Applied Operations Research

Solving applications of integer linear programming with Excel

Question 1.

Hiring rangers
The forest service of a country needs to set up sites for district rangers. The forest is made up of a number of districts, as illustrated in the following figure (Figure 1).

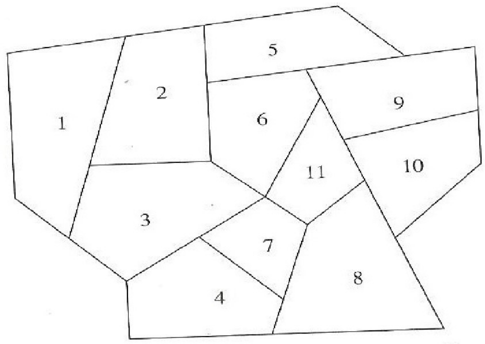

Figure 1: Forest.

A district ranger can be placed in any district and is able to handle the job of protecting the forest resources for future generations and to protect visitors for both its district and any adjacent districts. Consider that two districts are adjacent if they share one point at least. The objective is to minimize the number of district rangers hired.

1. Indicate the districts that a ranger can protect.

2. Formulate and solve the problem.

Question 2.

Sheet cutting planning
A pulp mill cuts sheets of 48 cm $\times$ 96 cm paper into smaller sheets. This company received an order with the characteristics indicated in Table 1. Table 2 and Figure 2 indicate the possible cutting patterns on a 48 cm $\times$ 96 cm sheet. The goal is to determine the cutting plan in order to minimize the number of 48 cm $\times$ 96 cm sheets used. Formulate this problem as an IP model and solve the model.

Type	Sheet of paper Dimensions cm $\times$ cm	Number
1	36 $\times$ 50	800
2	24 $\times$ 36	1300
3	20 $\times$ 60	500
4	18 $\times$ 30	1500

Table 1: Characteristics of the order.

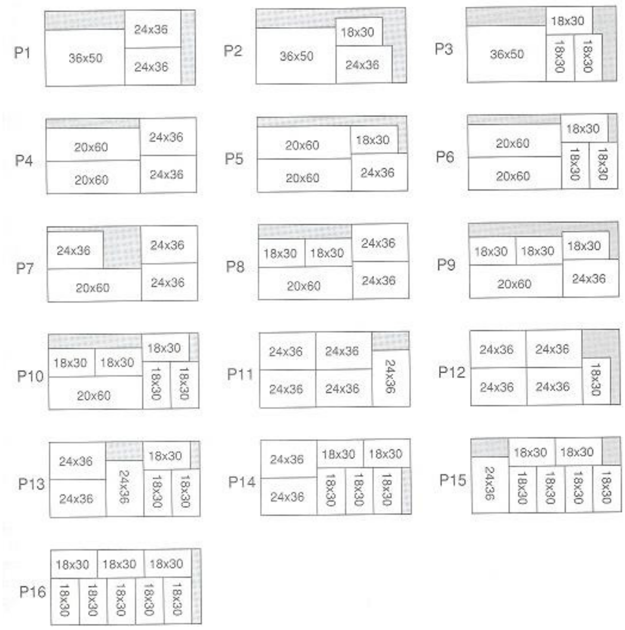

Figure 2: Possible cutting patterns on a 48 cm × 96 cm sheet.

Sheet of paper	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	1	0	2	1	0	3	2	1	0	5	4	3	2	1	0
3	0	0	0	2	2	2	1	1	1	1	0	0	0	0	0	0
4	0	1	3	0	1	3	0	2	3	5	0	1	3	5	6	8

Table 2: Possible cutting patterns on a 48 cm × 96 cm sheet.

Question 3.

