CHAPTER 3: FINANCIAL ANALYSIS WITH INFLATION

Up to now, we have mostly ignored inflation. However, inflation and interest are closely related. It was noted in the last chapter that interest rates should generally cover more than inflation. In fact, the amount of interest earned over inflation is the only real reward for an investment. Consider an investment of $1,000 that earns 4% over a one-year period, and assume that during that year the inflation rate is also 4%. At the end of the year, the investor receives $1,040. Has the investor gained anything from this investment? On the surface, she has earned $40, so it seems like she has. However, because of inflation, the purchasing power of the $1,040 that she now has is the same as the purchasing power that the initial investment of $1,000 would have had a year earlier. Essentially, the investor has earned nothing. In this case, the *real rate of return*, the rate of return after inflation, was zero.

It is easy to confuse discounting and compounding, which account for the time value of money, with deflating and inflating, which account for the changing purchasing power of money. *Inflation* reduces the purchasing power of a unit of currency, such as the dollar, so a dollar at one point in time will not have the same purchasing power as a dollar at another point in time. *Inflating* and *deflating* are used to adjust an amount expressed in dollars associated with one point in time to an amount with equivalent amount of purchasing power expressed in dollars associated with another point in time. On the other hand, discounting and compounding adjust the time at which the value is assumed to occur. Changes over time in the purchasing power of a unit of currency such as the dollar are not the same as differences in the values of costs and revenues that result from their timing.

To help you see that inflating and deflating are truly different from compounding and discounting, try to imagine a world without inflation; i.e., assume a world where the prices of goods never change. If the interest rate merely accounts for inflation, then the interest rate would also be zero in such a world. But what would happen if the interest rate was also zero? Would people still save and invest their money? On first thought, it seems like some people would. They would want to save so they would have money for future purchases, right? But would that be necessary? If they need money for future purchases, they would be able to borrow the money when they need it with no interest – after all, the interest rate is zero. Thinking about it this way, it seems likely that in a world with no inflation and with an interest rate of zero, few would bother to save and nearly everyone would want to borrow. There would be little to no supply of savings and a very high demand for money to borrow. This would be a classic case of a market out of equilibrium. In order to have an equilibrium in the market for money, the price of using money – and the reward for lending money – would have to rise. The rate of interest that is earned in excess of the inflation rate is this price of money. This rate, called the *real interest rate*, is determined by the balance between the demand for money to borrow and the supply of money from lenders. Even without inflation it would still be necessary to charge interest to achieve a balance in the market for money. In a world <u>with</u> inflation, the interest rate must cover <u>both</u> inflation and the cost of capital.

Because the true profitability of an investment is determined by the amount earned over inflation, it is very important for you to be able to account for inflation when doing a financial analysis. Since inflation is nearly always present in real-world situations, you will not be able to reliably apply the financial analysis techniques you learned in Chapter 2 without accounting for inflation. Generally, the best way to account for inflation is to remove it from all of the values in your analysis by converting all values to current dollar values and by using a real interest rate – that is, an interest rate with inflation removed from it. Furthermore, removing inflation usually simplifies the analysis of forestry investments. This chapter is intended to show you how to do this.

1. What is Inflation?

Inflation is an increase in the average price level, reducing the purchasing power of the dollar. The *inflation rate* is the average annual rate of increase in the price of goods. Inflation is measured by a variety of indexes. The broadest and most commonly-used are the *consumer price index* (CPI) and the *producer price index* (PPI). The CPI tracks the average increase in the cost of a standard collection of consumer goods, and the PPI tracks the average increase in the cost of a standard collection of production inputs. Actually, there are dozens of CPI’s and PPI’s. Generally, when we use the CPI we will use the CPI for “all items.” For the PPI, there are both industry and commodity series. For some purposes, we will use “all commodities” while for other purposes, we might be interested in the PPI for a particular industry or commodity, such as “sawmills” or “lumber.” The standard collections of goods that define these indices – called *market baskets* by economists – are modified and updated over time to reflect changes in consumption and production patterns and technology.

The values of the CPI and the PPI are arbitrarily set to 100 at some point in time. The United States Bureau of Labor Statistics (BLS) is responsible for calculating the values of these indices for the U.S. dollar. You can find the values of these indices for different points in time by visiting the BLS web page at http://stats.bls.gov. To calculate the inflation rate over a period of time, you need to choose an appropriate index and obtain the index value at the beginning of the time period and at the end of the time period. The ratio of the two index values indicates how much the average price level increased over the time period between the two index values. The average annual inflation rate over the period between the two index values is calculated using a formula that is analogous to solving for the interest rate in the single value discounting formula:

$$k = \left[\sqrt[(t_2 - t_1)]{\frac{CPI_{t_2}}{CPI_{t_1}}} \right] - 1$$

In the above equation, k represents the average inflation rate between times t_1 and t_2. The PPI would be used in an analogous way to calculate the average inflation rate for producer goods.

Example: Calculating the average annual inflation rate over a given time period.

What was the average inflation rate, as measured by the CPI, between 1975 and 2005?

Answer: If you visit the BLS web page at http://stats.bls.gov you will find that there are many versions of the CPI. We will use the new (revised) Consumer Price Index – All Urban Consumers – U.S. All items, 1982-84=100 (Series ID: CUUR0000SA0). The CPI was 53.8 in 1975 and 195.3 in 2005. Note that this means that the average price level almost quadrupled during this 30-year period.

Let k represent the inflation rate and CPI_{75} and CPI_{05} represent the Consumer Price Index in 1975 and 2005, respectively. The interest rate version of the single-value discounting formula can be used to calculate the annual inflation rate, as follows:

$$k = \left[\sqrt[(05-75)]{\frac{CPI_{05}}{CPI_{75}}} \right] - 1 = \left[\sqrt[30]{\frac{195.3}{53.8}} \right] - 1 = 0.043912 - 1 = 4.3912\%$$

Thus, the average annual inflation rate averaged about 4.4% over this period.

2. <u>Components of the Interest Rate</u>

It may help you develop a better understanding of the relationship between interest rates and inflation to think about breaking the interest rate down into the different components that influence interest rate levels. This discussion will also develop some of the basic terminology related to interest rates. The interest rate that most people usually talk about is called the nominal interest rate.

The *nominal interest rate* (sometimes simply called the *nominal rate*) is the interest rate that is quoted by banks, credit cards, stock brokers, etc. The nominal rate includes both the cost of capital and inflation. It is the rate that is used to discount actual, inflated future values.

Part of the nominal interest rate goes to cover inflation, and the rest is what is "really" earned on an investment. What is left over after inflation is called the real interest rate.

