CHAPTER 2: FINANCIAL ANALYSIS

This chapter introduces the basic concepts and formulas of financial analysis. Financial analysis is the process of evaluating the cash flows associated with different management scenarios in order to determine their relative profitability. This is an important factor to consider in evaluating forest management alternatives, but not necessarily the only one. At first glance, you may think it should be fairly obvious that alternatives that generate the most money, after expenses, are the most profitable. Generally speaking, this is true; however, it can and does get a bit more complicated than that.

The primary factor that complicates financial analysis is the fact that the timing of a cost or revenue can dramatically affect the value of the cost or revenue. As an example, consider which of the following lottery prizes you would rather win: 1) a payment of $10,000 that you get in cash today, or 2) ten equal payments of $1,000 that you will receive each year for 10 years. Most people would take the immediate payoff of $10,000. This is not just because over time inflation will reduce the purchasing power of $1,000, although this is part of the reason. Even if the lottery were to promise to increase the $1,000 payment with inflation over the 10-year period, most people would still choose $10,000 today. Some people would even take the $10,000 today if the alternative was ten equal payments of $1,200. Why is this? How can we judge whether it is better to take a payment today or wait for a higher value at some point in the future? These are the kinds of questions that this chapter will help you address. Questions like these are extremely relevant in forest management because of the long time periods involved in growing trees and the large capital values embodied in forests.

The basic tools of financial analysis are discounting and compounding formulas. This chapter covers five basic financial analysis formulas which you need to learn how to use. However, before diving into the formulas, you should have a clear understanding of the concepts behind the formulas. This will make it much easier for you to use and apply them. After the formulas have been presented, the next section presents some detailed examples and some tips for solving financial analysis problems.

1. What Is Discounting?

Let's begin with some simple definitions:

> *Discounting* is the process of converting future values to present values.
> *Compounding* is the reverse process: converting present values to future values.

These are simple – and mostly correct – definitions of discounting and compounding, but you could say that these definitions "beg the question." This means that the definitions really don't help us understand what discounting and compounding are because they don't explain what future and present values are, and without such an explanation the definitions are meaningless.

So, what are present and future values, and why are present values different from future values? Consider some additional definitions:

> A *present value* is a value that is expressed in terms of dollars received immediately. A *future value* is a value that is expressed in terms of dollars received at some future time.

Ok, so now you know what present and future values are, but you probably still feel like the basic question remains unanswered. Perhaps some questions still in your mind are: Why does it matter whether a value is received now or later? Why is a present value different from a future value? Why do values that occur at different times have to be converted?

To answer these questions, consider the example mentioned in the introduction to this chapter. Your lottery prize gives you a choice between 1) a payment of $10,000 which you get in cash today, or 2) 10 payments of $1,200 that you will receive each year for the next 10 years. Why is it that, for some people at least, $10,000 is better than $12,000? Clearly it is because of the timing of the values. Receiving $10,000 can be better than receiving $12,000, if there is a big enough difference in the timing of the two values.

As another example, consider how you might feel if a rich uncle offered to give you $20,000 to buy a new car when you graduate from college. For many of you, that date should not be too far off, but, in spite of your gratitude and excitement, you probably would also think “I wish I could get that new car <u>right now</u>.” You would obviously be more excited about the windfall if you were receiving it today. So the time when you receive something affects it’s value. What if the rich uncle says you will get the money when he dies – and he’s only in his 50's and in good health? Now, how would you feel about his offer? Time matters; doesn’t it? Often we are willing to settle for less if we can have it now instead of waiting. Economists call this aspect of human nature *time preference*.

Besides people’s preference for having things right now, the timing of a cost or revenue matters because money can be invested and used to earn more money. If you have some money today that you don’t need for immediate use, you can invest it and earn a larger amount of money in the future. Conversely, if you don’t have enough money for an immediate purchase, you can usually borrow what you need; but, not only will you have to pay back what you borrow, you will have to pay for using the money that you borrow.   Money can earn money. The corollary to this is that there is a cost to using money. So, what does this have to do with present and future values? Well, a given amount of money today – a present value – is equivalent to a larger amount of money in the future – a future value – because the money today can be converted to a larger amount of money in the future through investing, if it is invested wisely. The flip side of this idea is that if you have to borrow money to use today, the future amount you will pay will be more than the amount you borrowed. From either perspective, it takes a larger future value to equal a given present value. Discounting is the process of calculating a dollar amount today that is equivalent to a given dollar amount at some point in the future. Conversely, compounding is the process of calculating a dollar amount at a later date that is equivalent to a dollar amount today.

The above discussions have focused on the differences between present and future values. You probably have realized by now, however, that two identical future values are not equivalent if they do not occur at the same time. For example, $1,000 earned in five years is better than $1,000 earned in ten. The choice of the present as a unique reference point in time is somewhat arbitrary. The earlier definitions of discounting and compounding need to be generalized to recognize that these procedures can be used to convert the time reference of a value between any two points in time. For example, compounding can include converting a value that occurs in year 5 to an equivalent value in year 10, and discounting can include converting a value that occurs in year 25 to an equivalent value in year 15. The following definitions, although less intuitive, are therefore more complete and general than those given earlier.

> *Discounting* is the process of converting a value that is expressed in terms of dollars received at one point in time to an equivalent value expressed in terms of dollars received at an earlier point in time.
>
> *Compounding* is the process of converting a value that is expressed in terms of dollars received at one point in time to an equivalent value expressed in terms of dollars received at a later point in time.

2. <u>What Is Interest?</u>

Interest is the money that you pay to borrow money and the money that your investments earn when you lend money. The *interest rate* is the percentage of the amount invested or borrowed that is paid in interest after one unit of time – usually a year, but it could be monthly, or even daily. Why are people willing to pay interest to use other people's money? One reason, as discussed earlier, is time preference: often, when people want to have things today rather than wait until a later time they are willing to pay extra to avoid waiting. People also borrow money so they can invest it. In fact, most businesses operate on at least some borrowed money. In this case, people borrow money because they believe they can use the money to make more money than what it costs to borrow it. This type of activity is the foundation of capitalism. Forestry is a very capital-intensive business because a lot of money is usually tied up in the land and trees. As future managers of forests, it is important that you understand capital costs and interest.

The interest rate determines the relationship between current and future values. Consider money you might invest in a savings account. Let

> V_0 = the amount in the account today,
> V_1 = the amount in the account in one year, and
> i = the interest rate earned on the account in one period.

Now, if you make no further deposits or withdrawals, the amount you will have in the account in one year is a function of the current amount in the account and the interest rate:

$$V_1 = V_0 + iV_0 = principal + interest$$

This can be rearranged as follows:

$$V_1 = (1+i)V_0$$

This equation basically says that you can convert 1 dollar today to 1+ *i* dollars one year from now by investing the dollar at an interest rate of *i*. The formulas can also be rearranged to say that you can invest 1/(1+ *i*) dollars today to get one dollar one year from now. That is, 1/(1+ *i*) is the price of one dollar next year in terms of today's dollars. So, one way of thinking about the interest rate is that 1/(1+ *i*) is the exchange rate for converting dollars received one year from now to an equivalent amount in dollars received today. If you are converting money received in one year to money received now, this is the rate at which the future dollars are exchanged for current dollars – just like when you convert money from one currency to another. Discounting can therefore be viewed as converting future dollars to current dollars using the appropriate exchange rate, which is based on the interest rate.

