Ex. 2.33

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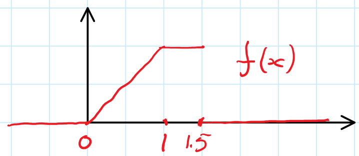
X V.A. VENDAS MENSAIS (100 kg)

a É A CAPACIDADE DE ARMAZENAMENTO $a = 1.5$ ($\times 100$ kg)

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 1, & 1 < x \leq 1.5 \\ 0, & \text{outros valores} \end{cases}$$



a) $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt =$

$$\begin{cases} 0, & x < 0 \\ \int_0^x t\,dt, & 0 \leq x \leq 1 \\ \int_0^1 t\,dt + \int_1^x 1\,dt, & 1 < x \leq 1.5 \\ 1, & x > 1.5 \end{cases} = \begin{cases} 0, & x < 0 \\ \left[\frac{t^2}{2}\right]_0^x = \frac{x^2}{2}, & 0 \leq x \leq 1 \\ \frac{1}{2} + [t]_1^x = \frac{1}{2} + (x-1), & 1 \leq x \leq 1.5 \\ 1, & x \geq 1.5 \end{cases}$$

b) $P(X > 1 \,/\, X > 0.5) = \dfrac{P(X > 1 \cap X > 0.5)}{P(X > 0.5)} = \dfrac{P(X>1)}{P(X>0.5)} = \dfrac{1 - F(1)}{1 - F(0.5)} = \dfrac{1 - \frac{1}{2}}{1 - \frac{\frac{1}{4}}{2}} = \dfrac{\frac{1}{2}}{1 - \frac{1}{8}} = \dfrac{4}{7}$

c) $E[X] = \int_{-\infty}^{+\infty} x\, f(x)\,dx = \int_0^1 x^2\,dx + \int_1^{3/2} x\,dx = \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^{3/2} = \frac{1}{3} + \frac{\frac{9}{4}}{2} - \frac{1}{2} = \frac{1}{3} + \frac{9}{8} - \frac{1}{2} = \frac{8+27-12}{24} = \frac{23}{24}$

$\chi_{0.5}$ É O MENOR VALOR DE x TAL QUE $F(x) = 0.5$

PELA EXP. DE $F(x)$, $F(x) = 0.5$ SSE $x = 1$. ENTÃO $\chi_{0.5} = 1$

d) $Y = 50X - 25$

i) $E[Y] = 50\,E[X] - 25 = 50 \times \frac{23}{24} - 25$

ii) $P(Y \geq 0) = P(X \geq 0.5) = 1 - 0.125 = 0.875$