The *real interest rate* (also called the *real rate*) is the rate earned on a capital investment after accounting for inflation. Inflation has been removed from the real interest rate. The real interest rate should be used to discount future values that are expressed in current dollar values.

A nominal interest rate can be broken out into two components: the inflation rate and the real interest rate. That is,

the nominal rate ≈ the inflation rate + the real rate.

(The "≈" symbol is used here to indicate that this relationship is only approximately correct.) This relationship is a critical one, as it shows simply and intuitively how nominal interest rates include components to cover both inflation and the cost of capital.

The real interest rate is the real return for investing one's money; it measures how much the actual purchasing power of your investment has increased. As mentioned in the last chapter, all investments are not alike, and different investments should be expected to earn different real rates of return. Five factors that affect the expected real rate of return of an investment are:

- risk – the degree of uncertainty regarding the return on the investment,
- illiquidity – how difficult it is to withdraw cash from the investment outside the investment's usual payment schedule,
- transactions costs – appraisal fees, loan origination fees, commissions, etc.,
- taxes – different investments have different tax implications: such as capital gains treatment, property taxes, etc., and
- time period – longer investments tend to be less liquid and more uncertain.

Each of these factors generally adds to the real rate of return that an investment must earn in order to be attractive to investors. That is, all other things being equal, a riskier investment must promise a higher expected real rate of return than a safer investment. Similarly, an investment that locks up investors' money would be expected to produce a higher real rate of return, relative to an investment where investors can get their money out at any time.

The real rate of return on an investment can be broken down into components that account for each of these factors, plus a component reflecting the pure cost of using money – the pure interest rate.

> The *pure interest rate* is the real rate of return earned by an imaginary, risk-free, perfectly liquid, tax-free investment with no transactions costs and a very short time period.

Obviously, no such investment exists, but it is useful to think of such an investment as a standard of comparison for other investments. One of the closest actual investments to this idealized standard is the 3-month U.S. Treasury bill. The U.S. Treasury is unlikely to default on it's debt; there is an active market for trading these bonds; the only taxes on the returns are income taxes; in large denominations, the transactions costs on these bonds are small; and the time-period of the bonds is relatively short. Average historical real rates of return on 3-month treasury bills have averaged about one percent over the past 100 years and have ranged mostly between minus one percent and three percent.

All investments involve some risk, varying degrees of illiquidity, taxes, transactions costs, and varying time periods. The expected real rate of return of an investment consists of a pure rate of return plus differing components, or *premiums*, for each of these factors. That is,

> the real rate ≈ the pure rate + a risk premium + an illiquidity premium + a tax premium + a transactions cost premium + a time period premium.

Each of these premiums is additional compensation investors require in order to assume the additional risk, illiquidity, taxes, transactions costs, and time period of a particular investment. For example, riskier investments require a risk premium to compensate investors for assuming a greater level of risk associated with those investments. The more risky an investment, the higher this risk premium will have to be. Without this premium, people would not be willing to place their money in more risky investments.

In summary, nominal interest rates are made up of many components, each of which is necessary to compensate investors for different aspects of the investment. A key component is inflation. After inflation, the remaining interest is called the real interest rate, as this is the "real" return on the investment. The real rate, in turn, includes several components, including a pure rate – which accounts for the opportunity costs of using money – plus premiums to account for the riskiness, illiquidity, taxes, transactions costs, and time period of the particular investment.

3. Combining Interest Rates

So far in this chapter the components of interest rates have been treated as if they were additive. For example, it was stated that the nominal interest rate is basically the sum of the inflation rate and the real interest rate. This view, however, is only approximately correct. Just as the compounded interest over two years is not exactly 2 times the annual interest rate, when two interest rate factors are combined to give another interest rate, the combined interest rate is not exactly the sum of the two other interest rates. Thus, while the nominal rate is a combination of the real rate and the rate of inflation, it is not precisely correct to say that the nominal rate is equal to the sum of the real rate and the inflation rate. It is a pretty good approximation, but not exactly correct.

To see how interest rates should be combined properly, consider first the case of the interest earned over two periods, i.e., the two-period interest rate, i_2. For two periods, the rate earned is:

$$i_2 = (1+i)(1+i) - 1 = 1 + 2i + i^2 - 1 = 2i + i^2 \cong 2i$$

Note that i_2 is approximately equal to $2i$, but not quite. There is the extra term i^2. Note that i^2 will generally be small, however.

Combining two rates (such as the real rate and the inflation rate) to get another rate (in this case, the nominal rate) requires an analogous formula. Before introducing the formula, some new notation is needed. Let:

i = the nominal interest rate,
r = the real interest rate, and
k = the inflation rate.

Now, the formula for combining the real interest rate and the inflation rate to get the nominal interest rate is:

$$i = (1 + r)(1 + k) - 1$$

Note the similarity of this relationship to the relationship above for i_2. This expression can be expanded as follows:

$$i = (1 + r)(1 + k) - 1 = r + k + rk \cong r + k$$

So, just as with the two-period rate, the combined rate here – in this case, the nominal interest rate – equals the sum of the two component rates ($r + k$), plus the product of the two component rates (rk).

The simplest form of the equation expressing the relationship between the nominal rate, the real rate, and inflation, is:

$$(1 + i) = (1 + r)(1 + k)$$

This can be rearranged to solve for i:

$$i = (1 + r)(1 + k) - 1$$

or, to solve for r:

$$r = \frac{(1 + i)}{(1 + k)} - 1$$

or, to solve for k:

$$k = \frac{(1 + i)}{(1 + r)} - 1$$

Example: Solving for the Nominal Rate.

If you wish to earn a real rate of at least 3% on an investment and you expect the inflation rate will be 4% during the investment period, what is the minimum nominal rate you must earn?

Answer:

$$i = (1 + r)(1 + k) - 1 = (1.03)(1.04) - 1 = 0.0712 = 7.12\%$$

Note that this also equals *(.03) + (.04) + (.03)(.04) = .07 + .0012*

Example: Solving for the Real Rate.

If the nominal interest rate earned on an investment was 8% and the inflation rate was 3% during the investment period, what real rate was earned on the investment?

Answer:

$$r = \frac{(1+i)}{(1+k)} - 1 = \frac{(1.08)}{(1.03)} - 1 = 0.0485 = 4.85\%$$

4. Nominal Values and Real Values

You've undoubtably heard the phrase "learning the value of a dollar." However, as you also have probably learned, the value of a dollar is not constant. When my father started his career in the late 1940s, he earned a salary of about $4,000. At the time, this was a fairly respectable salary for someone just graduating from college. In terms of buying power, it corresponded roughly to a starting salary of $32,400 today. That is, the buying power of $32,400 in 2005 is roughly the same as the buying power that $4,000 had in 1948. If in 1948 someone was considering an investment that would pay them $32,400 in 2005, they would want to account for the fact that the purchasing power of $32,400 in 2005 would be about the same as the purchasing power of $4,000 in 1948. In other words, they would want to know that the promised *nominal value* – in 2005 dollars – of $32,400 would have a corresponding *real value* – in 1948 dollars – of only $4,000.