Interest is an important concept that cannot be ignored when dealing with significant sums of money. Because money can be used to earn more money, there are opportunity costs whenever money (or any other form of capital) is used for a significant period of time. Say you want to borrow $1,000 for a year. Outside your family, it is unlikely that you would be able to borrow such an amount of money for that long without paying some kind of interest. Even though people might trust you completely and fully expect you to pay them back, they could always find something else to do with the money. At a minimum, they could invest the money in a certificate of deposit (CD) earning, say 5%. This means they would have to forego earning at least $50 to lend you the money. Investing the money at 5% is their alternative "opportunity." If they forgo that opportunity, they forgo the $50 they could have earned. So, whether they charge you for using the money or not, it costs them $50 to lend it to you. If they actually charge you $50, then they break even because their revenues equal their (opportunity) costs ($50 revenues = $50 costs).

The interest rate can be interpreted as the cost of capital because the use of capital (money for investment) always has opportunity costs (you can always invest it somewhere). Put another way, the interest rate is really just the price of using money. In this sense, the interest rate can be thought of as the price that results in an equilibrium between the aggregate amount of money people are willing to lend – the supply of capital – and the aggregate amount that people want to borrow – the demand for capital. If the interest rate is too high, lots of people will want to save, fewer people will want to borrow; the supply of capital will be higher than the demand for capital. Conversely, if the interest rate is too low, the aggregate amount people will want to borrow will be more than the aggregate amount available to lend.

One of the most confusing things about interest rates is that there are so many of them. If the interest rate is just the cost of borrowing money, why isn't there just one price? For example, why are the interest rates on credit cards so high, while the interest rate on savings accounts are so low? The answer to this question is that each investment is different. *Risk* is perhaps the most important difference between different investments that affects the particular interest rate for each investment. Risk has to do with the likelihood that the invested money will be lost, or that the rate of return will be lower than expected, versus the likelihood that the expected return will actually be earned or even exceeded. In general, people prefer to avoid risk. Economists call this *risk aversion*. In order to get people to put their money in an investment with a higher risk, it is reasonable to expect riskier investments to pay a higher interest rate relative to alternative investments with lower risks. The difference between the interest rate that a risky investment would be expected to pay and the interest rate that a risk-free investment would pay is called a *risk premium*. Since the future is inherently uncertain, all investments involve some risk. Some investments, like government bonds and CDs, involve minimal risk, and the risk premium on such investments is quite small. Other investments, like junk bonds, venture capital and futures options, are quite risky. These investments usually offer a higher expected rate of return. The extra interest is the premium investors earn for bearing the risks associated with these investments.

Investments also differ in terms of their *liquidity* – that is, the ease with which investors can withdraw their money prematurely from the investment. For example, when you put your money in a savings account in a bank, you can withdraw it at any time. However, when the bank loans you money, the best that they can expect is to get their money back according to the pre-determined payment schedule. Thus, your savings account is an almost perfectly liquid investment, while the loan that a bank makes to you is relatively illiquid. (Actually, they can sell your loan to other banks or real estate investors. So they can get their money out, it's just a bit more complicated.)

Financial institutions, such as banks, almost always make you pay a higher interest rate when you are borrowing money from them than they will pay you when you let them use your money. Risk is part of the explanation for this – it is more likely that borrowers will default on their payments to a bank than that the bank will default on their obligations to their customers. Another reason for this differential between borrowing and lending rates is *transactions costs*. Banks and other financial institutions generally pay the majority of their operating costs with the money they earn by borrowing money at one rate and lending the money at higher rates.

Each investment is different in terms of riskiness, liquidity, and other factors. As a result, each investment will have a uniquely appropriate interest rate. Investments are not a uniform commodity, and just as the price of a car varies from one model to the next, different investments earn different interest rates.

In addition to varying from one investment to another, interest rates vary a lot over time. Temporal changes in the interest rate are driven largely by changes in the inflation rate and shifts in the aggregate amount of money people want to lend and the aggregate amount people want to borrow for investments and for consumption. The next chapter focuses on the relationship between the interest rate and inflation, but you should know that interest generally must cover more than inflation. Even in a world with no inflation, people would still charge interest to cover the opportunity costs of using the money, a possible risk premium, transactions costs, etc. The amount of interest earned over the inflation rate is the true price of using money.

A note on terminology: there are many terms that are used somewhat interchangeably for the interest rate. They include: the discount rate, the alternate rate of return, the expected rate of return, and others. There are subtle differences in the meanings of these terms, but they generally mean the same thing. They will all be used in this text, as students should learn to recognize them.

3. Discounting Formulas

This section presents and discusses five basic formulas used for discounting and compounding cash flows. The use of each formula is discussed, examples are given, and a brief derivation of each formula is also given. Some formulas can be rearranged to find a present value or a future value. In those cases, both versions are given. Students often want to think of the different versions of the formulas as different formulas. I believe, however, that it is easier to learn only a few basic formulas and learn how to rearrange them than to learn many formulas.

The derivations of the formulas are included for completeness and because some students will find it easier to use the formulas if they understand where they came from. I have tried to make the derivations of the formulas as intuitive as possible; they are not rigorous proofs.

All of the formulas discussed here are for *discrete-time* problems – i.e., problems that treat each year as a distinct point in time, rather than as a continuous interval. The alternate approach, *continuous discounting*, is not addressed in this book. Generally, continuous discounting is more useful for deriving theoretical results, while discrete-time discounting is more appropriate for practical problems.

The Single Value Formula

The single value formula, also sometimes called the *single-sum* formula, is used to discount a single future value to the present or to compound a single present value to a future date. It can also be used to convert a value that occurs at one future date to an equivalent future value at another future date – for example to convert a future value that occurs in year 10 to an equivalent future value for year 20. In addition, the single value formula can be rearranged to solve for the interest rate that is earned on a given initial investment with a single final return.

This formula is the most fundamental of all the discounting formulas. You will use it frequently, and you could calculate the present or future value of just about any project using this formula only. While you cannot calculate the exact value of projects with infinite series of cash flows using this formula only, in practice the present values of cash flows in the far distant future are highly uncertain and small enough to safely ignore.

Note that the formula has four variables: 1) the present value, V_0; 2) the future value, V_n; 3) the interest rate, *i*; and 4) the number of years, *n*. In general, you need to know any three of these values and the different versions of the formula can be used to calculate the fourth. Thus, we could potentially have four different versions of the formula. It is unlikely that you will need to solve for the number of years, however, so that version is not presented here.

<u>Present Value</u>

This formula gives the present value (V_0) of a value that occurs in year *n* (V_n), given the interest rate *i*.

$$V_0 = (1+i)^{-n} V_n = \frac{V_n}{(1+i)^n}$$

As mentioned earlier, discounting can also include converting a value from a given date to any date before that time. Thus, a more general version of the formula would be:

$$V_t = (1+i)^{-n} V_{t+n}$$

where V_t is the value expressed in terms of dollars earned at time *t*, and
V_{t+n} is the value expressed in terms of dollars earned at time *t+n*, *n* years later.

Note that all of the formulas presented here can be written in a more general form like this. They are not presented this way primarily to keep them in as simple a form as possible. However, you should be able to make this type of adjustment to the formulas when appropriate.

