Solving applications of Integer Linear Programming with Excel

Isabel Martins

ISA - Applied Operations Research - 2020/2021

Outline

1. Hiring rangers
2. Sheet cutting planning

Hiring rangers

The problem

The forest service of a country needs to set up sites for district rangers. The forest is made up of a number of districts, as illustrated below. A district ranger can be placed in any district and is able to handle the job of protecting the forest resources for future generations and to protect visitors for both its district and any adjacent districts. Consider that two districts are adjacent if they share one point at least. The objective is to minimize the number of district rangers hired.

The problem

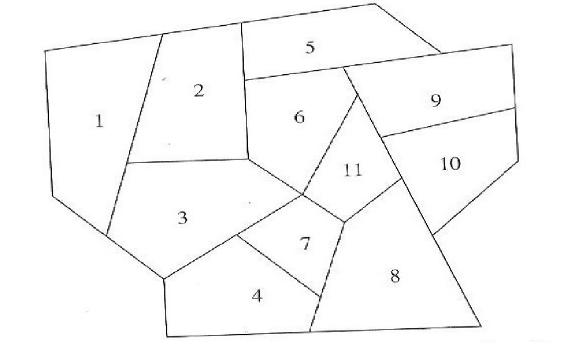


Figure 1: Forest.

Indicate the districts that a ranger can protect

Districts	Districts where the rangers can be placed										
	1	2	3	4	5	6	7	8	9	10	11
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											
11											

Table 1: Districts that a ranger can protect.

Indicate the districts that a ranger can protect

Districts	Districts where the rangers can be placed										
	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	0	0	0	0	0	0	0	0
2	1	1	1	0	1	1	0	0	0	0	0
3	1	1	1	1	0	1	1	0	0	0	1
4	0	0	1	1	0	0	1	1	0	0	0
5	0	1	0	0	1	1	0	0	1	0	0
6	0	1	1	0	1	1	1	0	1	0	1
7	0	0	1	1	0	1	1	1	0	0	1
8	0	0	0	1	0	0	1	1	0	1	1
9	0	0	0	0	1	1	0	0	1	1	1
10	0	0	0	0	0	0	0	1	1	1	1
11	0	0	1	0	0	1	1	1	1	1	1

Table 2: Districts that a ranger can protect.

Formulate and solve the problem

The decision variables are as follows:

Formulate and solve the problem

The decision variables are as follows:

$$x_j = \begin{cases} 1 & \text{if a ranger is placed in district } j \\ 0 & \text{otherwise.} \end{cases}$$

Formulate and solve the problem

number of rangers hired

Formulate and solve the problem

number of rangers hired

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

Formulate and solve the problem

number of rangers hired

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

number of rangers that protect district 1

Formulate and solve the problem

number of rangers hired

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

number of rangers that protect district 1

$x_1 + x_2 + x_3$

Formulate and solve the problem

number of rangers hired

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

number of rangers that protect district 1

$x_1 + x_2 + x_3$

number of rangers that protect district 2

Formulate and solve the problem

number of rangers hired

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

number of rangers that protect district 1

$x_1 + x_2 + x_3$

number of rangers that protect district 2

$x_1 + x_2 + x_3 + x_5 + x_6$

Formulate and solve the problem

number of rangers hired

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

number of rangers that protect district 1

$$x_1 + x_2 + x_3$$

number of rangers that protect district 2

$$x_1 + x_2 + x_3 + x_5 + x_6$$

number of rangers that protect district 3

Formulate and solve the problem

number of rangers hired

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

number of rangers that protect district 1

$x_1 + x_2 + x_3$

number of rangers that protect district 2

$x_1 + x_2 + x_3 + x_5 + x_6$

number of rangers that protect district 3

$x_1 + x_2 + x_3 + x_4 + x_6 + x_7 + x_{11}$

....