> A *nominal value* is a value expressed in the currency of the year in which the value occurs, i.e., a value that is expressed in dollars that have the purchasing power of dollars in the year when the value occurs.

For example, a nominal value that will be paid or earned in the year 2020 is expressed in terms of dollars with the purchasing power that dollars will have in the year 2020. Because of inflation, it is likely that a dollar will have less purchasing power in 2020 than it does today.

> A *real value* is a value that is expressed in terms of dollars with the same purchasing power as dollars today, or at any other meaningful reference point in time.

Note that from the perspective of people living in 1948 a real value would be one that is expressed in the dollars of their time – 1948 dollars. Generally, however, real values are expressed in current dollars – i.e., dollars with an equivalent amount of purchasing power as dollars today. Current dollars are the relevant reference point for most analyses since they are the dollars we are familiar with.

Because of inflation, when speaking of future values, it is generally necessary to clarify whether values are real future values – i.e., values expressed in dollars with the same purchasing power as dollars today – or nominal future values – i.e., values expressed in terms of future dollars. Although both situations are common, if someone does not expressly state whether they are referring to real or nominal future values, you should assume that they are referring to nominal future values. For example, in legal contracts – such as the mortgage on a house – future dollar transactions are nearly always spelled out in nominal terms. Thus, the monthly payment on my house is a fixed, nominal amount. If I am still making payments on it in 30 years when it is scheduled to be paid off, the nominal dollar value of my payments will be the same as they are today.

On the other hand, we often think of future values in real terms. For example, in estimating the future revenue from a timber sale – possibly twenty, thirty, or even more years from now – it is common to assume that the revenue can be calculated by multiplying the expected future harvest volume times a stumpage price similar to today's price. It would be very difficult to predict stumpage prices many years from now, so it is common to simply assume that the future price is likely to be similar to the price today. This may be a reasonable assumption, although not necessarily the best. It is only reasonable, however, if we mean that the real future price will likely be the same as today's price. Because of inflation, the nominal future price is almost certain to be higher than today's price. In fact, over the last several decades in many areas stumpage prices for sawtimber have increased much faster than inflation. It may, therefore, be more reasonable to assume that stumpage prices will increase faster than inflation. The next section discusses ways to handle this. In any case, it is highly unlikely that stumpage prices will remain constant in nominal dollars. Unless the inflation rate is zero or negative, that would be the same as assuming that real stumpage prices will decline.

It is useful to convert nominal values to real values so that we can judge the value of a dollar amount relative to the currency that we are familiar with – today's dollars. For example, suppose you are promised $1,000 (a nominal value) that you will receive in the year 2020. Since this is a nominal value, it is expressed in terms of actual dollars that you will receive in 2020. Because of inflation, it is likely that $1,000 will have less purchasing power in 2020 than it does today. It may be hard for you to judge, then, the true value of the promised amount. It would be useful for you to at least be able to estimate what the $1,000 will be worth in today's dollars – a currency you are familiar with.

5. Deflating and Inflating

Deflating and inflating are used to convert nominal values to real values and vice versa. Deflating and inflating are different from discounting and compounding because they do not change the time at which a value is assumed to occur; they only change the kinds of dollars that those values are expressed in.

> *Deflating* is the process of converting a value expressed in the currency of a given point in time into a value with an equivalent amount of purchasing power expressed in the currency of an earlier time; for example, converting a value expressed in 2020 dollars to an equivalent value expressed in 2005 dollars.
>
> *Inflating* is the process of converting a value expressed in the currency of a given point in time into a value with an equivalent amount of purchasing power expressed in the currency of a later time; for example, converting a value expressed in 2005 dollars to an equivalent value expressed in 2020 dollars.

There is a fundamental difference between inflating past values to the present and deflating future values to the present. For past values, we have a history of the inflation rate since the past value occurred, and an index such as the CPI or the PPI can be used to convert past dollar values to current dollar values or vice versa. These indices account for variations in the inflation rate over time and indicate the appropriate amount of inflation for any historical period. In order to inflate a past value to a current dollars, the past value should be multiplied by the ratio of the current price index over the price index for the time at which the past value occurred. For example, to convert the $4,000 that my father earned in 1948, use the CPI for 1948 – 24.1 – and the CPI for 2005 – 193.2 – as follows:

$$Salary\,(\$2005) = Salary\,(\$1948) \times \left(\frac{CPI_{'05}}{CPI_{'48}}\right) = \$4,000 \times \left(\frac{195.3}{24.1}\right) = \$32,415$$

For future values, we can only guess what the inflation rate will be between now and the time when the future value will occur – future values of the CPI or PPI obviously are not available. Since we can hardly guess what the average future inflation rate will be, it doesn't make much sense to project a different inflation rate for each year. Thus, a single, average inflation rate is usually projected. A mathematical formula which looks a lot like the formula for calculating the present value of a single sum is used to deflate future values. First, some notation; let:

V_n^* = a nominal future value occurring in year *n*, and
V_{n}^* = a real future value occurring in year *n*.

The formula for deflating a nominal future value (i.e., calculating the equivalent real future value) is:

$$V_n = (1+k)^{-n} V_n^* = \frac{V_n^*}{(1+k)^n}$$

This formula can be rearranged to give the formula used to inflate a real future value and convert it into a nominal future value:

$$V_n^* = (1+k)^n V_n$$

These equations are very similar to the different versions of the basic formula for discounting and compounding a single sum, but they are <u>not</u> the same. First, these equations do not change the time at which the values are assumed to occur. That is, the values on both the left and the right sides of the equations – V_n and V_n^* – are future values; one is a real future value and the other is a nominal future value. Thus, while deflating and inflating appear to be mathematically the same as discounting and compounding, they are conceptually quite different. The second key difference between the formulas presented here and the single-sum discounting formulas is the rate variable. The inflation rate is used to deflate and inflate values, and a discount rate (i.e., interest rate) is used to discount or compound values.

Example: The Price of Milk in 1972

A gallon of milk cost about $1.00 in 1972. What did a gallon of milk cost then, in current (2005) dollars?

