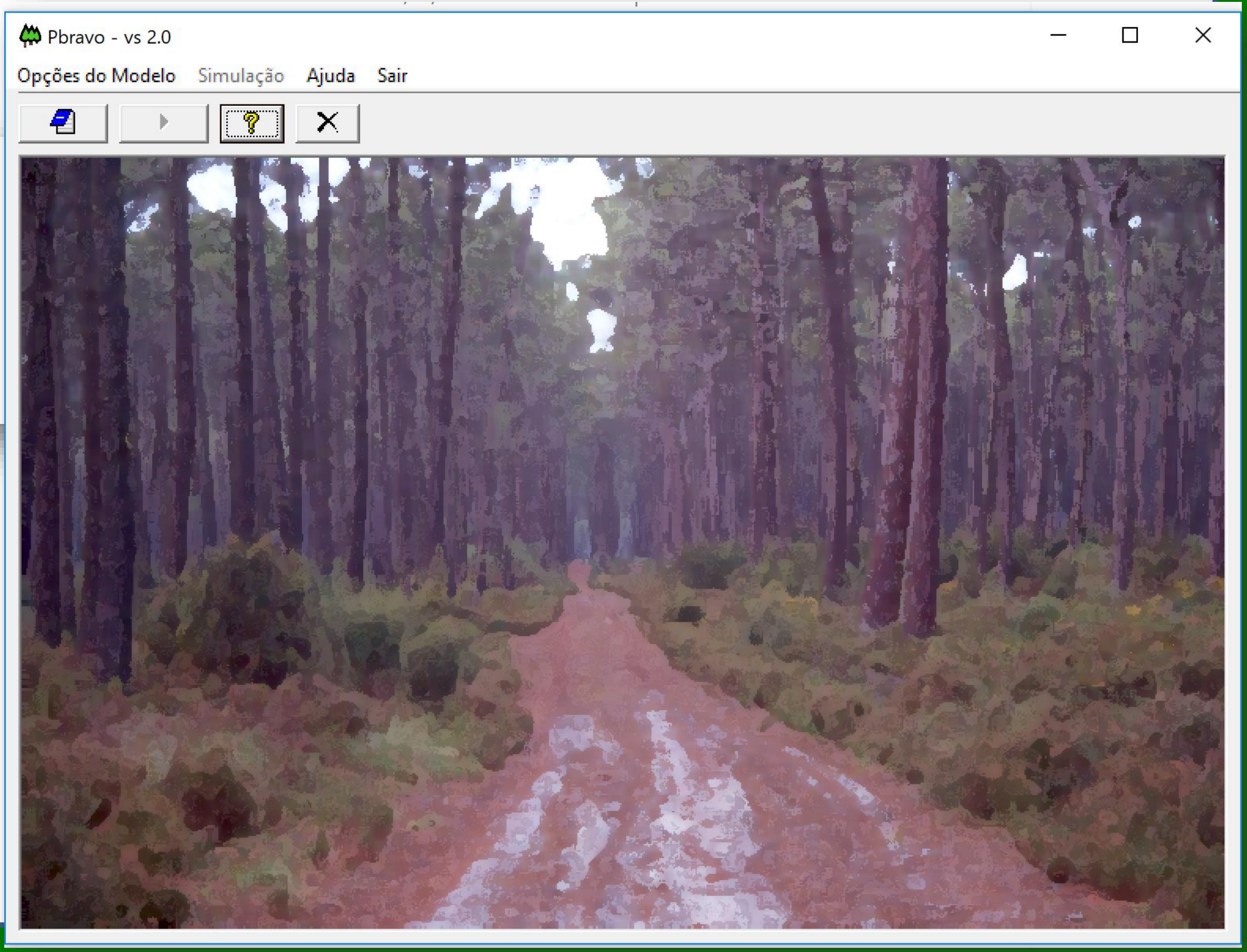
PBRAVO Model

1) Copy the folder PBRAVO from the memory stick

2) Go to PBRAVO\PBRAVO-FPFP

3) Click on the setup (NOT on the SETUP1)