Formulate and solve the problem

number of rangers hired

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

number of rangers that protect district 1

$x_1 + x_2 + x_3$

number of rangers that protect district 2

$x_1 + x_2 + x_3 + x_5 + x_6$

number of rangers that protect district 3

$x_1 + x_2 + x_3 + x_4 + x_6 + x_7 + x_{11}$

....

number of rangers that protect district 11

Formulate and solve the problem

number of rangers hired

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

number of rangers that protect district 1

$x_1 + x_2 + x_3$

number of rangers that protect district 2

$x_1 + x_2 + x_3 + x_5 + x_6$

number of rangers that protect district 3

$x_1 + x_2 + x_3 + x_4 + x_6 + x_7 + x_{11}$

....

number of rangers that protect district 11

$x_3 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$

Formulate and solve the problem

$$\min\ Z = \sum_{j=1}^{11} x_j \tag{1}$$

subject to

Formulate and solve the problem

$$\min\ Z = \sum_{j=1}^{11} x_j \tag{1}$$

subject to

$$
\begin{array}{ccccccccccccc}
x_1 & +x_2 & +x_3 & & & & & & & & & \geq 1 \\
x_1 & +x_2 & +x_3 & & +x_5 & +x_6 & & & & & & \geq 1 \\
x_1 & +x_2 & +x_3 & +x_4 & & +x_6 & +x_7 & & & & +x_{11} & \geq 1 \\
 & & x_3 & +x_4 & & & +x_7 & +x_8 & & & & \geq 1 \\
 & x_2 & & & +x_5 & +x_6 & & & +x_9 & & & \geq 1 \\
 & x_2 & +x_3 & & +x_5 & +x_6 & +x_7 & & +x_9 & & +x_{11} & \geq 1 \\
 & & x_3 & +x_4 & & +x_6 & +x_7 & +x_8 & & & +x_{11} & \geq 1 \\
 & & & x_4 & & & +x_7 & +x_8 & & +x_{10} & +x_{11} & \geq 1 \\
 & & & & x_5 & +x_6 & & & +x_9 & +x_{10} & +x_{11} & \geq 1 \\
 & & & & & & & x_8 & +x_9 & +x_{10} & +x_{11} & \geq 1 \\
 & & x_3 & & & +x_6 & +x_7 & +x_8 & +x_9 & +x_{10} & +x_{11} & \geq 1 \\
x_1, & x_2, & x_3, & x_4, & x_5, & x_6, & x_7, & x_8, & x_9, & x_{10}, & x_{11} & \in \{0, 1\}
\end{array}
$$

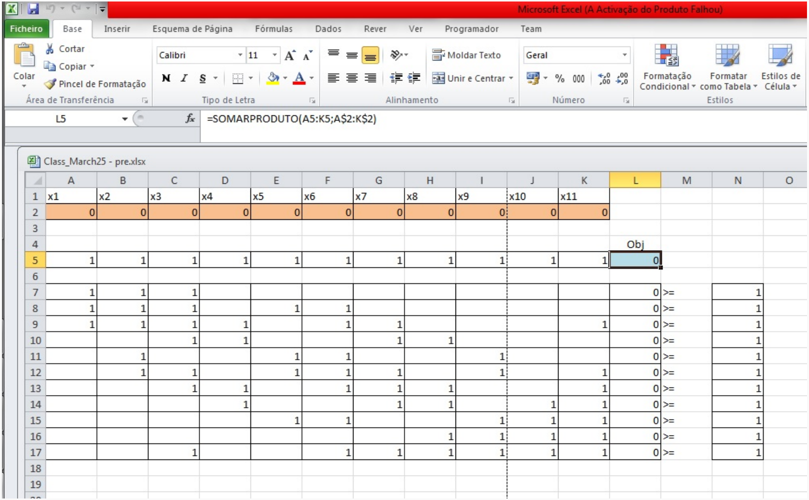
Formulate and solve the problem

Expression (1) minimizes the number of rangers hired

All constraints bebore the last ensure that each district is protected

The last constraints state the nature of the variables

Formulate and solve the problem



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11				
2	0	0	0	0	0	0	0	0	0	0	0				
3															
4												Obj			
5	1	1	1	1	1	1	1	1	1	1	1	0			
6															
7	1	1	1									0	>=	1	
8	1	1	1		1	1						0	>=	1	
9	1	1	1	1		1	1				1	0	>=	1	
10			1	1			1	1				0	>=	1	
11		1			1	1			1			0	>=	1	
12		1	1		1	1	1		1		1	0	>=	1	
13			1	1		1	1	1			1	0	>=	1	
14				1			1	1		1	1	0	>=	1	
15					1	1			1	1	1	0	>=	1	
16								1	1	1	1	0	>=	1	
17			1			1	1	1	1	1	1	0	>=	1	
18															
19															

Figure 2: Excel.