Project selection
Consider the following integer LP problem to select a combination of projects from projects 1 through 6:

$$\max\ 4x_1 + 8x_2 + 6x_3 + 3x_4 + 4x_5 + 7x_6 \tag{1}$$

s.t.

$$500x_1 + 700x_2 + 550x_3 + 400x_4 + 450x_5 + 750x_6 \leq 2200 \tag{2}$$

$$10x_1 + 7x_2 + 9x_3 + 9x_4 + 8x_5 + 5x_6 \leq 35 \tag{3}$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \in \{0, 1\}. \tag{4}$$

1. What is the optimal solution for this problem?
2. Formulate constraints and find the optimal solutions for the following conditions:
   (a) Exactly two projects out of projects 2, 3, 4 and 5 must be selected.
   (b) Project 1 may be selected if and only if project 6 is selected.
   (c) If project 2 is selected, projects 4 and 5 must both be selected.
   (d) If project 1 and 2 are both selected, 6 must be selected.

Question 4.

Road network design
Figure 3 shows a road network serving four potential multiple-use forestry projects P_1, P_2, P_3 and P_4. The roads have not yet been built; the map represents only the possible ways the projects can be connected to the existing county road, represented by a bold line.

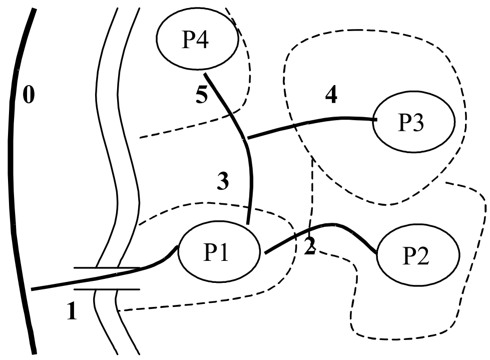

Figure 3: Road network. Dotted lines indicate the project boundaries.

Each project must be done completely or not at all. But not all four projects need to be done. On the other hand, the projects that are done must be connected to the existing county road because they will require some form of road access. The cost of building each road section depends on its length, its topography and necessary work of art. The civil engineers attached to the project have estimated the costs of each road section, shown in Table 3 . These costs are the cumulative discounted cost of building and maintaining the roads over the entire life of the project.

	Section				
	1	2	3	4	5
Cost (10^6 €)	0.8	0.4	0.3	0.2	0.4

Table 3: Cost of building each road section.

On completion, each project is expected to produce the amounts of timber and recreational shown in Table 4. The expected cost of each project is also shown in this table. Again, this is the discounted value of expected costs over the entire life of the project.

	Project			
Result	P_1	P_2	P_3	P_4
Recreational (10^3 rvd)	1	1	2	3
Timber (10^3 m^3)	6	8	13	10
Cost (10^6 €)	0.7	0.1	0.5	0.8

Table 4: Output and cost by project.

The following objectives have been set for the set of projects and for the road network linking them: all camps taken together must be able to accommodate at least 2000 recreational visitor days (rvd) per year; the timber production from all projects should be at least 17000 m^3 per year. These goals must be efficient. That is, we seek the set of projects and the road network that meet the timber and recreational goals at least cost.

1. Formulate this problem as an IP model.
2. Solve the model.
3. P_2 and P_3 project areas share a long common boundary. Because all projects involve building a road and timber harvesting, it may be desirable, for aesthetic reasons, to avoid doing projects on adjacent land areas. Formulate and solve the new model.

4. Consider that the management objectives are to meet the timber production goal of 17000 m^3/y at least or the recreational goal of 2000 rvd/y at least (or both). Formulate and solve the new model.
5. Consider that the management objectives are to meet either the timber production goal of 17000 m^3/y at least or the recreational goal of 2000 rvd/y at least (but not both). Formulate and solve the new model.
6. Consider that the management objectives are if timber production is greater than or equal to 17000 m^3/y, all camps taken together must be able to accommodate at least 2000 rvd/y. Consider also that one project at least should be selected. Formulate and solve the new model.
7. Consider that P_3 can be partially realized (in this case, the project is done using a fraction of the land that could be allocated to the project). Assume that if the project is started, 0.3 million of euros is needed for basic infrastructure. Thereafter, costs increase in proportion to the level of completion. The annual productions of wood and recreation are relative to the project carried out in full, being directly proportional to the fraction of the project carried out. Formulate and solve the new model.