Answer: First, you will need the CPI for 2005 and 1972. The index values can be obtained from the BLS web site, and they are: CPI $_{'72}$ = 41.8 and CPI $_{'05}$ = 195.3. Now, the real price ($2005) of milk can be calculated as follows:

$$P_{m,72}(\$2005) = P^{*}_{m,72}(\$1972)\times\left(\frac{CPI_{05}}{CPI_{72}}\right) = \$1.00(\$1972)\times\left(\frac{195.3}{41.8}\right) = \$4.67(\$2005)$$

Example: The Real Value of $1,000 in 2020

If inflation averages 3% per year between 2005 and 2020, what will be the real value, in $2005, of a nominal value of $1,000 in 2020?

Answer: Deflate the $1,000 value, using a 3% inflation rate, for 15 years:

$$V_{'20} = \frac{V^{*}_{'20}}{(1+k)^{n}} = \frac{\$1{,}000}{(1.03)^{15}} = \$641.86$$

6. Accounting for Inflation When Discounting

Since there are two kinds of future values – nominal future values and real future values – it is necessary to discuss the appropriate way to discount each. The rule is straightforward:

☞ Discount real future values with a real interest rate, and discount nominal future values with a nominal interest rate.

The corollary to the last rule is that when a present value is compounded with a nominal interest rate the result is a nominal future value, and when a present value is compounded with a real interest rate the result is a real future value. These rules should seem fairly obvious, but they are very commonly violated by students new to financial analysis – and even sometimes by professional analysts. Unfortunately, when these rules are broken, the resulting analysis is likely to be quite wrong. Usually such mistakes occur because the student or analyst is not absolutely clear whether the future value being discounted is a real value or a nominal value. Serious errors will occur if this distinction is not clear and explicit.

Table 3.1 summarizes the use of the different rates for different types of operations. (Figure 3.1, below, also helps show the relationships between these operators and the different rates, present and future values.) Here are a few basic points to keep in mind about real future values, nominal future values, and accounting for inflation in discounting and compounding:

☞ Real future values are uninflated; nominal future values are inflated.

- ☞ Converting a nominal future value into a real future value is an example of *deflating*.
- ☞ Converting a real future value to a nominal future values is an example of *inflating*.
- ☞ To convert real future values to nominal future values or vice versa, you must use the inflation rate.
- ☞ To discount real future values, use a real interest rate.
- ☞ To discount nominal future values, use a nominal interest rate.
- ☞ Compounding a present value with a real interest rate results in a real future value.
- ☞ Compounding a present value with a nominal interest rate results in a nominal future value.

Table 3.1. Formulas for deflating, inflating, discounting, and compounding real and nominal future values.

Formula	**Variables** (replace *x*, *y*, and *z* in the formula)		
	k, V_n , V_n^*	r, V_0 , V_n	i, V_0 , V_n^*
$y = z / (1 + x)^n$	Deflating	Discounting real future values	Discounting nominal future values
$z = (1 + x)^n y$	Inflating	Compounding; results in real future values	Compounding; results in nominal future values

As mentioned earlier, sometimes the nominal price of a specific product will increase at a rate different than the general inflation rate. So, things can, and will, get even more complicated. For now, make sure you understand these relationships. Changing real prices are discussed in the next section.

Example: Inflation Example 1

A bond that you buy today for \$508.35 matures in 10 years, when it will be worth \$1,000. (Can you verify that the bond earns 7%?) This \$1,000 is an actual amount that you will get paid; therefore it is a nominal amount, and the 7% you will earn on the bond is a nominal rate.

a. If you expect inflation to average 3% during the 10-year period until the bond matures, what real rate of return will you will earn on the bond?

Answer:

$$r = \frac{(1+i)}{(1+k)} - 1 = \frac{1.07}{1.03} - 1 = 0.0388 = 3.8835\%$$

b. What will be the real future value of the bond?

Answer: Note that there are two ways to calculate the real future value of the bond: 1) by deflating the nominal future value ($1,000):

$$V_n = \frac{V_n^*}{(1+k)^n} = \frac{\$1{,}000}{(1.03)^{10}} = \$744.09$$

or 2) by compounding the present value using the real interest rate we calculated above:

$$V_n = (1+r)^n V_0 = (1.038835)^{10}\$508.35 = \$744.09$$

In summary, the investment of $508.35 today earns $1,000 in nominal dollars ten years from now. Due to expected inflation during that period, the buying power of the $1,000 earned in ten years will equal the buying power of $744.09 today. In other words, in ten years a nominal value of $1,000 will equal a real value (in today's dollars) of $744.09. Figure 3.1 summarizes these relationships.

Example: Inflation Example 2

Assume a short-rotation hybrid poplar plantation can be established for about $600 per acre. Good land will be required, which can currently be rented for $100 per acre per year (paid at the beginning of the year). You expect the land rent to go up at about the same rate as the inflation rate, which you expect to be about 4% a year. After 7 years, you project the plantation will produce 20 tons of chips per acre. The current price for chips is $100 per ton, and you expect this price to go up at about the rate of inflation. You would like to earn a real rate of 8% on your investment. At this interest rate, what is the present value per acre of the hybrid poplar plantation venture?

Answer: At time zero, $700 dollars will be required to establish the plantation and pay the first year's rent. Since the land rent is expected to go up at the same rate as inflation, the real rent paid in years 1 through 6 will be constant, at $100/ac. (Think about this, as it is an important concept.) In year seven, $2,000/ac (20 tons of chips at $100/ton) will be earned. The table below shows the net revenue stream over time, in real and inflated values. It also shows the present value of each period's net revenue. Below the table are some example calculations.

To calculate the values for year 5:

The real annual rent is $100 per acre. Because of the 4% inflation rate, it has become (nominally):

$$V_n^* = (1+k)^n V_n = 1.04^5 \times \$100 = \$121.67$$

This value is a nominal future value, so it should be discounted with a nominal discount rate. The nominal discount rate is calculated as follows:

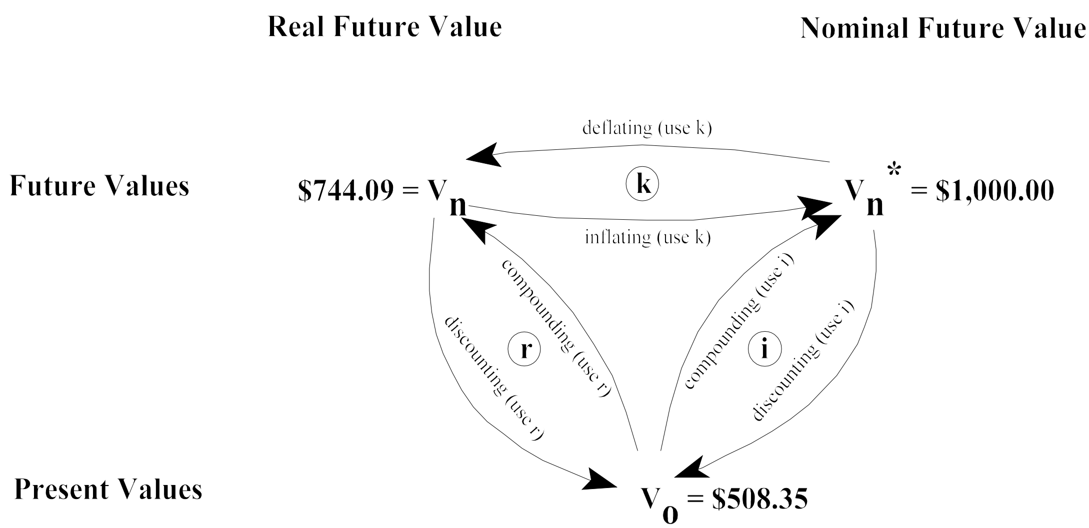

Figure 3.1. The relationship between the real future value, the nominal future value, and the present value using the values from Example 1.

$$i = (1 + k)(1 + r) - 1 = (1.04)(1.08) - 1 = 0.1232$$

Now, discount the nominal value of $121.67 dollars for five years using this nominal discount rate:

$$V_0 = \frac{\$121.67}{(1.1232)^5} = \$68.06$$

Thus, the present value of the rent for year 5 is $68.06. Note that you should get the same present value if you discount the real future value ($100) using the real discount rate (8%):

$$V_0 = \frac{\$100}{(1.08)^5} = \$68.06$$

The present value of the project is just the sum of the present values from each year. You should verify some of the other numbers in the table for yourself, going through these same steps.

Table 3.2. Real and inflated cash flows for hybrid poplar example, and present values.

Year	Real Net Revenue	Inflated Net Revenue	Present Value
0	-700	-700.00	-700.00
1	-100	-104.00	-92.59
2	-100	-108.16	-85.73
3	-100	-112.49	-79.38
4	-100	-116.99	-73.50
5	-100	-121.67	-68.06
6	-100	-126.53	-63.02
7	$2,000	$2,631.86	$1,166.98
Net Present Value	---	---	$4.70

Before leaving this example, consider for a moment what the present value of $4.70 per acre means. This seems like a pretty small number. Does it mean that this investment is not very profitable? No. The fact that the present value of the project is positive indicates that the investment does produce at least the desired alternate rate of return; i.e., it earns at least a real rate of return of 8%. This is a good real alternate rate of return, so it means the investment is actually reasonably profitable. The fact that the net present value is small indicates that the investment does not earn much more than the 8% real alternate rate of return. In other words, the investment earns almost exactly 8% (real).

If you worked through the above example, you may have noticed that it is easier to work with real values rather than nominal values. For example, because the real value of the rent is the same in each year the formula for a finite annual series could be used to calculate the present value of all of the rent payments at once. Also, it is easier to work with real values rather than nominal values in this example because they are easier to predict. It is really pretty hard to say what inflation will be over the seven years of the project, and it is not likely to be constant every year. However, for most products, it is relatively safe over the long run to assume that the prices of those products will increase at about the same rate as inflation – whatever it turns out to be. This should generally be your default assumption. For some products, you may have reasons to expect that their prices may increase somewhat faster, or slower, than inflation – this is discussed in the next section. In any case, it is usually easier to predict the changes in the future prices of products relative to the inflation rate (i.e., to predict real future prices) than to predict how the actual future prices of the products will change (i.e., to predict nominal future prices).

It is also easier to work with real values because it is easier to judge whether real future prices and costs seem reasonable than to judge the reasonableness of nominal future values. Real prices are expressed in today's dollars, and we are familiar with the value of a dollar today. We are just as unfamiliar with the nominal value of a dollar in 2020 as we are with the value of a yen or a franc. (For example, do you know whether ¥2,400 would be a reasonable price for a meal at a restaurant?) Finally, as discussed above, it is useful to work with real future values and real interest rates because, after all, it is the real rate of return from our investment that really matters. It doesn't do much good to make a nominal rate of return of 10% on an investment if the inflation rate is 12%. On the other hand, a nominal rate of return of 4% can be reasonably good when there is no inflation.

7. Real Changes in Prices and Costs

Inflation refers to the change in the average level of prices. As is usually the case with averages, the prices of some goods go up faster than this average, while the prices of other goods go up slower than the average or may even go down. University tuition is an example of a price that has gone up faster than inflation. One study estimated that the average annual rate of increase in tuition at public universities over the past five years was more than six percent. During the same time period, the general rate of inflation was about 2.5 percent. Thus, university tuition went up at a rate that was about 3 to 4 percentage points faster than the average rate of increase in the price of everything else. In other words, university tuition increased at a nominal rate of about 6 percent and at a real rate of about 3 to 4 percent. This "real rate" is the rate tuition went up relative to the price of everything else.

In order to assess the likely profitability of projects, it will usually be necessary to estimate future costs and output prices. As discussed earlier, in many cases the best assumption is that real costs and prices will stay more-or-less the same as they are now. (Note that because of inflation it is necessary to say "real costs and prices" here. Otherwise, the phrase "prices will stay the same" would be ambiguous, and, furthermore, it is generally not a good assumption that nominal prices will stay the same.) Sometimes, however, historical price series show consistent trends that suggest that an alternative assumption should be used. In these cases, it generally is best to project real price trends rather than nominal price trends because inflation has historically been quite variable and is difficult to predict. Projecting real price trends allows you to avoid having to predict inflation rates. Thus, price projections will often use phrases such as "prices (or costs) will stay constant in real terms," or "prices will increase at *X*% faster than inflation."

The following terminology will help clarify the discussion of real price changes.

> The *nominal future price* of a product is the expected price that will be paid for the product at a future date, expressed in future dollars – i.e., dollars with the purchasing power that dollars will have at that future date.

> The *real future price* of a product is the expected price that will be paid for the product at a future date, expressed in current dollars.
>
> The *nominal rate of price change* for a product is the expected or historical rate of change in the nominal price of the product over a particular period.
>
> The *real rate of price change* is the expected or historical rate of change in the real price of a product. In other words, it is the rate at which the price for that product has changed, or is expected to change, relative to the general rate of inflation.

Because inflation is almost always positive, nominal price changes are more likely to be positive than negative. That is, the nominal prices for most products usually increase. Real prices, on the other hand, are just as likely to go down as up. Since the general rate of inflation is the average rate of change for all products, the prices of about half of all products will go up faster than the inflation rate and the prices of about half will go up slower than the inflation rate. Real price changes have been adjusted for inflation, and, on average, real prices stay the same. Thus, the average real price change must be zero.