Formulate and solve the problem

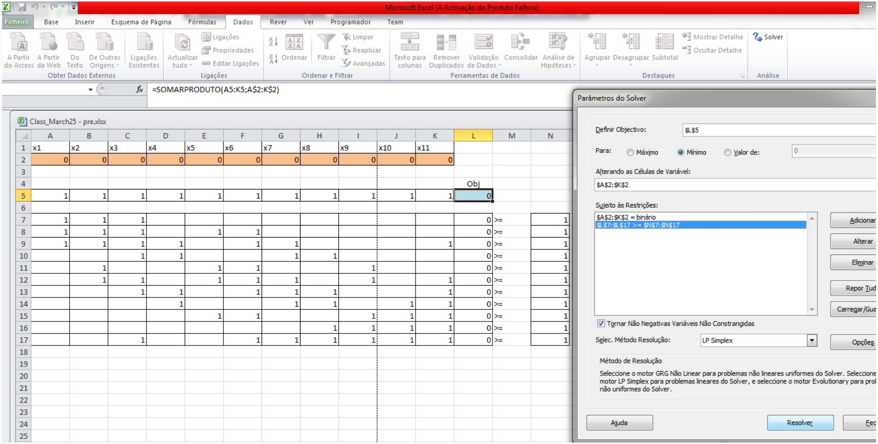


Figure 3: Excel.

Formulate and solve the problem

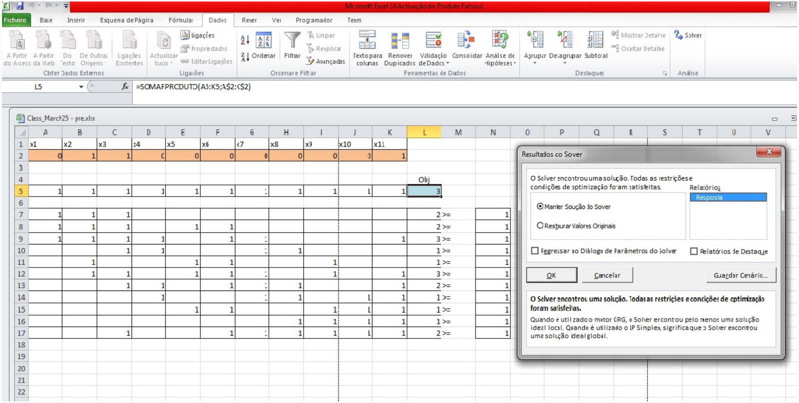
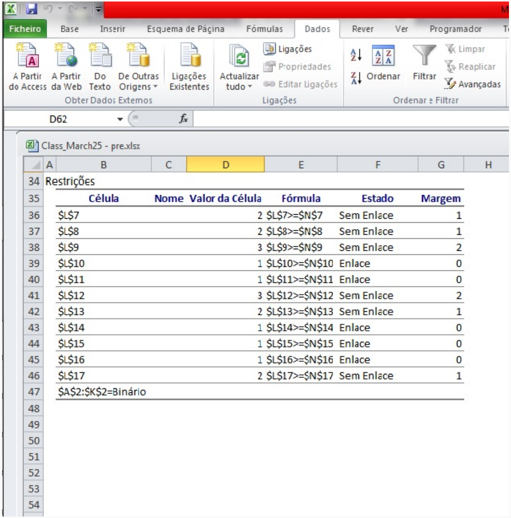


Figure 4: Excel.



Restrições

Célula	Nome	Valor da Célula	Fórmula	Estado	Margem
\$L\$7		2	\$L\$7>=\$N\$7	Sem Enlace	1
\$L\$8		2	\$L\$8>=\$N\$8	Sem Enlace	1
\$L\$9		3	\$L\$9>=\$N\$9	Sem Enlace	2
\$L\$10		1	\$L\$10>=\$N\$10	Enlace	0
\$L\$11		1	\$L\$11>=\$N\$11	Enlace	0
\$L\$12		3	\$L\$12>=\$N\$12	Sem Enlace	2
\$L\$13		2	\$L\$13>=\$N\$13	Sem Enlace	1
\$L\$14		1	\$L\$14>=\$N\$14	Enlace	0
\$L\$15		1	\$L\$15>=\$N\$15	Enlace	0
\$L\$16		1	\$L\$16>=\$N\$16	Enlace	0
\$L\$17		2	\$L\$17>=\$N\$17	Sem Enlace	1
\$A\$2:\$K\$2=Binário					

Figure 5: Excel.

Formulate and solve the problem

The optimal solution is obtained with three rangers, placed in districts 2, 3 and 11.

This problem has alternative optimal solutions. Can you list some?

