CHAPTER 10: AREA AND VOLUME CONTROL

In this chapter we shift our perspective on forest management from the stand level to the landscape, or forest level. The decisions discussed in previous chapters have applied almost exclusively to individual stands. Stand-level decisions include things like when to harvest a stand, what kind of regeneration treatments to apply, whether to thin, what kind of thinning, etc. These treatments are applied on areas of 10 to maybe 100 acres, and the time frames considered tend to cover a few years to one or two decades. Landscape-level decisions apply at spatial scales of hundreds to hundreds of thousands of acres and typically involve time frames of decades to centuries. Questions addressed include things like "what volume of timber can be sustainably harvested from this landscape over several decades?" and "what future condition would we like to move the landscape towards over the next several decades?" Thinking about landscape-level management often also requires a significant mental shift for forestry students; typically the majority of their forestry education prior to taking forest management classes has been focused at the stand level.

The age-class distribution is a fundamental conceptual model for thinking about the structure of a forested landscape where the dominant stand structure is even-aged. It shows the number of acres in the landscape that fall into each age class, where age classes might be, for example, acres in stands that are 0-10 years old, 11-20 years old, etc. An age-class distribution might also distinguish areas by other factors, such as forest type and site class. A common objective of forest-level management is balancing the age-class distribution, which means evenly distributing the acres in the forest among age classes. Such a structure is considered desirable because it makes it easier to produce a sustainable harvest of timber, and it provides a sustainable balance of the different types of habitats provided by forests of different ages. A forest with a balanced age-class distribution is called a *regulated forest*.[1]

The concept of a regulated forest provides a good starting point for studying landscape-level decisions. It is unlikely you will ever see such a forest, and in most situations it is no longer even considered desirable to manage for a strictly regulated forest structure. The concept of a regulated forest is useful, however, for starting our discussion management at the landscape level precisely because it is simple. Students can acquire an initial intuitive grasp of many landscape-level concerns by thinking about a regulated forest. Because of their simplicity and lack of flexibility, regulation techniques such as area control and volume control are of limited use in practice. The classic regulation methods are inadequate for addressing the more complex concerns of today, and more modern methods are available to find the right mix of management activities to efficiently achieve a more diverse set of forest-wide objectives. Nevertheless, area control and volume control are useful techniques to learn in an undergraduate forest management

[1] Note that the phrase "regulated forest," as used in this chapter, does not refer to the legal regulation of forest practices.

class because this exercise can help students build some basic intuition about managing forests on a larger scale, and they can learn how to do some the basic calculations that will be useful for thinking about landscape-level concerns without also having to also struggle with the complexity of more up-to-date methods such as linear programming.

We will rely a lot on mathematical models for thinking about landscape-level management decisions. Models have been used in discussing stand-level decisions in earlier chapters, so the idea of modeling should not be entirely new to you. However, models play a much more central role in strategic forest planning and landscape management. This is because models provide a framework for organizing both ideas and information about large-scale management problems. As you can imagine, there is a lot one could know about a forested landscape that may or may not be relevant for developing a strategic plan. Models are, by definition, simplifications of reality. They help us sift through all the possible information one could collect about a complex object like a forest in order to identify the most critical information that one needs to focus on in making decisions. The art of model building is to achieve a balance between one extreme, where so much detail is included that important general relationships are obscured, and the other extreme, where models are too simple and thus fail to recognize key information that must be accounted for in order to make sound decisions. A basic rule that you should keep in mind is that a model should be as simple as possible and yet complex enough to represent all the information and relationships that are important and relevant to the decision that must be made. In this regard, the regulation models we will study in this chapter are about as simple as strategic forest management models can be. This simplicity is useful when the objective is to understand the most basic concepts of forest-level management, but they fail in terms of addressing the full complexity of modern landscape level management decisions.

The type of management discussed in the remainder of this book is often referred to as forest planning. In this context, stand-level decisions are sometimes referred to as "operational" decisions, while landscape-level decisions are considered "strategic" decisions. Strategic and operational decisions require different types of management skills and different ways of thinking. Operational decisions require a hands-on understanding of how things are done. Strategic decisions are much more abstract. They require the ability to process, summarize, and synthesize data. They require an ability to recognize and understand processes that occur on large spatial and temporal scales – a sense of the "big picture." People responsible for strategic decision-making are also more likely to need strong communication skills for working with people with diverse backgrounds and interests.

1. Some Fundamental Forest Regulation Concepts

The transition from stand-level management to landscape-level management requires the introduction of some new fundamental concepts. As mentioned in the introduction to this chapter, one of the most basic is the age-class distribution, and the concept of a regulated forest refers to an idealized structure for the age-class distribution of the forest. The primary motivation for achieving a regulated forest is to achieve an even, sustained flow of forest products from the forest over time. Even flow and sustained yield are two of the oldest objectives of forest management. They are also reflected in the major laws prescribing the management principles of the federal forests of the United States.

Age-class Distributions

From a management perspective, forest type and stand age are the two most important attributes of an even-aged stand. A lot can be inferred about a stand if just these two things are known. With an uneven-aged stand, even a basic description of the stand requires more information. As in even-aged stands, the forest type is important, but age is meaningless for an uneven-aged stand. A description of the diameter class distribution of the stand is also needed. Admittedly, the state of an uneven-aged stand could be described by its basal area, but this still allows for a tremendous amount of variation. It would also be useful to know whether the trees in the stand are well distributed across diameter classes, or whether there are diameter classes that are understocked. Is there an appropriate balance between small and large-diameter trees? Does the stand consist mostly of small trees, or are there a fair number of large trees, too? A graph of the diameter class distribution is particularly useful in answering questions such as these.