In analyzing forestry investments, a key price is the price that is received for stumpage, that is, the price of trees "on the stump." For some products, stumpage prices have been going up faster than the rate of inflation. In northwestern Pennsylvania, for example, average black cherry sawtimber prices went up at a real, annual rate of over 5% between 1996 and 2006. For other products, prices have increased slower than the general rate of inflation. For example, real northern red oak prices in the same region decreased at a rate of almost 2% per year.[1] Most long-term sawtimber stumpage price series show real price increases of 1 to 2%. On the other hand, pulpwood prices have seldom gone up faster than inflation for long periods of time.

Just because the price of a product has been going up faster than inflation for the last few years does not mean this trend must continue. Sometimes the real price of a product will go up for a while and then go down. Depending on the period of time one looks at, the price of gasoline has sometimes increased faster than inflation and at other times slower than inflation. In 1976, I had a job pumping gas. The price then was 59.9¢ per gallon. Today, in 2006, the price of gas is about $2.30 per gallon. While this represents a 280% nominal increase, and about a 4.6% annual nominal increase, in real terms it represents only a very small average annual increase of about a quarter of a percent. (Expressed in 2006 dollars, the price of gas in 1976 was $2.12 per gallon.) On the other hand, in 1998, the price of gas was about $1.05 per gallon. This represented a nominal price increase of about 2.6% per year relative to the 1976 price but a real annual <u>decrease</u> of 2.2% per year.

[1] For Pennsylvania stumpage prices, see the Timber Market Report, published by the Penn State Cooperative Extension Service (http://www.sfr.cas.psu.edu/TMR/).

Let's take a minute to see how at least some of these numbers were derived. To convert (i.e., inflate) the 1976 price of gas to 1998 dollars use the 1998 CPI – 163 – and the CPI for 1976 – 56.9:

$$P_{g,1976}(\$1998/gal) = 0.599(\$1976/gal) \times \left(\frac{163}{56.9}\right) = \$1.72(\$1998/gal)$$

To convert the 1976 price to 2006 dollars, use the CPI for 2006[2] – 201.6:

$$P_{g,1976}(\$2006/gal) = 0.599(\$1976/gal) \times \left(\frac{201.6}{56.9}\right) = \$2.12(\$2006/gal)$$

We'll discuss in a minute how to calculate the real and nominal rates of price change. Before going on to those formulas, what other products can you think of whose real prices have fallen? Can you think of any products whose nominal prices have fallen?

We have discussed real and nominal discount rates, the general rate of inflation, and real and nominal rates of price change for particular products. Some notation would be useful for keeping track of all these rates; let:

r = real discount rate,
i = the nominal discount rate,
k = the expected (general) inflation rate,
k_p = the expected nominal rate of price change for the price of a particular product (product p), and
r_p = the expected real rate of price change for the price of a particular product (product p).

These different rates are related to each other. First, note that the nominal rate of price change for a particular product is approximately equal to the inflation rate plus the real rate of price change for the product. That is, $k_p \approx k + r_p$. The exact relationship between these different rates is given by the following equations:

$$(1+k_p) = (1+k)(1+r_p)$$

or,

$$k_p = (1+k)(1+r_p) - 1$$

or,

$$r_p = \frac{(1+k_p)}{(1+k)} - 1$$

[2] Note: the value of the PPI for 2006 is provisional at the time this is being written; i.e., it may be revised slightly.

Now, let

$P^*_{p,n}$ = the nominal future price of product *p* in period *n*,
$P_{p,n}$ = the real future price of product *p* in period *n*, and
$P_{p,0}$ = the current price of product *p*.

To calculate the expected nominal future price of product *p* in period *n*, it is necessary to inflate the current price using the expected nominal inflation rate for product *p* for *n* periods. The formula for this operation is:

$$P^*_{p,n} = (1+k_p)^n P_{p,0}$$

To calculate the expected real future price of product *p* in period *n*, inflate the current price using the expected real inflation rate for product *p* for *n* periods. The formula for this operation is:

$$P_{p,n} = (1+r_p)^n P_{p,0}$$

These last two equations can be rearranged to calculate the nominal and real rates of price change, respectively, when you know the prices at any two points in time (t_1 and t_2). That is,

$$k_p = \left[\sqrt[(t_2-t_1)]{\frac{P^*_{p,t_2}}{P_{p,t_1}}}\right] - 1$$

and

$$r_p = \left[\sqrt[(t_2-t_1)]{\frac{P_{p,t_2}}{P_{p,t_1}}}\right] - 1$$

Example: Calculating Real and Nominal Changes in the Price of Gas

It was noted above that the nominal price of gas was 59.9¢ per gallon in 1976, \$1.05 in 1998, and \$2.30 today. In 1998 dollars, the 1976 price of gas was \$1.72 per gallon, and in 2005 dollars, the 1976 price of gas was \$2.12. Calculate the nominal and real rates of price change between 1976 and 1998 and between 1976 and 2006.

Answer: First, calculate the nominal rate of price change between 1976 and 1998:

$$k_{g,1976-1998} = \left[\sqrt[(98-76)]{\frac{P^*_{g,1998}}{P^*_{g,1976}}}\right] - 1 = \left[\sqrt[22]{\frac{1.05}{0.599}}\right] - 1 = 0.02584 = 2.584\%$$

Now, calculate the real rate of price change between 1976 and 1998:[3]

$$r_{g,1976-1998} = \left[\sqrt[(98-76)]{\frac{P^{*}_{g,1998}}{P^{1998}_{g,1976}}} \right] - 1 = \left[\sqrt[22]{\frac{1.05}{1.72}} \right] - 1 = -0.02208 = -2.208\%$$

Now, calculate the nominal rate of price change between 1976 and 2006:

$$k_{g,1976-2006} = \left[\sqrt[(2006-1976)]{\frac{P^{*}_{g,2006}}{P^{*}_{g,1976}}} \right] - 1 = \left[\sqrt[30]{\frac{2.30}{0.599}} \right] - 1 = 0.04587 = 4.587\%$$

Now, calculate the real rate of price change between 1976 and 2006:

$$r_{g,1976-2006} = \left[\sqrt[(2006-1976)]{\frac{P^{*}_{g,2006}}{P^{2006}_{g,1976}}} \right] - 1 = \left[\sqrt[22]{\frac{2.30}{2.12}} \right] - 1 = 0.00288 = 0.268\%$$

Example: Calculating Real Hard Maple Stumpage Price Changes in NW Pennsylvania

In the first quarter of 1992, hard maple stumpage in northwestern Pennsylvania sold for an average price of $104/mbf. In the first quarter of 2006, the average hard maple stumpage price in the region was $480. The PPI for for lumber and wood products for the first quarter of 1992 was 139.9, and in the first quarter of 2006 it was 200.2.

a. What were the real and nominal rates of change in hard maple prices in the region over this 5-year period?