Sheet cutting planning

A pulp mill cuts sheets of 48 cm $\times$ 96 cm paper into smaller sheets. This company received an order with the characteristics indicated in Table 3. Figure 6 and Table 4 indicate the possible cutting patterns on a 48 cm $\times$ 96 cm sheet. The goal is to determine the cutting plan in order to minimize the number of 48 cm $\times$ 96 cm sheets used.

The problem

Type	Sheet of paper Dimensions cm × cm	Number
1	36×50	800
2	24×36	1300
3	20×60	500
4	18×30	1500

Table 3: Characteristics of the order.

The problem

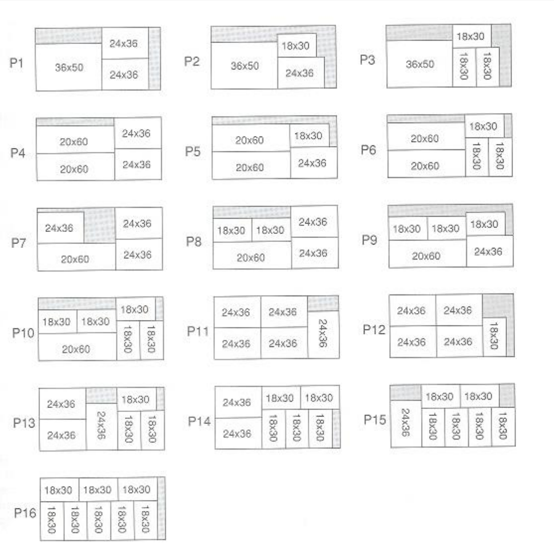


Figure 6: Possible cutting patterns on a 48 cm × 96 cm sheet.

The problem

Sheet of paper	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	1	0	2	1	0	3	2	1	0	5	4	3	2	1	0
3	0	0	0	2	2	2	1	1	1	1	0	0	0	0	0	0
4	0	1	3	0	1	3	0	2	3	5	0	1	3	5	6	8

Table 4: Possible cutting patterns on a 48 cm $\times$ 96 cm sheet.

Formulate this problem as an IP model and solve the model

The decision variables are as follows:

Formulate this problem as an IP model and solve the model

The decision variables are as follows:

x_j– number of 48 cm $\times$ 96 cm sheets assigned to cutting pattern P_j, $j = 1, ..., 16$.

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm $\times$ 96 cm

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm $\times$ 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm $\times$ 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm $\times$ 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

$$x_1 + x_2 + x_3$$

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm $\times$ 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

$x_1 + x_2 + x_3$

no. of sheets of type 2

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm $\times$ 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

$$x_1 + x_2 + x_3$$

no. of sheets of type 2

$$2x_1+x_2+2x_4+x_5+3x_7+2x_8+x_9+5x_{11}+4x_{12}+3x_{13}+2x_{14}+x_{15}$$

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm $\times$ 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

$$x_1 + x_2 + x_3$$

no. of sheets of type 2

$$2x_1+x_2+2x_4+x_5+3x_7+2x_8+x_9+5x_{11}+4x_{12}+3x_{13}+2x_{14}+x_{15}$$

no. of sheets of type 3

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm $\times$ 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

$x_1 + x_2 + x_3$

no. of sheets of type 2

$2x_1+x_2+2x_4+x_5+3x_7+2x_8+x_9+5x_{11}+4x_{12}+3x_{13}+2x_{14}+x_{15}$

no. of sheets of type 3

$2x_4 + 2x_5 + 2x_6 + x_7 + x_8 + x_9 + x_{10}$

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm $\times$ 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

$x_1 + x_2 + x_3$

no. of sheets of type 2

$2x_1+x_2+2x_4+x_5+3x_7+2x_8+x_9+5x_{11}+4x_{12}+3x_{13}+2x_{14}+x_{15}$

no. of sheets of type 3

$2x_4 + 2x_5 + 2x_6 + x_7 + x_8 + x_9 + x_{10}$

no. of sheets of type 4

Formulate this problem as an IP model and solve the model

total number of sheets of 48 cm $\times$ 96 cm

$$Z = \sum_{i=1}^{16} x_i$$

no. of sheets of type 1

$x_1 + x_2 + x_3$

no. of sheets of type 2

$2x_1 + x_2 + 2x_4 + x_5 + 3x_7 + 2x_8 + x_9 + 5x_{11} + 4x_{12} + 3x_{13} + 2x_{14} + x_{15}$

no. of sheets of type 3

$2x_4 + 2x_5 + 2x_6 + x_7 + x_8 + x_9 + x_{10}$

no. of sheets of type 4

$x_2 + 3x_3 + x_5 + 3x_6 + 2x_8 + 3x_9 + 5x_{10} + x_{12} + 3x_{13} + 5x_{14} + 6x_{15} + 8x_{16}$

