

Solving applications of Integer Linear Programming with Excel

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ISA - Applied Operations Research - 2020/2021

1. Hiring rangers
2. Sheet cutting planning

Hiring rangers

The problem

The forest service of a country needs to set up sites for district rangers. The forest is made up of a number of districts, as illustrated below. A district ranger can be placed in any district and is able to handle the job of protecting the forest resources for future generations and to protect visitors for both its district and any adjacent districts. Consider that two districts are adjacent if they share one point at least. The objective is to minimize the number of district rangers hired.

The problem

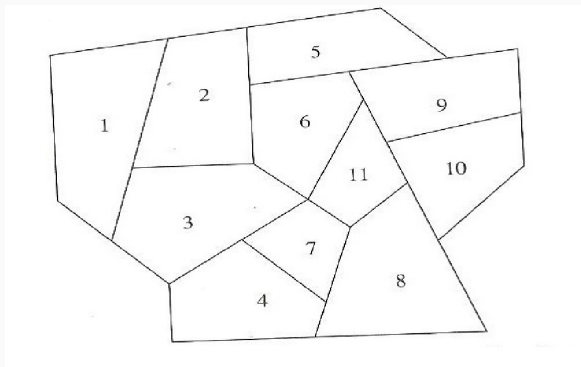


Figure 1: Forest.

Indicate the districts that a ranger can protect

Districts	Districts where the rangers can be placed										
	1	2	3	4	5	6	7	8	9	10	11
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											
11											

Table 1: Districts that a ranger can protect.

Indicate the districts that a ranger can protect

Districts	Districts where the rangers can be placed										
	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	0	0	0	0	0	0	0	0
2	1	1	1	0	1	1	0	0	0	0	0
3	1	1	1	1	0	1	1	0	0	0	1
4	0	0	1	1	0	0	1	1	0	0	0
5	0	1	0	0	1	1	0	0	1	0	0
6	0	1	1	0	1	1	1	0	1	0	1
7	0	0	1	1	0	1	1	1	0	0	1
8	0	0	0	1	0	0	1	1	0	1	1
9	0	0	0	0	1	1	0	0	1	1	1
10	0	0	0	0	0	0	0	1	1	1	1
11	0	0	1	0	0	1	1	1	1	1	1

Table 2: Districts that a ranger can protect.

Formulate and solve the problem

The decision variables are as follows:

Formulate and solve the problem

The decision variables are as follows:

$$x_j = \begin{cases} 1 & \text{if a ranger is placed in district } j \\ 0 & \text{otherwise.} \end{cases}$$

Formulate and solve the problem

number of rangers hired

Formulate and solve the problem

number of rangers hired

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11}$$

Formulate and solve the problem

number of rangers hired

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11}$$

number of rangers that protect district 1

Formulate and solve the problem

number of rangers hired

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11}$$

number of rangers that protect district 1

$$X_1 + X_2 + X_3$$

Formulate and solve the problem

number of rangers hired

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11}$$

number of rangers that protect district 1

$$X_1 + X_2 + X_3$$

number of rangers that protect district 2

Formulate and solve the problem

number of rangers hired

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11}$$

number of rangers that protect district 1

$$X_1 + X_2 + X_3$$

number of rangers that protect district 2

$$X_1 + X_2 + X_3 + X_5 + X_6$$

Formulate and solve the problem

number of rangers hired

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11}$$

number of rangers that protect district 1

$$X_1 + X_2 + X_3$$

number of rangers that protect district 2

$$X_1 + X_2 + X_3 + X_5 + X_6$$

number of rangers that protect district 3

Formulate and solve the problem

number of rangers hired

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11}$$

number of rangers that protect district 1

$$X_1 + X_2 + X_3$$

number of rangers that protect district 2

$$X_1 + X_2 + X_3 + X_5 + X_6$$

number of rangers that protect district 3

$$X_1 + X_2 + X_3 + X_4 + X_6 + X_7 + X_{11}$$

....