Answer: First, outline what is known:

- $P^{*}_{HM,'92}$ = $104/mbf
- $P^{*}_{HM,'06}$ = $480/mbf
- $PPI_{Q1'92}$ = 139.9
- $PPI_{Q1'06}$ = 200.2

Note that the first subscript, HM, stands for hard maple. The second subscript indicates the year in which the price was observed. The asterisk (*) indicates that a price is a nominal price. Calculating the nominal price change is a straightforward application of the equation given just above:

$$k_{HM} = \left[\sqrt[(2006-1992)]{\frac{P^{*}_{HM,2006}}{P^{*}_{HM,1992}}} \right] - 1 = \left[\sqrt[14]{\frac{\$480}{\$104}} \right] - 1 = 0.1154 = 11.54\%$$

[3] Note: If you try to reproduce my answer here, your answer may differ from mine because I carried more significant digits than are shown here in the $1998 price of gas in 1976. The answer shown here does not include round-off error.

Thus, nominal hard maple prices increased at an average rate of 11.5% per year.

Several approaches could be used to calculate the real rate of price change. One approach would be to calculate the average inflation rate over the 5-year period using the two PPI values. Then, the following relationship can be used to solve for the real rate of price increase:

$$r_{HM} = \frac{(1+k_{HM})}{(1+k)} - 1$$

Calculate the average inflation rate as follows:

$$k_{HM,1992-2006} = \left[\sqrt[2006,1992]{\frac{PPI_{Q1,'06}}{PPI_{Q1,'92}}} \right] - 1 = \left[\sqrt[14]{\frac{200.2}{139.9}} \right] - 1 = 0.02593 = 2.593\%$$

Now, calculate the rate of real price change:

$$r_{HM,1992-2006} = \frac{1.1154}{1.02593} - 1 = 0.8724 = 8.724\%$$

Thus, the real stumpage price for hard maple sawtimber increased, on average, by 8.7% over the 12-year period. In other words, hard maple sawtimber stumpage prices increased at a rate of 8.7% over inflation – a rather healthy rate!

Note that the real rate of price increase could also have been calculated by converting the 1992 price to 2006 dollars, as follows:

$$P_{HM,'92} = P^*_{HM,'92} \frac{PPI_{'06}}{PPI_{'92}} = \$104/mbf \ \frac{200.2}{139.9} = \$148.83/mbf \ (\$2006)$$

Now, the real price change can be calculated using the following equation:

$$r_{HM,1992-2006} = \left[\sqrt[(2006-1992)]{\frac{P_{HM,'06}}{P_{HM,'92}}} \right] - 1 = \left[\sqrt[14]{\frac{\$480}{\$148.83}} \right] - 1 = 0.08724 = 8.724\%$$

Note that it was assumed here that 1997 is the “base year,” and that real prices are expressed in 1997 dollars. However, we could just as well have assumed that 1992 is the base year. In that case, we could have converted the price in 1997 to 1992 dollars. Then we could have used the same formula, with prices for both years expressed in 1992 dollars. The result would have been the same (except for round-off error).

b. What will the nominal and real hard maple stumpage prices be in 2020 if these trends continue?

Answer: The information you will need is:

- The hard maple stumpage price was $480/mbf in 2006.
- Inflation is assumed to be 2.593%.
- The nominal hard maple stumpage price is assumed to increase by 11.5433% each year.
- The real hard maple stumpage price is assumed to increase by 8.7241% each year.

To find the nominal expected price in 2005, inflate the current price by the projected nominal rate of price increase, as follows:

$$P^*_{HM,2020} = \$480/mbf \times (1.115433)^{14} = \$2,215.39/mbf \ (\$2020)$$

To find the real expected price in 2005, inflate the current price by the projected real rate of price increase, as follows:

$$P_{HM,2020} = \$480/mbf \times (1.087241)^{14} = \$1,548.11/mbf \ (\$2006)$$

Example: The Hybrid Poplar Project with Changing Real Prices

Here we will reconsider the price assumption from the hybrid poplar example from the previous section. Recall that the plantation can be established for about $600 per acre and the current land rent is $100 per acre per year. After 7 years, the stand is projected to produce 20 tons of chips per acre. The current price for chips is $100 per ton, and inflation is expected to be about 4% each year. This time, we won't assume that the price per ton of chips will rise at the same rate as inflation, however. Instead, consider what will happen if the price of chips goes up 1 percent faster than inflation. (I.e., what if there is a 1 percent real annual increase in the price of chips?) The real alternate rate of return on the investment is still 8%. What is the present value of the hybrid poplar plantation venture per acre under this new assumption?

Answer: Table 3.3 shows the new net revenue stream over time, in real and inflated values. It also shows the present value of each period's net revenue.

Compare the values in the table with those in the earlier example. Note that only the values in row 7 and the NPV have changed. This is because none of the assumptions concerning the net revenues for years 0 through 6 have changed. The only thing that changed was the expectation regarding the price of chips in year 7.

To calculate the values for year 7, consider first what will happen to the real price of chips. The price of chips is now assumed to go up at a rate 1 percent over the rate of inflation. This is the same thing as saying that the real price of chips will go up by 1% each year. Thus, to calculate the real future price, inflate the current price of chips by 1% over 7 years (note that the subscript *c* is for "chips" – i.e., P_{7c} is the real price of chips in year 7):

$$P_{7c} = (1+r_c)^n P_{0c} = 1.01^7 \times \$100 = \$107.21$$

Table 3.3. Real and inflated cash flows and present values for the hybrid poplar example with a 1% real price increase.

Year	Real Net Revenue	Inflated Net Revenue	Present Value
0	-700	-700.00	-700.00
1	-100	-104.00	-92.59
2	-100	-108.16	-85.73
3	-100	-112.49	-79.38
4	-100	-116.99	-73.50
5	-100	-121.67	-68.06
6	-100	-126.53	-63.02
7	$2,144.27	$2,821.71	$1,251.16
Net Present Value	---	---	$88.87

Thus the projected real price of a ton of chips in year 7 (P_{7c}) is $107.21. The real future value of the revenue from the sale of chips is this price times the projected yield (Y_7):

$$V_7 = Y_7 \times P_{7c} = 20 \times \$107.21 = \$2{,}144.27$$

This is the real future value of the revenue from the sale of chips in year 7.