Formulate this problem as an IP model and solve the model

$$\min \; Z = \sum_{i=1}^{16} x_i$$

Formulate this problem as an IP model and solve the model

$$\min \; Z = \sum_{i=1}^{16} x_i$$

subject to

$$
\begin{array}{llllllllllll}
x_1+ & x_2+x_3 & & & & & & & & & & = 800 \\
2x_1+ & x_2+ & 2x_4+x_5+ & & 3x_7+ & 2x_8+x_9+ & & 5x_{11}+ & 4x_{12}+3x_{13}+ & 2x_{14}+x_{15} & & = 1300 \\
 & & 2x_4+2x_5+ & 2x_6+x_7+ & & x_8+x_9+ & x_{10} & & & & & = 500 \\
 & x_2+3x_3+ & x_5+ & 3x_6+ & & 2x_8+3x_9+ & 5x_{10}+ & & x_{12}+3x_{13}+ & 5x_{14}+6x_{15}+ & 8x_{16} & = 1500
\end{array}
$$

$x_j \in \mathbb{N}_0 \;\; j = 1, \ldots, 16$

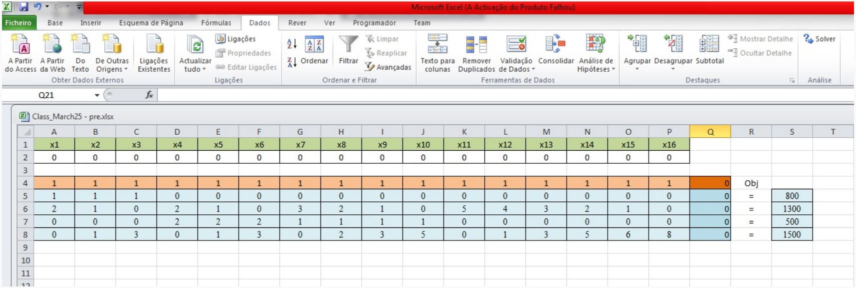
Formulate this problem as an IP model and solve the model

The first expression minimizes the number of 48 cm $\times$ 96 cm sheets used.

All constraints before the last ensure that the characteristics of the order are satisfied.

The last constraints state the integer requirements on the variables.

Formulate this problem as an IP model and solve the model



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16			
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			
3																			
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	Obj	
5	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	=	800
6	2	1	0	2	1	0	3	2	1	0	5	4	3	2	1	0	0	=	1300
7	0	0	0	2	2	2	1	1	1	1	0	0	0	0	0	0	0	=	500
8	0	1	3	0	1	3	0	2	3	5	0	1	3	5	6	8	0	=	1500

Figure 7: Excel.

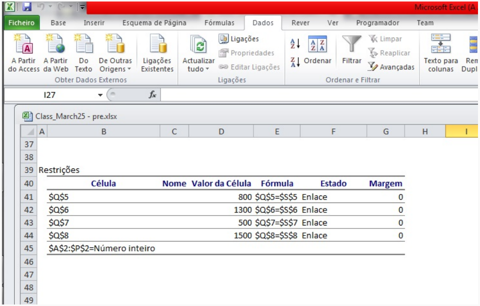
Formulate this problem as an IP model and solve the model

The optimal number of 48 cm $\times$ 96 cm sheets used is 1088.

Number of 48 cm $\times$ 96 cm sheets															
P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}
642	4	154	1	0	249	0	0	0	0	1	0	0	1	3	33

Table 5: Optimal solution.

Formulate this problem as an IP model and solve the model



Célula	Nome	Valor da Célula	Fórmula	Estado	Margem
$Q$5		800	$Q$5=$S$5	Enlace	0
$Q$6		1300	$Q$6=$S$6	Enlace	0
$Q$7		500	$Q$7=$S$7	Enlace	0
$Q$8		1500	$Q$8=$S$8	Enlace	0
$A$2:$P$2=Número inteiro					

Figure 8: Excel.

Homework

Exam1_2017.pdf Exercise 1 in Extra Support Material - Integer Linear Programming from 2019/2020.

Bom estudo!