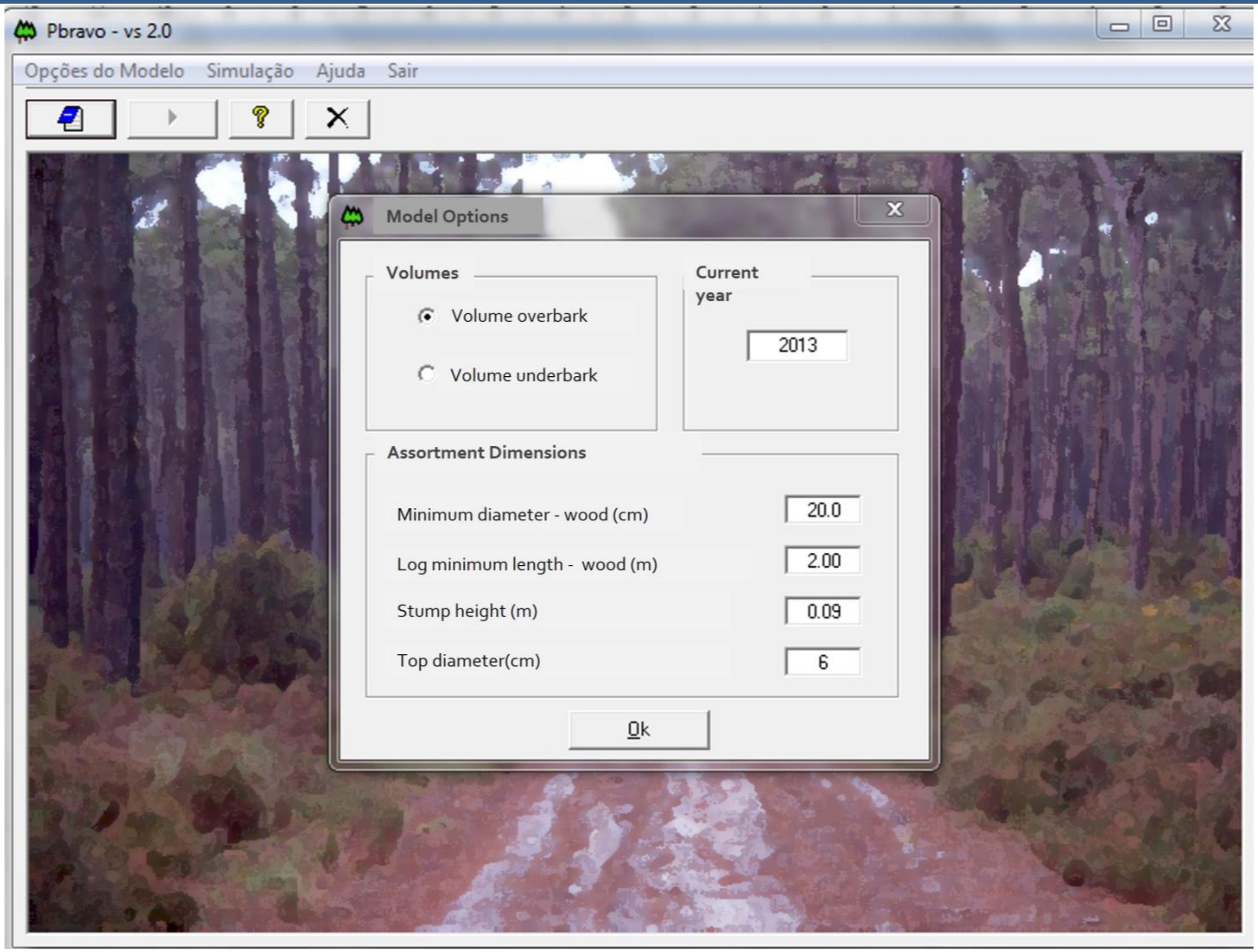
4) After installing the setup, click on the Pbravo application