Question 5.

Route selection

A company that sells apples had an overall harvest of 1400 tons of apples. The company has several routes to sell these apples. They can export these apples to foreign countries, sell them to supermarkets, transport them to some companies in the food industry for reprocess, or directly sell them in the local market. If some apples cannot be sold or handled by these routes, the company should dispose all the remaining apples under law regulation. For each of these selling routes, certain operations are needed for preparation. The apples selling and distribution processes for the company is shown in the following figure.

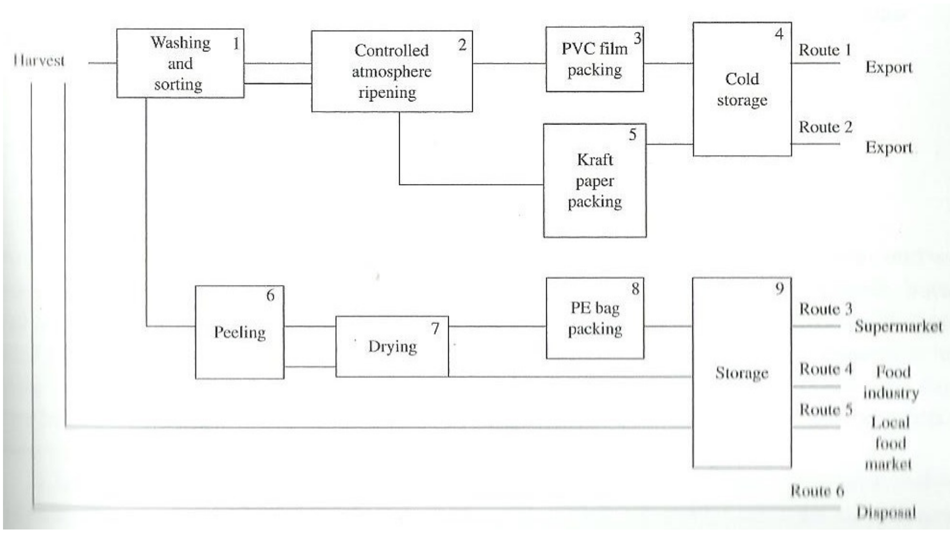

Figure 4: Apples selling and distribution processes.

For each operation, the fixed charge for the equipment (a_i, €/year), the cost for the capacity of operation (b_i, €/t), the processing cost for the apples (d_i, €/t) and the capacity of the operation (LB: LB_i; UB: UB_i) are shown in Table 5.

Table 6 contains the final selling price (S_r), the distribution cost (I_r) and the capacity (LB: L_r; UB: U_r) for each route.

The supply cost is fixed in $200/t$. We do not need to consider transportation cost in this process. The labor has been accounted into the cost in operations. As the whole process is in a short time period, inflation is not considered.

i	a_i (€/year)	b_i (€/t)	d_i (€/t)	LB_i (t/year)	UB_i (t/year)
1	1300	40	42	60	1400
2	7200	70	117	300	1400
3	4200	43	120	300	1400
4	7000	105	80	150	1400
5	2000	95	88	150	1400
6	600	30	32	100	1400
7	1200	20	75	150	1400
8	3600	60	126	120	1400
9	700	26	18	100	1400

Table 5: Operation information.

r	I_r (€/t)	S_r (€/t)	L_r (t/year)	U_r (t/year)
1	70	1200	100	500
2	80	1100	200	400
3	80	1000	25	120
4	90	700	70	150
5	30	400	100	200
6	0	0	0	-

Table 6: Route information.

1. Compute the profit obtained with one ton of apples for each route.
2. Compute the annual equipment cost associated to each route.
3. Find the optimal route selection so that the company can maximize profits.