<u>Future Value</u>

This formula gives the equivalent future value at time *n* (V_n) of a value that occurs today (V_0), given the interest rate *i*.

$$V_n = (1+i)^n V_0$$

<u>Solving for the Interest Rate</u>

This formula gives the interest rate *i* earned on an initial investment of V_0 yielding a single final return in time *n* of V_n.

$$i = \left[\sqrt[n]{\frac{V_n}{V_0}}\right] - 1 = \left[\frac{V_n}{V_0}\right]^{1/n} - 1$$

Note that the two ways of writing the n^{th} root are equivalent.

Example

What is the present value, using a discount rate of 8%, of $1,000 to be received in 30 years?

Answer:

$$V_0 = \$1{,}000 / (1.08)^{30} = \$99.38$$

Similarly, $1,000 is the future value in 30 years (V_{30}) of $99.38 today, at 8%.

If 8% is the appropriate interest rate for you, then you should be essentially indifferent about the choice of getting $100 today versus $1,000 in 30 years. If your discount rate is higher than 8%, then you would prefer the $100 today. If your discount rate is lower than 8%, then you would prefer the $1,000 in 30 years.

Example

A release treatment that occurs in a pine stand at age 4 costs $60/ac. With or without the release treatment, the stand in question would be harvested at age 25. How much must the release treatment add to the harvest revenue at age 25 in order to return a rate of return of at least 6% on the $60/ac investment?

Answer: In order to earn at least 6%, the additional revenue at age 25 that results from the release treatment must be equal to or greater than the future value of the $60/ac cost at age 4, compounded over 21 years at 6%.

$$V_{25} = V_4 (1+i)^{(25-4)} = \$60(1.06)^{21} = \$203.97$$

Thus, if the release treatment conducted at age 4 increases the harvest revenue at age 25 by at least $203.97, then the investment of $60/ac will have earned a rate of at least 6% over the 21-year period. (Note: if the release treatment changes the optimal harvest age, or *rotation*, then the problem is more complicated. You will learn how to solve this problem in Chapter 6, which discusses *Land Expectation Values*.)

Example

What rate of return is earned on an investment that doubles your money in ten years?

Answer: Assume that we invest $1 today and earn $2 in 10 years. Then apply the interest rate version of the formula:

$$i = \left[\sqrt[10]{2/1}\right] - 1 = 0.0718 = 7.18\%$$

An investment that doubles your money in 10 years will earn a 7.18% rate of return.

☞ There is a rule-of-thumb that is very helpful for estimating the rate of return earned by an investment. It is called the *Rule of 72*. The rule says that you can estimate the rate of return on an investment if you know the time it takes for an investment to double your money. The rule can also be used to determine how long it takes to double your money when you know the interest rate.

- To estimate the rate of return, divide 72 by the number of years to double your money.
- To estimate how long it takes to double your money, divide 72 by the interest rate.

Derivation

The n-period single-sum formula is a straight-forward generalization of the one-period formulas discussed previously. Recall that if you invest V_0 for one year at an interest rate i, the formula for the future value after one year is:

$$V_1 = (1+i)V_0$$

Similarly, if you reinvest the value earned after one year (V_1) for another year, the value after the second year will be:

$$V_2 = (1+i)V_1$$

Now, substituting the formula for V_1 given above into this formula gives the following:

$$V_2 = (1+i)(1+i)V_0 = (1+i)^2 V_0$$

For the n-period formula, simply repeat the one-period formula n times and combine terms as done here for the two-period formula. For example, consider the 3-period case:

$$V_3 = (1+i)V_2 = (1+i)(1+i)V_1 = (1+i)(1+i)(1+i)V_0 = (1+i)^3 V_0$$

Infinite Annual Series

The infinite annual series formula is used to calculate the present value of a series of equal, annual payments that continues forever. An example of such a series would be the expected net revenues from a farming activity that is repeated each year. In such a case, the formula could be used to estimate the value of the farmland. There is no future value version of the formula, since the future in this case is infinitely distant.

The infinite annual series formula has three variables: 1) the present value, V_0; 2) the interest rate, *i*, and 3) the annual payment, *R*. Thus, if you know any two of these, you can solve for the third. Thus, the formula can be rearranged to calculate the net revenue (payment) that must be earned annually, forever, from an initial investment in order to earn a given interest rate, or to calculate the interest rate that will be earned for a given infinite annual payment and initial investment.

Few investments can realistically be expected to produce infinite cash flows. Land ownership, however, comes pretty close to this as long as the land is not abused. One of the most fundamental uses of this formula, therefore, is for calculating land values. Because of this use of the formula, it is an important one for foresters, who, as land managers, need to understand the fundamental determinants of land values.

<u>Present Value</u>

This formula gives the present value (V_0) of an infinite series of annual payments of *R* dollars, given the interest rate *i*.

$$V_0 = \frac{R}{i}$$

<u>Future Value</u>

The future value of any infinite series is infinite.

<u>Calculating the Payment</u>

This formula gives the value of the annual payment (*R)* that an investment should generate if the initial investment (V_0) were expected to earn an interest rate of *i*.

$$R = iV_0$$

<u>Calculating the Interest Rate</u>

This formula gives the interest rate (*i*) earned by annual payment (*R)*, given an initial investment (V_0).

$$i = \frac{R}{V_0}$$

Example

You wish to endow your alma matter with a fund that will generate $1,000 each year for scholarships. A conservative estimate of the interest rate that the fund will earn is 6%. How much money will you have to put in the fund?

Answer:

$$V_0 = \$1{,}000 / 0.06 = \$16{,}667$$

You will need to donate $16,667 for the fund.

Example

If you purchase some farm land for $1,800/ac and your alternate rate of return is 8%, how much should you expect to earn from the farm per year?

Answer:

$$R = 0.08 \times \$1{,}800 / ac = \$144 / ac$$

<u>Derivation</u>

Say that you have a bank account with $1,000 in it earning 5% per year. If you take out the interest every year, you should receive $50 annually. This is calculated by simply multiplying the interest rate times the principal. In the example of your bank account,

$$R = \$1{,}000(0.05) = \$50$$

The general formula is, therefore:

$$R = iV_0$$

Conversely, if you receive $50 per year, and the appropriate interest rate for you is 5%, what is this investment be worth to you today? We can rearrange the last equation to give us the answer:

$$V_0 = \frac{R}{i}$$

For the example, then,

$$V_0 = \$50 / 0.05 = \$1{,}000$$

This example would be applicable to a piece of land that could earn (net, after expenses and taxes) $50 per acre each year. At 5%, the land would be worth $1,000 per acre.

Question: How much would the land be worth to you if the appropriate interest rate for you was 7%?

Note: all of the formulas for series of payments that are discussed in this chapter assume that payments come at the end of their associated period. If an infinite annual series of payments starts immediately, then the formula for the present value of the series would be:

$$V_0 = \frac{R}{i} + R$$

The first payment is already in current dollars, so we can just add this payment to the infinite series which accounts for all of the payments except for the initial one. This basic approach can be used to modify all of the series formulas to handle a payment that starts immediately.