Formulate and solve the problem

number of rangers hired

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11}$$

number of rangers that protect district 1

$$X_1 + X_2 + X_3$$

number of rangers that protect district 2

$$X_1 + X_2 + X_3 + X_5 + X_6$$

number of rangers that protect district 3

$$X_1 + X_2 + X_3 + X_4 + X_6 + X_7 + X_{11}$$

....

number of rangers that protect district 11

Formulate and solve the problem

number of rangers hired

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11}$$

number of rangers that protect district 1

$$X_1 + X_2 + X_3$$

number of rangers that protect district 2

$$X_1 + X_2 + X_3 + X_5 + X_6$$

number of rangers that protect district 3

$$X_1 + X_2 + X_3 + X_4 + X_6 + X_7 + X_{11}$$

....

number of rangers that protect district 11

$$X_3 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11}$$

Formulate and solve the problem

$$\min Z = \sum_{j=1}^{11} x_j \quad (1)$$

subject to

Formulate and solve the problem

$$\min Z = \sum_{j=1}^{11} x_j \quad (1)$$

subject to

x_1	$+x_2$	$+x_3$									≥ 1
x_1	$+x_2$	$+x_3$		$+x_5$	$+x_6$						≥ 1
x_1	$+x_2$	$+x_3$	$+x_4$		$+x_6$	$+x_7$				$+x_{11}$	≥ 1
		x_3	$+x_4$			$+x_7$	$+x_8$				≥ 1
	x_2			$+x_5$	$+x_6$			$+x_9$			≥ 1
	x_2	$+x_3$		$+x_5$	$+x_6$	$+x_7$		$+x_9$		$+x_{11}$	≥ 1
		x_3	$+x_4$		$+x_6$	$+x_7$	$+x_8$			$+x_{11}$	≥ 1
			x_4			$+x_7$	$+x_8$		$+x_{10}$	$+x_{11}$	≥ 1
				x_5	$+x_6$			$+x_9$	$+x_{10}$	$+x_{11}$	≥ 1
							x_8	$+x_9$	$+x_{10}$	$+x_{11}$	≥ 1
		x_3			$+x_6$	$+x_7$	$+x_8$	$+x_9$	$+x_{10}$	$+x_{11}$	≥ 1
$x_1,$	$x_2,$	$x_3,$	$x_4,$	$x_5,$	$x_6,$	$x_7,$	$x_8,$	$x_9,$	$x_{10},$	x_{11}	$\in \{0, 1\}$

Formulate and solve the problem

Expression (1) minimizes the number of rangers hired

All constraints before the last ensure that each district is protected

The last constraints state the nature of the variables

Formulate and solve the problem

The screenshot shows Microsoft Excel with the following data in the spreadsheet:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11				
2	0	0	0	0	0	0	0	0	0	0	0				
3															
4												Obj			
5	1	1	1	1	1	1	1	1	1	1	1	0			
6															
7	1	1	1									0	>=		1
8	1	1	1			1	1					0	>=		1
9	1	1	1		1		1	1				0	>=		1
10				1	1			1	1			0	>=		1
11			1			1	1			1		0	>=		1
12			1			1	1	1		1		0	>=		1
13				1	1			1	1			0	>=		1
14					1			1	1		1	0	>=		1
15						1	1			1	1	0	>=		1
16									1	1	1	0	>=		1
17					1			1	1	1	1	0	>=		1
18															
19															

Figure 2: Excel.

Formulate and solve the problem

The screenshot displays the Microsoft Excel interface with a linear programming problem set up in a spreadsheet and the Solver Parameters dialog box open.

Spreadsheet Data:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11			
2	0	0	0	0	0	0	0	0	0	0	0	0		
3														
4														
5	1	1	1	1	1	1	1	1	1	1	1	Obj	0	
6														
7	1	1	1										0 >=	1
8	1	1	1		1	1	1						0 >=	1
9	1	1	1	1	1	1	1	1					0 >=	1
10			1	1	1	1	1	1	1				0 >=	1
11		1	1		1	1	1	1	1				0 >=	1
12		1	1	1	1	1	1	1	1				0 >=	1
13		1	1	1	1	1	1	1	1				0 >=	1
14			1	1	1	1	1	1	1				0 >=	1
15					1	1	1	1	1	1	1		0 >=	1
16								1	1	1	1		0 >=	1
17				1			1	1	1	1	1		0 >=	1
18														
19														
20														
21														
22														
23														
24														
25														