Read the PBRAVO_Model.pdf

Class Materials \ PowerPoints

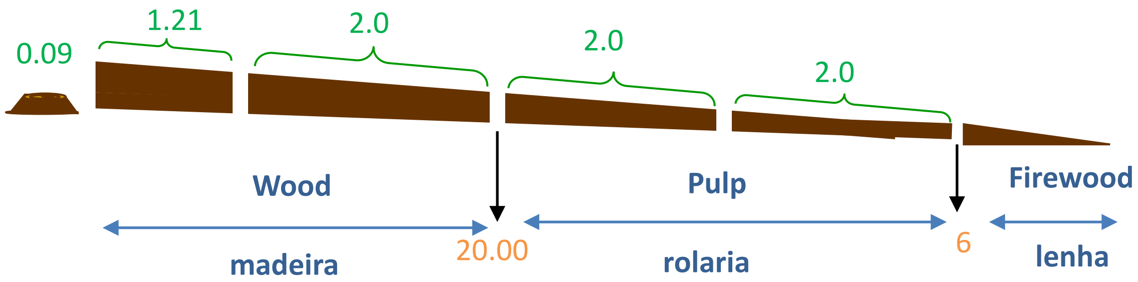
PBRAVO Model



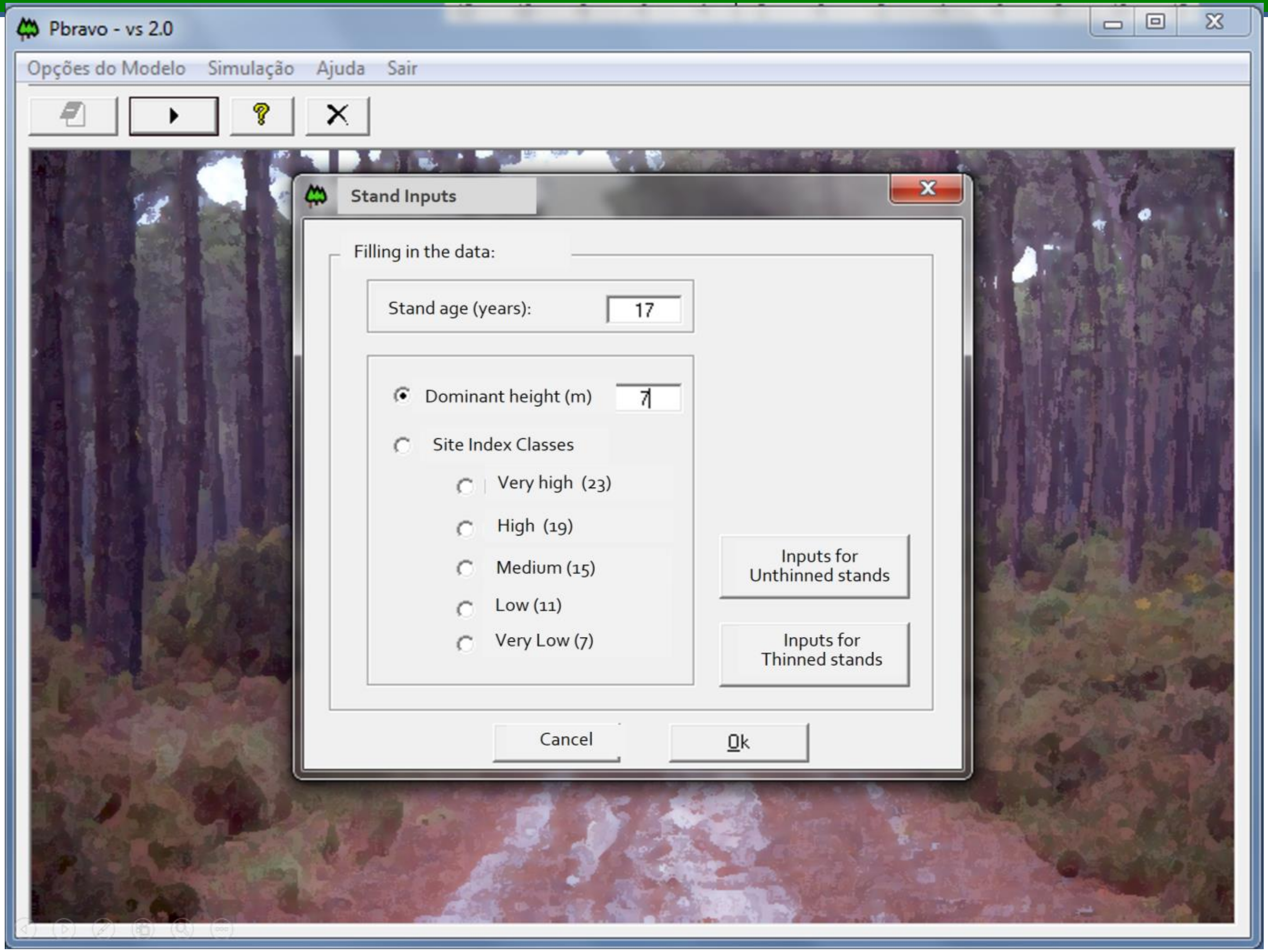
The current year defined should be consistent with stand age