Finite Annual Series

The finite annual series formula is used to discount or compound a regular annual payment or cost, *R*, for a fixed number of years, *n* (instead of forever, as in the infinite annual series formula). There are a lot of situations where this formula can be used. One example is calculating regular, annual payments you might make on a loan. Or, it could be used for discounting rent payments you might make on a piece of land each year over a fixed-term contract. Similarly, it might be used for annual tax payments, or an annual management costs, made over a fixed period of time.

The formula can also be used for series of monthly payments, such as you would make on a mortgage loan or an auto loan. If you use a monthly interest rate instead of an annual interest rate, the number of months can be substituted for the number of years. The monthly interest rate (i_m) that is equivalent to a given annual interest rate (i) can be calculated as follows:

$$i_m = \left[\sqrt[12]{(1+i)}\right] - 1 = (1+i)^{1/12} - 1$$

The finite annual series formula has a present value version and a future value version. In either version, there are four variables. They are 1) the interest rate, *i*; 2) the payment, *R*; 3) the number of periods, *n*; and 4) either the present value, V_0, or the future value, V_n. Note that the formula cannot be solved directly for the interest rate or the number of periods. It can, however, be solved for the payment needed to achieve either a given present value or a given future value. (Note: as with all of the other series formulas discussed here, the first payment in the series is assumed to come at the end of the first year.)

Present Value

This formula is used to calculate the present value, at an interest rate *i*, of a regular, annual payment (R) that is made for a fixed number of years (*n*).

$$V_0 = \frac{R[(1+i)^n - 1]}{i(1+i)^n}$$

Future Value

This formula is used to calculate the future value in year n (V_n), at an interest rate i, of a regular, annual payment (R) that is made for a fixed number of years (n).

$$V_n = \frac{R[(1+i)^n - 1]}{i}$$

Payment to Achieve a Given Present Value

This formula is used to calculate the annual payment (R) that would be required over a period of n years in order to achieve a given present value (V_0) at a given interest rate i.

$$R = \frac{V_0\, i\,(1+i)^n}{[(1+i)^n - 1]}$$

Payment to Achieve a Given Future Value

The derivation of this formula has been left as an exercise for the student.

Example

You want to rent a piece of land for seven years to grow Christmas trees. The annual rent (paid at the end of each year) is $100 per acre, and your cost of capital (discount rate) is 4%. What is the present value of the seven years' land rent?

Answer:

$$V_0 = \frac{\$100[(1.04)^7 - 1]}{.04\,(1.04)^7} = \$600.21$$

The present value of the seven payments is $600.21.

Example

Consider the case of an even-aged northern hardwood stand that is managed on an 80-year rotation with no intermediate revenues – i.e., the only revenue earned from the stand is from the final harvest at age 80. At an interest rate of 8%, how much revenue (per acre) must be earned from this final harvest (at age 80) to pay for an annual property tax of $2/ac that is paid each year from age 1 to age 80?

Answer:

$$V_{80} = \frac{\$2/ac[(1.08)^{80} - 1]}{0.08} = \$11{,}774/ac$$

Think about that! The stand will need to earn $11,774/ac at harvest just to cover an annual property tax of $2/ac.

Example

You want to buy a new pickup truck that costs $24,000. You have $4,000 for a down payment. How much should your monthly payment be if you get a 5-year loan at 8%?

Answer: After the down payment, your loan balance will be $20,000, and you will make 60 monthly payments. Now, calculate the monthly interest rate:

$$i_m = (1.08)^{1/12} - 1 = 0.6434\%$$

Now, calculate the payment:

$$R = \frac{\$20{,}000(0.006434)(1.006434)^{60}}{[(1.006434)^{60} - 1]} = \$402.86$$

Your monthly payment should be $402.86.

Derivation

The present value of a finite annual series can be thought of as the difference between the present values of two infinite annual series: one that starts at the end of the current year (year 1) and one that starts at the end of year $n+1$. The infinite annual payment formula above can be used to give the present value of the infinite series that starts now. Next, subtract the present value of an infinite series with the same payment that starts at the end of period $n+1$. (Note that the end of year n is equivalent to time 0 for this series. Thus, the infinite annual series formula calculates the value of the series at time n. Discounting this value for an additional n years results in a present value, at time 0.)

$$V_0 = \frac{R}{i} - \frac{1}{(1+i)^n}\frac{R}{i}$$

Multiply the first term above by $(1+i)^n / (1+i)^n$ (this is the same thing as multiplying by one) to get:

$$V_0 = \frac{R(1+i)^n}{i(1+i)^n} - \frac{1}{(1+i)^n}\frac{R}{i}$$

Now, both terms have the same denominator ($i(1+i)^n$), so they can be combined:

$$V_0 = \frac{R(1+i)^n - R}{i(1+i)^n}$$

The *R* can be factored out of the numerator, resulting in the present value version of the formula:

$$V_0 = \frac{R[(1+i)^n - 1]}{i(1+i)^n}$$

To obtain the future value version of the formula, multiply the present value version by $(1+i)^n$. This will cancel the term in parentheses in the denominator, leaving only *i*. To obtain the formula for the payment required to obtain a given present value, solve the present value version of the formula for *R*. Similarly, to obtain the formula for the payment required to obtain a given future value, solve the future value version of the formula for *R*.

Infinite Periodic Series

The infinite periodic series formula is used for situations where a fixed amount is earned periodically, for example every *t* years, instead of every year. As implied by the formula's title, the series of payments is assumed to continue forever. The formula is also sometimes called the *perpetual periodic series* formula, which means the same thing as *infinite periodic series*. As with the infinite annual payment formula, there is no future value version of this formula, since the future is infinitely distant in this case. The formula could be solved for the periodic payment that would be needed to produce a given present value at a given interest rate, but we would seldom use that version of the formula.

The most important use of this formula in forestry is for calculating the *Land Expectation Value* (LEV) of forest land, also sometimes called the *Soil Expectation Value*, or SEV. The LEV formula is one of the two or three most important formulas you will learn in this class. Chapter 6 discusses the LEV in depth, and you will see a lot more of this formula there.

<u>Present Value</u>

This formula is used to identify the present value (V_0) of an infinite series of regular payments (R) that are made every *t* years.

$$V_0 = \frac{R}{(1+i)^t - 1}$$

Example

Consider an aspen stand that regenerates naturally, so there is no regeneration cost. Also, for simplicity, assume that there are no taxes on the property. Every 40 years, the stand can be harvested to yield, on average, 30 cords of pulpwood at an estimated price of $6 per cord. That is, every 40 years an expected revenue of $180 per acre will be generated. Assuming there are no taxes or other costs, and that the discount rate is 3%, what is the value of the land (per acre) when used for growing aspen?

Answer: Note that the value of any asset is given by the present value of the expected present and future costs and revenues accrued by owning the asset. Thus, the value of the land on which the aspen stand is growing is the present value of the infinite annual series of $180/ac, earned every 40 years.

$$V_0 = \frac{\$180}{(1.03)^{40} - 1} = \$79.57$$

In this simple example, the aspen stand produces a revenue that is earned every 40 years. Thus, the value of the land is just the present value of these 40-year revenues. This is the LEV of the aspen stand. In Chapter 6, you will learn how to calculate the LEV for stands with more complex cash flow patterns.