Solver Parameters Dialog Box:

- Definir Objectivo: \$L\$5
- Para: Máximo Mínimo Valor de: 0
- Alterando as Células de Variável: \$A\$2:\$K\$2
- Sujeito às Restrições:
 - \$A\$2:\$K\$2 = binário
 - \$L\$7:\$L\$17 >= \$M\$7:\$M\$17
- Tígnar Não Negativas Variáveis Não Constringidas
- Sêlec. Método Resolução: LP Simplex
- Método de Resolução: Seleccione o motor GRG Não Linear para problemas não lineares uniformes do Solver. Seleccione motor LP Simplex para problemas lineares do Solver, e seleccione o motor Evolutionary para problemas não uniformes do Solver.

Figure 3: Excel.

Formulate and solve the problem

The screenshot shows Microsoft Excel with a linear programming problem set up in a spreadsheet. The Solver Results dialog box is open, indicating that a solution has been found.

Spreadsheet Data:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11											
2	0	1	1	0	0	0	0	0	0	0	0	1										
3																						
4																						
5																						
6																						
7																						
8																						
9																						
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13																						
14																						
15																						
16																						
17																						
18																						
19																						
20																						
21																						
22																						

Solver Results Dialog Box:

Resultados do Solver

O Solver encontrou uma solução. Todas as restrições e condições de otimização foram satisfeitas.

Manter Solução do Solver Restaurar Valores Originais

Regressar ao Diálogo de Parâmetros do Solver Relatórios de Detecção

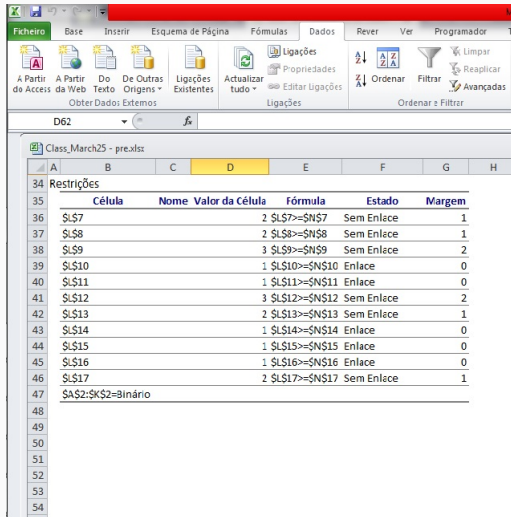
OK Cancelar Guardar Cenário...

O Solver encontrou uma solução. Todas as restrições e condições de otimização foram satisfeitas.

Quando é utilizado o motor GRG, o Solver encontrou pelo menos uma solução ideal local. Quando é utilizado o IP Simplex, significa que o Solver encontrou uma solução ideal global.

Figure 4: Excel.

Formulate and solve the problem



The screenshot shows the Microsoft Excel interface with the 'Dados' (Data) tab selected. The active cell is D62. The spreadsheet contains a table of constraints for a linear programming problem. The table is titled 'Restrições' and lists constraints for variables \$L\$7 through \$L\$17, including their respective formulas and states.

	Célula	Nome	Valor da Célula	Fórmula	Estado	Margem
34	Restrições					
35						
36	\$L\$7		2	\$L\$7>=\$N\$7	Sem Enlace	1
37	\$L\$8		2	\$L\$8>=\$N\$8	Sem Enlace	1
38	\$L\$9		3	\$L\$9>=\$N\$9	Sem Enlace	2
39	\$L\$10		1	\$L\$10>=\$N\$10	Enlace	0
40	\$L\$11		1	\$L\$11>=\$N\$11	Enlace	0
41	\$L\$12		3	\$L\$12>=\$N\$12	Sem Enlace	2
42	\$L\$13		2	\$L\$13>=\$N\$13	Sem Enlace	1
43	\$L\$14		1	\$L\$14>=\$N\$14	Enlace	0
44	\$L\$15		1	\$L\$15>=\$N\$15	Enlace	0
45	\$L\$16		1	\$L\$16>=\$N\$16	Enlace	0
46	\$L\$17		2	\$L\$17>=\$N\$17	Sem Enlace	1
47	\$A\$2:\$K\$2=Binário					
48						
49						
50						
51						
52						
53						
54						

Figure 5: Excel.