Now, consider the nominal future value of the revenue in year 7. First, calculate the nominal price of chips in year 7. Since the price of chips is assumed to go up at a rate 1 percent over the rate of inflation, the expected nominal rate of price increase for chips is approximately 4% + 1%. The following formula gives the precise rate of increase:

$$k_c = (1+k)(1+r_c) - 1 = (1.04)(1.01) - 1 = 1.0504 - 1 = 5.04\%$$

The price of chips is assumed to go up at a nominal rate of 5.04%. Now calculate the nominal price of chips in year 7 ($P_{c,7}^*$) under these assumptions:

$$P^*_{c,7} = (1+k_c)^7 \$100/ton = 1.0504^7 \times \$100/ton = \$141.09/ton$$

Thus, the projected nominal price of a ton of chips in year 7 is $141.09. The nominal revenue in year 7 can be obtained by multiplying this price times the projected yield:

$$\mathrm{V}^*_7 = Y_7 \times P^*_{7c} = 20 \times \$141.09 = \$2{,}821.71$$

Now, to calculate the present value of the revenue in year 7, either discount the nominal future value ($2,821.71) using the nominal discount rate (12.32%):

$$V_0 = \frac{\$2{,}821.71}{(1.1232)^7} = \$1{,}251.16$$

Or, discount the real future value ($2,144.27) using the real discount rate (8%):

$$V_0 = \frac{\$2{,}144.27}{(1.08)^7} = \$1{,}251.16$$

As always, you should calculate these numbers for yourself. As before, the net present value of the project is found by summing the present values from each year – which results in a net present value of $88.87/ac. Notice what the change in the assumptions did to the net present value in the example. With an assumed real price increase of one percent, the net present value per acre is $88.87, compared with $4.70 with no assumed price increase. Which is the "right" net present value? It all depends which scenario ends up being closer to the truth! In fact, neither of them is likely to be exactly correct because it is unlikely that either one will be a perfect prediction of the future.

8. Study Questions

1. In a world with no inflation, would interest rates still be positive? Why or why not?

2. In a world with no inflation, would it still be necessary to discount future values? Why or why not?

3. What is inflation?

4. How is inflation measured?

5. What is the difference between a real interest rate and a nominal interest rate?

6. What is the difference between deflating and discounting?

7. What characteristics of an investment influence the real interest rate that the investment must earn in order to attract investment funds?

8. Explain why a riskier investment would be expected to earn a higher rate of return than a safer investment.

9. What is the difference between a nominal future value and a real future value?

10. Why is it not necessary to distinguish between real present values and nominal present values?

11. List some situations where future values are likely to be expressed as nominal values. List some situations where future values are likely to be expressed as real values.

12. Explain why assuming that future nominal stumpage prices will remain constant is generally equivalent to assuming declining real stumpage prices.

13. Why is it not realistic in most cases to assume that the nominal price of a good will stay the same?

14. The formula for deflating a nominal future value is very similar to the formula for discounting a real future value. How are these formulas different?

15. Why is it usually simpler in financial analyses of forestry problems to express all future values in real terms?

16. Explain why the average real price change for all products is zero.

17. Explain why the real prices of particular products are just as likely to go down as up.

9. Exercises

1. If you want to earn a real rate of 7% on a 5-year investment, and you expect inflation to average 3% over the next five years, what nominal rate of return must you earn on the investment?

*2. You have invested $1,000 at a nominal rate of 9%.

   a. What will be the nominal value of your investment after 20 years?

   b. If inflation averages 3.5% over the next 20 years, what will the real value of your investment be after 20 years?

   c. What will be the real rate of return on your investment?

*3. The CPI for 1960 was 29.6 and the CPI for 1999 (so far) is 166.7.

a. What was the average rate of inflation for consumer goods between 1960 and 1999?

b. A loaf of bread sold for about 20 cents in 1960. What did a loaf of bread cost then, in current (1999) dollars?

*4. Assume that you invested $1,000 ten years ago and that now you have doubled your money in the investment; i.e., it is now worth $2,000 (nominal).

a. What (nominal) rate of return have you made on your investment?

b. If inflation has averaged 3.5% over the last 10 years, what real rate of return have you earned?

*5. You wish to endow your alma matter with a fund that will generate a real value of $1,000 each year, forever, for scholarships. The fund is expected to earn a nominal rate of 8%, and inflation is expected to average 3.5%.

a. What real rate of return is the fund expected to earn?

b. How much money will you need to place in the fund to ensure that a real value of $1,000 can be withdrawn each year?

*6. Your father-in-law is due to retire this year. He has saved up $250,000 for his retirement. He has the money invested in an account that earns, on average, a nominal rate of 9% per year. He wants to assume that either he or your mother-in-law will live for another 25 years, so he wants to spread the money out over a 25-year period. He has asked you to help him determine how much he can withdraw from this account each year so that the purchasing power of his withdrawals will remain approximately constant each year and so that the fund is used up after 25 years.

a. How much should he withdraw in the first year if he expects inflation to be about 4% on average over the next 25 years?
b. How much should he withdraw in the first year if he expects the average inflation rate to be about 3%?

*7. Assume that the average stumpage price for red oak sawtimber is now (in 1999) $450/mbf and that it was $150/mbf in 1976.

a. What was the nominal annual rate of price increase for red oak sawtimber between 1976 and 1999? (Give three significant digits in your answer.)

b. If inflation averaged 4% per year during this period, what was the real annual rate of price increase for red oak sawtimber? (Give three significant digits in your answer.)

8. Assume that southern pine sawtimber stumpage prices in your region now average about $350/mbf.

a. If, over the next 20 years these stumpage prices increase at a real rate of 2% per year, what will the real southern pine sawtimber stumpage price be in 20 years?

b. If inflation averages 4% over this period, what will the nominal southern pine sawtimber stumpage price be in 20 years?

*9. A family has just inherited 20 acres of forestland. They don't really want to keep the land, and they think they can get the most money from the land by clearcutting the timber and selling the bare land. The current inventory shows 9 mbf/ac of good oak timber, which, at the going rate of $450/mbf, should earn them $4,050 per acre. A developer is interested in the land, and has offered to pay $900/ac for the cleared land. You observe that the stand is still growing well, and, using your favorite growth simulator, you estimate that in ten years the yield should increase to 14 mbf/ac.

a. Assume constant real prices for both land and timber. If the family's real alternate rate of return is 7%, should they wait to sell the timber and the land?

b. Assume that oak sawtimber prices will increase at a real rate of 3% per year and that the bare land value will increase at a real rate of 5%. Under these assumptions, should the family wait to sell the timber and land?