Derivation

The simplest way I have found to explain the derivation of the infinite periodic series formula is to use the concept of a *t*-year interest rate. The *t*-year interest rate is the interest rate earned over a period of *t* years – instead of over a 1-year period. The notation i_t will be used here to represent the *t*-year interest rate. The formula for the *t*-year interest rate is:

$$i_t = (1+i)^t - 1$$

To see where this formula comes from, consider the future value of an investment made at an annual interest rate of *i* for *t* years:

$$V_t = (1+i)^t V_0$$

Subtracting the value of the initial investment gives the interest earned on the investment over the *t* years:

$$V_t - V_0 = (1+i)^t V_0 - V_0 = [(1+i)^t - 1]V_0$$

So, the interest earned on the investment over the *t* year period is $[(1+ i)^t - 1]$ times the initial investment. The interest earned over one year would be the interest rate, *i*, times the initial investment, so this expression, $(1+ i)^t - 1$, can be interpreted as a *t*-year interest rate.

If you are not convinced by this explanation, note that the *t*-year interest rate reduces to the simple, one-year interest rate when $t = 1$.

$$i_1 = (1+i)^1 - 1 = (1+i) - 1 = i$$

Also, consider the interest rate for a two-year period:

$$\begin{aligned} i_2 &= (1+i)^2 - 1 \\ &= (1+i)(1+i) - 1 \\ &= 1 + 2i + i^2 - 1 \\ &= 2i + i^2 \end{aligned}$$

Since i^2 is very small, this is approximately equal to $2i$.

Finally, recall the monthly interest rate that is equivalent to an annual interest rate i, given by the following equation:

$$i_m = \left[\sqrt[12]{(1+i)}\right] - 1 = (1+i)^{1/12} - 1$$

The case of a monthly interest rate is like setting t=1/12, so this formula, too, is just a special case of the *t*-period interest rate.

Hopefully, you are now convinced that the *t*-year interest rate really is $[(1+ i)^t - 1]$. Now, consider the formula for the infinite annual series:

$$V_0 = \frac{R}{i}$$

and substitute the *t*-period interest rate for the annual interest rate. The result is the infinite periodic series formula:

$$V_0 = \frac{R}{(1+i)^t - 1}$$

Thus, the infinite annual series and the infinite periodic series formulas are essentially the same, except that an annual interest rate is used in the former and a *t*-period interest rate is used in the latter.

Finite Periodic Series

The finite periodic series formula is for situations where you have a regular series of payments of a fixed amount (R) that you receive every t years, and that ends after n years. Note that n here should be a multiple of t and that the number of payments will be n/t. This is not a formula that you will use often, but it is included here for completeness. As with the finite annual series formula, we could discuss four versions of this formula: 1) the present value version, 2) the future value version, 3) the version giving the payment needed to obtain a certain present value, and 4) the version giving the payment needed to obtain a certain future value. The first three of these are presented here, and the fourth is left for the student to derive.

Present Value

This formula is used to solve for the present value, at an interest rate i, of a regular, periodic payment R that is received every t years, which ends after a fixed number of years n.

$$V_0 = \frac{R[(1+i)^n - 1]}{[(1+i)^t - 1](1+i)^n}$$

Future Value

This formula is used to solve for the future value, at an interest rate i, of a regular, periodic payment R that is received every t years, which ends after a fixed number of years n.

$$V_n = \frac{R[(1+i)^n - 1]}{[(1+i)^t - 1]}$$

Payment to Achieve a Given Present Value

This formula is used to calculate the periodic payment R that would be required every t years, over a period of n years, in order to achieve a given present value V_0, at an interest rate of i.

$$R = \frac{V_0[(1+i)^t - 1](1+i)^n}{[(1+i)^n - 1]}$$

Example

An intensively managed hybrid poplar stand can be coppiced to produce three crops at seven-year intervals. After 21 years the stand must be replanted. At years 7, 14, and 21 the harvested crop produces a revenue of $200 per acre. What is the present value, using a 5% discount rate, of these three harvests?

Answer:

$$V_0 = \frac{\$200[(1.05)^{21} - 1]}{[(1.05)^7 - 1](1.05)^{21}} = \$314.94$$

Derivation

To obtain this formula, replace the annual interest rate *i* in the finite annual series formula with the *t*-period interest rate $[(1 + i)^t - 1]$.

You could also derive this formula by subtracting the present value of an infinite periodic series that starts in year *n* from the present value of an infinite periodic series that starts now. The steps would be very similar to the derivation of the finite annual series formula given earlier.

To obtain the future value version of the formula, just multiply the present value version by $(1 + i)^n$. This will cancel the second term in the denominator of the present value version.

To obtain the periodic payment required to achieve a given present value, solve the present value version of the formula for the payment. Similarly, to obtain the periodic payment required to achieve a given future value, solve the future value version of the formula for the payment.

4. Tips for Solving Financial Analysis Problems

The previous section presented the basic financial analysis formulas and gave example problems for each formula. The examples show how to apply the individual formulas, but for many students the most difficult step in solving financial analysis problems is figuring out which formula to use and where to use the given information in the formula. Sometimes the most difficult part of working one of these problems is just to figure out what the question is asking for. This section presents several additional examples, ranging from fairly simple ones to more complicated ones, and describes basic procedures that can be used to help break down financial analysis problems into a set of relatively straight-forward steps.

Example Problems

Example

Say you invest $1,000 in a 3-year CD (certificate of deposit) that pays 8% simple interest. How much should you get back after 3 years?

Answer: The first two questions to ask yourself when working a problem like this are: “What do I know?” and “What do I need to know?” To answer the first question for this example, note the following: 1) the initial investment, a present value, is $1,000; 2) the interest rate, *i*, is 8%; and 3) the investment lasts for 3 years, so $n = 3$. The question is asking what the investment will be worth in 3 years. This is a future value, V_n, or, in this case, V_3, since $n = 3$. Often it is useful just to write out what you know and what you need to know as follows:

$$V_0 = \$1{,}000 \qquad i = 8\%$$
$$n = 3 \qquad V_3 = ?$$

The next question is: “How do I get from what I know to what I need to know?” To answer this question, you typically need to determine which formula, or formulas, to apply. (Yes, many problems will require more than one formula.) In this case, your clue is that only a single present value and a single future value are involved; i.e., there is no series of payments in the problem. This should tell you that the single value formula applies here. Furthermore, you know that the answer to the problem is a future value, so you can conclude that the future value version of the single value formula applies here. That is:

$$V_n = (1+i)^n V_0$$

The final step is to plug in the known values and solve for the future value:

$$V_3 = (1.08)^3 \$1{,}000 = \$1{,}259.71$$

So, the investment should be worth $1,259.71 after 3 years.

This was a fairly simple example. However, it illustrates the basic procedure for solving financial analysis problems:

1. Identify what you know and what you need to know.
2. Based on the information in step 1, determine which equation or equations you will need to use to solve the problem.
3. Plug the known values into the appropriate formula or formulas and calculate the solution.

As you might guess, step 2 is usually the most difficult. In the above example, we used the fact that only one future value and one present value were involved to select the single-value formula for solving this problem. You may find the following dichotomous key useful in selecting the appropriate formula.