Formulate and solve the problem

The optimal solution is obtained with three rangers, placed in districts 2, 3 and 11.

This problem has alternative optimal solutions. Can you list some?

Sheet cutting planning

The problem

A pulp mill cuts sheets of 48 cm \times 96 cm paper into smaller sheets. This company received an order with the characteristics indicated in Table 3. Figure 6 and Table 4 indicate the possible cutting patterns on a 48 cm \times 96 cm sheet. The goal is to determine the cutting plan in order to minimize the number of 48 cm \times 96 cm sheets used.

The problem

Sheet of paper		
Type	Dimensions cm × cm	Number
1	36 × 50	800
2	24 × 36	1300
3	20 × 60	500
4	18 × 30	1500

Table 3: Characteristics of the order.

The problem

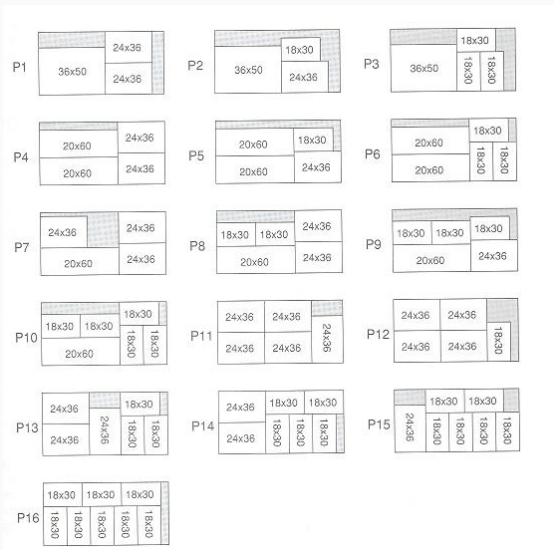


Figure 6: Possible cutting patterns on a 48 cm x 96 cm sheet.

The problem

Sheet of paper	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	1	0	2	1	0	3	2	1	0	5	4	3	2	1	0
3	0	0	0	2	2	2	1	1	1	1	0	0	0	0	0	0
4	0	1	3	0	1	3	0	2	3	5	0	1	3	5	6	8

Table 4: Possible cutting patterns on a 48 cm \times 96 cm sheet.

Formulate this problem as an IP model and solve the model

The decision variables are as follows:

Formulate this problem as an IP model and solve the model

The decision variables are as follows:

x_j – number of 48 cm \times 96 cm sheets assigned to cutting pattern P_j , $j = 1, \dots, 16$.

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm \times 96 cm

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm \times 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm \times 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm × 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

$$x_1 + x_2 + x_3$$

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm × 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

$$x_1 + x_2 + x_3$$

no. of sheets of type 2

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm × 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

$$x_1 + x_2 + x_3$$

no. of sheets of type 2

$$2x_1 + x_2 + 2x_4 + x_5 + 3x_7 + 2x_8 + x_9 + 5x_{11} + 4x_{12} + 3x_{13} + 2x_{14} + x_{15}$$

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm × 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

$$x_1 + x_2 + x_3$$

no. of sheets of type 2

$$2x_1 + x_2 + 2x_4 + x_5 + 3x_7 + 2x_8 + x_9 + 5x_{11} + 4x_{12} + 3x_{13} + 2x_{14} + x_{15}$$

no. of sheets of type 3

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm × 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

$$x_1 + x_2 + x_3$$

no. of sheets of type 2

$$2x_1 + x_2 + 2x_4 + x_5 + 3x_7 + 2x_8 + x_9 + 5x_{11} + 4x_{12} + 3x_{13} + 2x_{14} + x_{15}$$

no. of sheets of type 3

$$2x_4 + 2x_5 + 2x_6 + x_7 + x_8 + x_9 + x_{10}$$

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm × 96 cm

$$Z = \sum_{i=1}^{16} X_i$$

no. of sheets of type 1

$$X_1 + X_2 + X_3$$

no. of sheets of type 2

$$2X_1 + X_2 + 2X_4 + X_5 + 3X_7 + 2X_8 + X_9 + 5X_{11} + 4X_{12} + 3X_{13} + 2X_{14} + X_{15}$$

no. of sheets of type 3

$$2X_4 + 2X_5 + 2X_6 + X_7 + X_8 + X_9 + X_{10}$$

no. of sheets of type 4

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm × 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