For maritime pine trees bark represents 20-30% of stem volume

The assortment dimensions:



PBRAVO Model



The model can run for:

- stands that have hdom measured-> fill hdom value

- stands with no hdom measured -> fill site index class

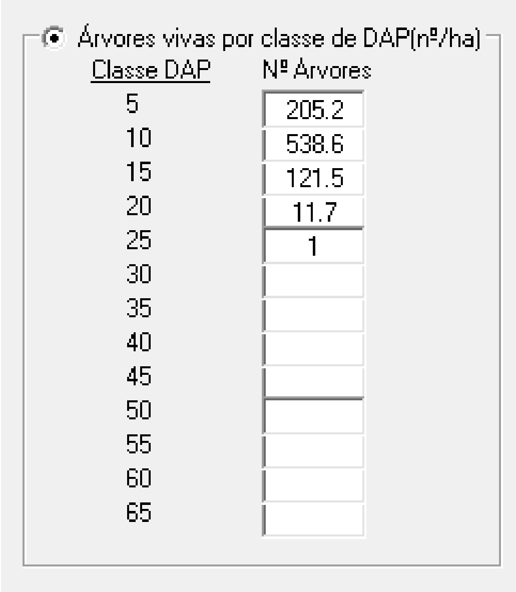
- unthinned stands - young stands (projections are not so good for unthinned old stands)

- For young stands, the user has to provide either the number of standing trees (ha^{-1}) or the number of trees planted (ha^{-1}). In the latest case a mortality model is applied to express the death of trees due to competition in early stages of stand development
- The model runs in 5-year time-steps, stopping at each step allowing to (re-)define the management for the next 5-years period

PBRAVO Model

- For older stands (already thinned), **the user has to provide:**

The number of trees by diameter class



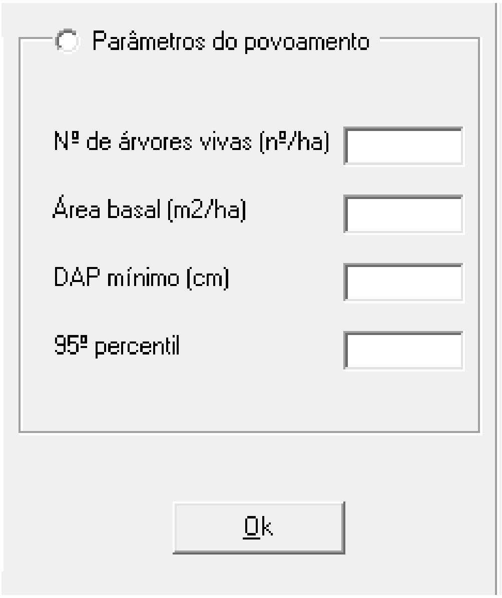
The dbh class value represents the midpoint of the diameter class

If the stand has trees with dbh greater than 67.5 cm these should be grouped under the 65 class

Class 5 includes not only the trees with dbh [2.5, 7.5[but also those with dbh < 2.5 cm

OR

The stand variables to calculate the Weibull parameters (a, b, c)



Whole stand models

- GLOBULUS 3.0
- MODISPINASTER
- PBRAVO
- Other at your choice (in the literature)

❑ Possible topics for the assignments:

- ✓ Choose a model
- ✓ Describe its state and control variables
- ✓ Describe its modules and how silvicultural treatments are taken in to account
- ✓ Make a simulation run with the models and present the results

- Bibliographic review on how:
  - silvicultural tretments are covered in growth models
    - ✓ Thinnings
    - ✓ Fertilizations
    - ✓ ...

The end!!