1. Does the problem involve a series of equal revenues or payments?
   Yes: Go to question 2.
   No: You will probably need to use the single-value formula. If multiple values are involved, you will probably need to apply the formula separately to each value.
2. Is the series of revenues or payments annual or periodic?
   Annual: Go to question 3.
   Periodic: Go to question 4.
3. Does the series of annual revenues or payments end at a specific time, or is it infinite?
   Specific ending point: Use the finite annual series formula.
   Infinite: Use the infinite annual series formula.
4. Does the series of periodic revenues or payments end at a specific time, or is it infinite?
   Specific ending point: Use the finite periodic series formula.
   Infinite: Use the infinite periodic series formula.

Once you have identified the basic formula that you will need, you can determine which version of the formula to use based on what you need to know. If you need a future value, you will need the future value version of the formula; if the answer you are looking for is a present value, you will need the present value version; etc.

Example

Your broker has suggested that you invest $5,000 in a company that says it will guarantee that you will earn a nominal interest rate of 9% over a 5-year period. The company promises to pay you a dividend of $450 at the end of years 1 through 4. In order to earn 9%, what should you expect the company to pay you after 5 years?

Answer: First, answer the questions: “What do I know?” There is an initial investment of $5,000. Also, there is a series of 4 revenues of $450 each, occurring at the end of years 1, 2, 3, and 4. The total number of years that the investment lasts, however, is 5. The amount of the last payment, in year 5, is what you need to determine. The interest rate is 9%. Summarizing this, you have:

$$V_0 = -\$5{,}000 \quad R_{1,2,3,4} = \$450$$
$$n = 5 \qquad i = 9\%$$

Note that the $5,000 is negative here to differentiate payments from revenues.

Now, consider the question “What do I need to know?” What you want to know in this example is a the amount that the company will have to pay you in year 5 in order to give you a 9% rate of return. This is a future value in year 5, so call it V_5

. A key idea that will be useful here is that if the investment earns exactly 9% then the net present value of the investment, calculated at a 9% discount rate, must be zero. Note, also, that if the net present value is zero, then the net future value at year 5 will also be zero. (The future value of zero is also zero.) To summarize, you need to solve for a future value – a revenue – such that the future (and present) value of the entire project is zero when evaluated at a 9% discount rate.

Now, determine which formula or formulas to use to obtain this amount. You will need to calculate the future values of the known costs and revenues. If you work through the dichotomous key given earlier, you should observe that the answer to the first question in the key is "both." This tells you that you will need to use more than one formula. The first will be the single value formula to handle the single initial $5,000 investment. Either by working through the key or simply by studying the problem, you should be able to determine that you will also need the finite annual series formula to handle the four $450 payments.

The next step is to calculate the future values of the initial investment and the four revenues using these formulas. The future value, in year 5, of the initial investment is:

$$V_5 = (1.09)^5(-\$5{,}000) = -\$7{,}693.12$$

There is a bit of a complication with simply calculating the future value of the series of four revenues. Since the series of revenues ends in year 4, the future value version of the finite annual series formula will give a future value <u>in year 4</u>. Thus, you will need to compound this value forward one more year to get the future value of the series <u>in year 5</u>. That is,

$$V_4 = \frac{\$450[(1.09)^4 - 1]}{0.09} = \$2{,}057.91$$

$$V_5 = (1+i)^1 V_4 = (1.09)\$2{,}057.91 = \$2{,}243.12$$

Now, the net future value in year 5 of the known quantities in the investment is -$5,450 (-$7693.12 + $2,243.12). Since at the end of year 5 you will be $5,450 in the red, you will need a revenue of $5,450 to bring the net future value of the investment in year 5 to zero.

Again, why do you want a net value of zero in year 5? This means that the investment earns exactly a 9% rate of return. If the net future value was positive, it would mean that the investment will earn <u>more</u> than 9%. If it was negative, that would mean that the investment will earn <u>less</u> than 9%.

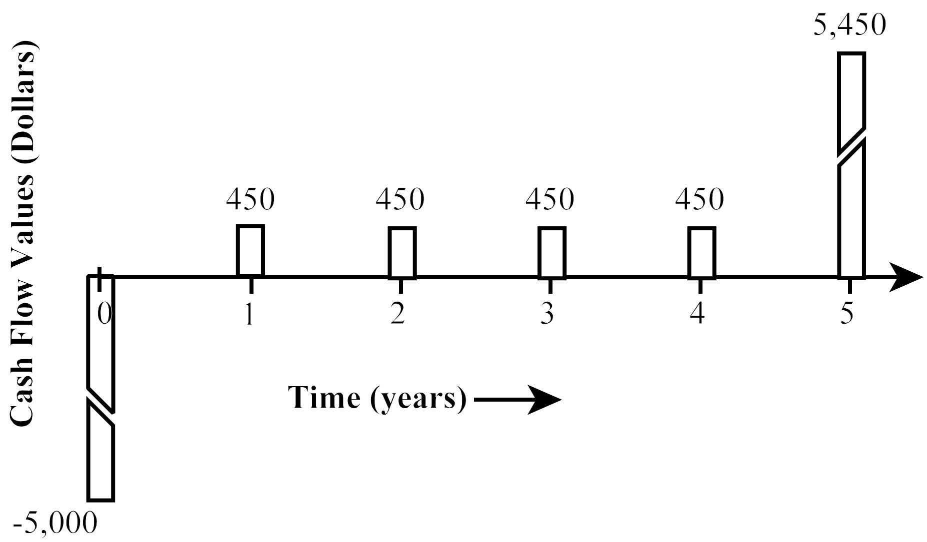

Figure 2.1. Cash flow diagram for $5,000 investment problem.

This example problem is more difficult than the first example in this section for a couple of reasons. First, it is difficult to determine just what to solve for. Second, it involves more than one type of cash flow, so more than one equation is needed to solve it. Third, the inconsistency between the fact that the finite annual series ends in year 4 and the fact that the project ends in year 5 is potentially confusing. As financial analysis problems become more complex, you will find it increasingly useful to draw a *cash flow diagram* of the problem. A cash flow diagram shows the timing of the costs and revenues of a problem. Figure 2.1 shows the cash flows associated with this last example.

Example: Tracy Treefarmer's Christmas Trees

Tracy Treefarmer wants to grow Christmas trees. She has identified a 120 acre field that she can rent for $90 per acre, paid at the beginning of each year. She plans to plant 800 trees per acre with the expectation that only 75 percent of the trees will survive and be of high enough quality to market. She estimates that site prep and planting will cost $400 per acre and that her annual management expenses – mostly pesticide treatments and shearing – will be $500 per acre. (Note: since she will need this money throughout the year, treat it as if it is spent at the beginning of the year.) In 6 years she hopes to sell the trees on the stump for $12 each. Her uncle, who is the president of the local bank, will lend her all she needs at 8 percent interest, and he will not expect to be repaid until the final sale of the trees.

a. What is the *net present value* (NPV) per acre of Tracy's (6-year) investment?

Answer: As always, start by summarizing what you know. Tracy will use 120 acres, and her interest rate is 8%. Here is a list of her expenses and revenues:

- rent: $90/ac for years 0 through 5;
- site prep and planting: $400/ac at time 0;
- annual management expenses: $500/ac for years 0 through 5;
- final return: 800 trees/ac × 75% survival rate × $12/tree = $7,200.

Since they occur at the same time each year, the rent payments and the annual management expenses can be combined. Thus, Tracy will have total annual expenses of $590/ac. These will occur in years 0 through 5. Since the first annual expense occurs at time 0, it can be combined with the initial site prep and planting expense. Thus, at time 0, Tracy will need $990 per acre. At the end of year 6, Tracy will earn $7,200/ac. Figure 2.2 illustrates these costs and revenues in a cash flow diagram.