$$x_1 + x_2 + x_3$$

no. of sheets of type 2

$$2x_1 + x_2 + 2x_4 + x_5 + 3x_7 + 2x_8 + x_9 + 5x_{11} + 4x_{12} + 3x_{13} + 2x_{14} + x_{15}$$

no. of sheets of type 3

$$2x_4 + 2x_5 + 2x_6 + x_7 + x_8 + x_9 + x_{10}$$

no. of sheets of type 4

$$x_2 + 3x_3 + x_5 + 3x_6 + 2x_8 + 3x_9 + 5x_{10} + x_{12} + 3x_{13} + 5x_{14} + 6x_{15} + 8x_{16}$$

Formulate this problem as an IP model and solve the model

$$\min Z = \sum_{i=1}^{16} x_i$$

Formulate this problem as an IP model and solve the model

The first expression minimizes the number of 48 cm \times 96 cm sheets used.

All constraints before the last ensure that the characteristics of the order are satisfied.

The last constraints state the integer requirements on the variables.

Formulate this problem as an IP model and solve the model

Microsoft Excel (A Activação do Produto Falhou)

Ficheiro Base Inserir Esquema de Página Fórmulas Dados Reverter Ver Programador Team

Obter Dados Externos A Partir do Access A Partir da Web Do Texto De Outras Origens * Ligações Existentes Atualizar tudo * Editar Ligações Ligações

Ordenar e Filtrar Ordenar Filtrar Avançadas Limpar Reaplicar

Textos para colunas Remover Duplicados de Dados Ferramentas de Dados Validação Consolidar Análise de Hipóteses

Agrupar Desagrupar Subtotal Destaque

Mostrar Detalhes Ocultar Detalhes Solvers Análise

Q21

Class_March25 - pre.xlsx

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16				
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
3																				
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	Obj		
5	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	=	800	
6	2	1	0	2	1	0	3	2	1	0	5	4	3	2	1	0	0	=	1300	
7	0	0	0	2	2	2	1	1	1	1	0	0	0	0	0	0	0	=	500	
8	0	1	3	0	1	3	0	2	3	5	0	1	3	5	6	8	0	=	1500	
9																				
10																				
11																				

Figure 7: Excel.

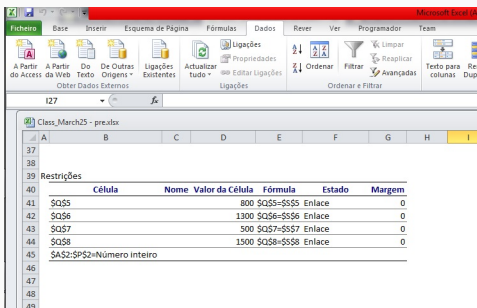
Formulate this problem as an IP model and solve the model

The optimal number of 48 cm \times 96 cm sheets used is 1088.

Number of 48 cm \times 96 cm sheets															
P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}
642	4	154	1	0	249	0	0	0	0	1	0	0	1	3	33

Table 5: Optimal solution.

Formulate this problem as an IP model and solve the model



The screenshot shows the Microsoft Excel interface with the 'Fórmulas' ribbon selected. The active sheet is 'Class_March25 - pre.xlsx'. The table below is located in the range A40:I45.

	Célula	Nome	Valor da Célula	Fórmula	Estado	Margem
40						
41	\$Q\$5		800	\$Q\$5=\$S\$5	Enlace	0
42	\$Q\$6		1300	\$Q\$6=\$S\$6	Enlace	0
43	\$Q\$7		500	\$Q\$7=\$S\$7	Enlace	0
44	\$Q\$8		1500	\$Q\$8=\$S\$8	Enlace	0
45	\$A\$2:\$P\$2=Número inteiro					

Figure 8: Excel.

Exam1_2017.pdf Exercise 1 in Extra Support Material -
Integer Linear Programming from 2019/2020.

Bom estudo!