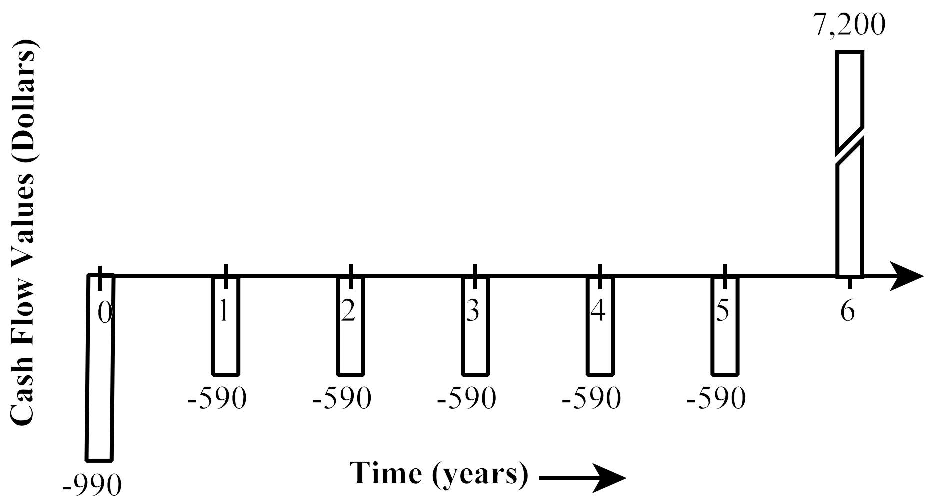

Figure 2.2. Cash flow diagram for Tracy Treefarmer problem (part a.).

The problem now is to calculate the present value of these cash flow items. Since the initial investment is already a present value, you do not need to do anything further with it. The annual management costs and rents are a finite annual series lasting five years. The final revenue can be discounted with the single value formula.

The present value of the five annual costs is:

$$V_0 = \frac{-\$590/ac\ [(1.08)^5 - 1]}{0.08(1.08)^5} = -\$2{,}355.70/ac$$

The present value of the final revenue is:

$$V_0 = \frac{\$7{,}200/ac}{(1.08)^6} = \$4{,}537.22/ac$$

Thus, the net present value (NPV) of Tracy Treefarmer's Christmas tree plantation project is:

$$NPV = V_0 = \$4{,}537.22 - \$2{,}355.70 - \$990 = \$1{,}191.52/ac$$

If Tracy earns this on 120 acres, the NPV for the total project will be $142,982.

b. What is the net present value per acre of her investment if 90% of the trees live and are good enough to market?

Answer: Note that this will change only the final revenue; none of the other values need to be re-calculated. If 90% of her trees are marketable, instead of earning $7,200/ac in year 6, Tracy will earn $8,640/ac. The present value of this return is:

$$V_0 = \frac{\$8{,}640/ac}{(1.08)^6} = \$5{,}444.67/ac$$

Now, her NPV is

$$NPV = V_0 = \$5{,}444.67 - \$2{,}355.70 - \$990 = \$2{,}098.97/ac$$

In this case, the NPV for the entire 120 acres will be $251,876. Note that increasing her survival rate from 75% to 90% increases Tracy's NPV by over 75%.

c. Tracy has not figured in her salary yet. How much can she afford to pay herself at the end of each year and still break even on the investment? Assume a 75 percent survival rate.

Answer: The difficult question here is "What do I want to know?" Tracy's annual salary will be an additional annual expense for the project. These annual expenses will occur at the end of years 1 through 6. The problem is to determine the amount of this annual payment. What does the phrase "break even on the investment" mean? In this case, it means that the NPV of the investment, at 8%, should equal zero. Remember that this means that the investment earns exactly 8%, which is the rate Tracy must pay her uncle on the series of loans for the project. Thus, the problem is to calculate an annual payment that Tracy could make to herself at the end of each year so that the present value of the entire project equals zero using a discount rate of 8%. Since the NPV of the project equals $142,982 without the salary payments (assuming a survival rate of 75%), an annual payment for a 6-year finite annual series that has a present value of $142,982 will bring the NPV of the project to zero. This value can be calculated using the payment version of the finite annual series formula.

$$R = \frac{V_0\,(i)(1+i)^n}{[(1+i)^n - 1]} = \frac{\$142{,}982\,(0.08)(1.08)^6}{[(1.08)^6 - 1]} = \$30{,}929$$

Note that this salary is based on the assumption that 75% of the trees will be marketable. If Tracy does well and a larger proportion survive, then she will be able to give herself a bonus in the last year. She will have a problem, however, if her survival rate is lower than 75%. Thus, she would be prudent to base her projections on a conservative survival assumption.

d. Tracy has the option of buying the land. If she pays $1,000 per acre for the land and sells it for $1,000 per acre after 6 years, what will be the NPV of her tree-planting project? (Assume a 75% survival rate and that she will <u>not</u> pay herself an annual salary.)

Answer: Consider how buying the land changes Tracy's cash flows. First, it adds a cost of $1,000 per acre at time zero. It adds a similar revenue ($1,000/ac) at the end of the project. Also, since she will own the land in this case, she will not have to pay rent. (She will, however, probably have to pay taxes, but we will ignore that here in the interest of simplicity.) Figure 2.3 summarizes Tracy's cash flows in this case.

The present value of the five annual $500/ac management costs is:

$$V_0 = \frac{\$500/ac[(1.08)^5 - 1]}{0.08\,(1.08)^5} = \$1{,}996.36/ac$$

The present value of the final revenue of $8,200/ac is:

$$V_0 = \frac{\$8{,}200/ac}{(1.08)^6} = \$5{,}167.39/ac$$

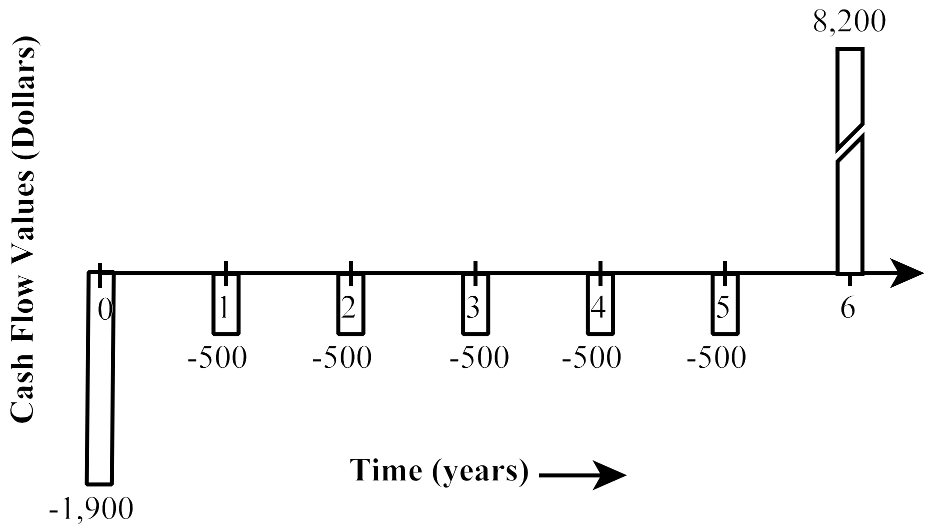

Figure 2.3. Cash flow diagram for Tracy Treefarmer problem (part d.).

Thus, the NPV per acre of the Christmas tree project, assuming a 75% survival rate and that she buys the land, is:

$$NPV = V_0 = \$5{,}167.39 - \$1{,}996.36 - \$1{,}900 = \$1{,}271.03/ac$$

The NPV for all 120 acres, with these assumptions, is $152,524. This is higher than the $142,982 that she would earn by renting the land, so this analysis indicates that it would be best for her to buy the land rather than renting. (Of course, in order to keep the problem relatively simple, we have ignored some of the costs of land ownership – for example, taxes.)

Significant Digits

One issue that frequently comes up with calculations such as those discussed in this chapter is the number of *significant digits* that an answer should have to be "right." The answer to this question depends on the problem and on the precision of the information that went into the calculation. For example, answers to problems involving money (e.g., US dollars) usually should be precise to the smallest unit of the currency (e.g., cents). However, you should always consider your audience and not give them more detail than they want. For example, few taxpayers would care whether the federal budget for 1996 was $2,972,348,643.91 or whether it was $3 trillion. Furthermore, if some of the data used to calculate a number are only accurate to one or two digits, it would not make sense to report an answer with more than a couple of significant digits. A key point to understand, however, is that if you round off answers that are *intermediate calculations* – i.e., numbers that will be inputs to another problem – you could lose a lot of precision in your final answer. Therefore, you should maintain enough significant digits in your intermediate calculations so that the answers to subsequent calculations are also correct to the desired number of significant digits. Usually, it is best to maintain a high level of precision throughout all of your calculations and then round off only the final result to the desired number of significant digits. For example, if you want your final answer to be accurate to at least 3 significant digits, you probably should carry at least 5 significant digits in all your intermediate calculations, and more would be even better.

How can you tell how many significant digits are in a number? The best answer is the minimum number of digits that would be required to express the same number using scientific notation. Generally, zeros between a non-zero digit and the decimal point are not significant digits. For example, 1,300,000 has two significant digits, as does 0.0000013. The first number can be expressed as 1.3×10^{6}, and the second number can be expressed as 1.3×10^{-6}. The number 1,020 has 3 significant digits. The first zero is a significant digit because it is between two other non-zero digits.

5. Study Questions

1. Explain what discounting and compounding are.

2. Explain why a dollar earned in the future does not have the same value as a dollar earned today.

3. What are present values and future values and why are they different?

4. What is time preference?

5. List two fundamental reasons why people are willing to borrow money, knowing that they will have to pay interest on the borrowed money.

6. Why are there opportunity costs when you use money?

7. Would a higher interest rate give you a better price or a worse price if you wanted to buy dollars for next year with dollars today?

8. What if you wanted to buy dollars for today with next year's dollars, would you prefer a higher or lower interest rate?

9. Think of the interest rate as a price in the market for money to borrow and invest. What would happen if the interest rate was too high?

10. What is the most significant use of the infinite annual series formula for foresters?

11. Give an example where you might use the finite annual series formula.

12. Derive the formula for the annual payment (*R*) needed to achieve a given future value (V_n) after *n* years at an interest rate *i*. (Start with the future value version of the formula.)

13. What is the relationship between the value of an asset and the expected present and future costs and revenues associated with owning the asset?

14. What is a cash flow diagram?

15. What guidelines should you consider when determining the number of significant digits to report in your results?

17. How many significant digits does each of the following numbers have?

    a. 96,100 **b.** 0.029 **c.** 3.140 **d.** 1,050,00 **e.** 12.103

6. <u>Exercises</u>

1. If you expect a timber harvest to yield $235,000 in 60 years, and your minimal acceptable rate of return is 6%, what is the harvest worth to you today? (What is its present value?)

2. You wish to harvest a 40-acre oak stand. You want to regenerate the stand with oak, but there is currently no advance regeneration of any species – just a lot of hay-scented fern. Your best bet is to partially harvest the stand, then kill the ferns with herbicides and put a fence around the stand. The fence and the herbicide treatment will cost $8,500. In five years, you should have enough advance regeneration to allow you to harvest the rest of the stand. It will probably be another eighty years before the next stand can be harvested. Since the current investment of $8,500 is an investment in the future stand – the current stand can be harvested with or without the investment – how much additional value, per acre, must this investment generate in 85 years in order to earn a 5% rate of return?

3. Assume that the value of an asset is based on the present value of the expected revenues and costs associated with owning the asset. At an alternate rate of return of 7%, how should a $1 increase in the property tax affect the value of a property?

4. The 30-year mortgage on a $100,000 house carries a 8% annual interest rate.

   a. What is the equivalent monthly interest rate?

   b. What is the monthly payment on the mortgage? (For your information, a house payment usually also includes taxes and insurance, which are not included here.)

   c. How much would the monthly payment be if the interest rate was 7%?

d. How much would the monthly payment be, at the 8% rate, if the loan period was 15 years?

5. Assume that it costs $250/ac to successfully regenerate a black cherry stand, that the annual costs include $2/ac in property taxes plus $1/ac for management, and that the stand can produce 11 mbf/ac, valued at $1,000/mbf, and 13 cords of pulpwood, valued at $10/cord, on an 80-year rotation.

   a. What is the present value at the beginning of the rotation, at a 4% discount rate, of the costs and revenues from one 80-year rotation of this stand?

   b. What is the present value at the beginning of the rotation, at a 5% discount rate, of the costs and revenues from one 80-year rotation of this stand?

6. If you invest $2,200 in a stock which grows at 11% annually, how much would it be worth in 22 years?

7. If an investment triples in value in 10 years, what (annual) rate of return will the investment have earned?

8. a. At 8% interest, what is the present value of annual tax payments of $10, beginning in one year and continuing in perpetuity?

   b. At 8% interest, what is the present value of annual tax payments of $10, beginning today and continuing in perpetuity?

9. What is the future value (in year 12) of 12 annual hunting lease payments of $200 each, assuming they are reinvested at 5%? The hunting leases are paid at the end of each year.

10. a. Using a 6% interest rate, find the present value of timber harvest income of $3,300 that occurs every 40 years, starting 40 years from now.

    b. What percentage of the present value calculated in part a comes from the first harvest and what percentage comes from the remaining harvests?

    c. Assume the timber stand in part a is already 20 years old. Find the present value of timber harvest income of $3,300 every 40 years, starting 20 years from now.

11. You are planning an even-aged forest management program for red pine to be planted this year and harvested for pulp in 35 years. You plan to spend $100 per acre to fertilize the trees at age 20. What increase in the harvest income per acre (at age 35) must the fertilization produce in order for you to earn 6% on your fertilization expenditure?

12. You are thinking about buying a new pickup. The best interest rate you can get on a loan is 9% (annual). You can afford a monthly payment of $350.00.

    a. What is the equivalent monthly interest rate. (*Hint*: $i_m = (1 + i_a)^{(1/12)} - 1$.)

    b. How much can you afford to pay for the pickup if you get a 3-yr loan?

    c. How much can you afford to pay for the pickup if you get a 5-yr loan?

    d. How much can you afford to pay for the pickup if you save $350 each month for 3 years before buying?