In a similar way, a key way of describing the state of an even-aged forest the forest's *age-class distribution*. The age-class distribution lists the number of acres in each age class in the forest. This information is compiled by processing a list of the stands in the forest and adding up the acreage of each stand to determine the cumulative area in the age class. The age-class distribution can be represented as a table (Tables 10.1 and 10.2) or as a histogram (Figures 10.1 and 10.2). Usually, a separate age-class distribution would be developed for each forest type in a forest. However, forest types may be aggregated, for example, into such broad categories as softwood and hardwood types. More detailed information can be conveyed about the forest by recognizing other variables. Site quality is arguably the third most important characteristic of an even-aged stand (after forest type and age). Thus, an age-class distribution that also breaks out areas by site class, as in Table 10.2 and Figure 10.2, conveys more detailed information about the forest.

The Regulated Forest

The regulated forest is defined in terms of its age-class distribution. For the purposes of this chapter, a regulated forest will be defined as follows:

> A *regulated forest* is one with an equal number of acres in each age class.
>
> *Forest regulation* is the process of converting a forest with an unbalanced age-class distribution into a regulated forest – one with an even distribution of acres in each age class.

Table 10.1. A sample age-class distribution.

Age Class	Acres
0 to 10 years	1,350
11 to 20 years	2,480
21 to 30 years	1,730
Over 30 years	4,210

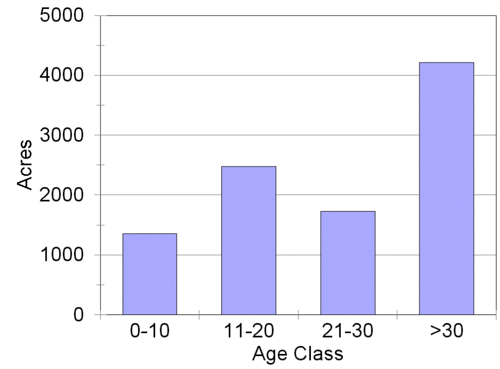

Figure 10.1. A graph of a sample age-class distribution.

Table 10.2. A sample age-class distribution, broken out by site class.

	Acres		
Age Class	Site I	Site II	Site III
0 to 10 years	440	560	350
11 to 20 years	880	930	670
21 to 30 years	460	650	620
Over 30 years	1,760	1,115	1,335

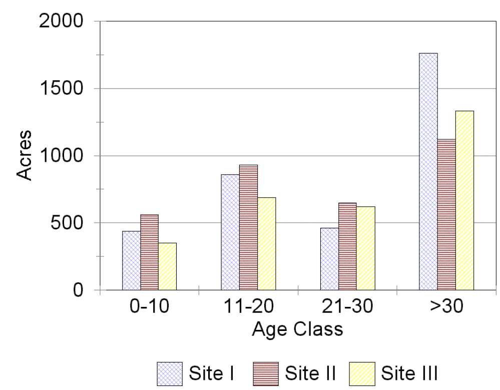

Figure 10.2. A graph of a sample age-class distribution with three site classes.

When there is more than one site class, simply having an equal area in each age class does not guarantee that the area in each age class will be capable of producing an equal amount of wood. Davis and Johnson (1997, p. 540) offer this more generalized definition of the regulated forest:

> The essential requirements of a regulated forest are that age and size classes must be represented in such proportion and be consistently growing at such rates that an approximately equal annual or periodic yield of products of desired sizes and quality may be obtained in perpetuity. A progression of size and age classes must

> exist such that an approximately equal volume and size of harvestable trees are regularly available for cutting.

As you can see, the simple definition of the regulated forest given earlier can quickly become complicated and fuzzy when all of the variability that exists in the real world is recognized. However, the question of what regulation means when there are multiple site and stocking classes, as well as age classes, is only important to someone who is actually considering implementing a regulation plan on an actual forest. Since the purpose here is solely to gain insight, this goal can be best achieved by keeping things as simple as possible. Therefore, it will generally be assumed in the remainder of this chapter that there is only one forest type, with one site class, and that all acres are well-stocked, and we will work with age-class distributions like the one in Table 10.1.

As suggested by Davis and Johnson's (1997) definition, the primary purpose of regulating a forest is to achieve a state where an even flow of products can be produced in perpetuity. This is desirable for two fundamental reasons: sustainability and stability. The flow of products from a regulated forest will be sustainable as long as the basic structure of the regulated forest is maintained and the productivity of the soil itself is not degraded. Once a forest is regulated, the oldest age class can be harvested each year, each age class will then grow one year older, and the acres that were harvested will be regenerated and replace the acres that grew out of the youngest age class. This process maintains the age-class distribution in a steady state where harvest and growth are balanced and the age-class distribution always remains the same at the end of each period. Because the age and the area of the harvested acres are the same each year, the harvest from a regulated forest is constant from year to year. This guarantees a consistent flow of products to wood processing facilities, which, in turn, allows them to provide a steady supply of wood products and employment. Thus, a regulated forest helps ensure stability in the wood products industry and in the economic sectors that use their products.

A regulated forest has both desirable and undesirable properties for wildlife habitat. In a regulated forest all age classes – up to the rotation age – are equally represented at all times, so the range of habitat conditions provided by all of the ages in the age-class distribution will be provided at all times. If a forest is not regulated, any gaps in the age-class distribution will result in shortages of the type of habitat provided by the age-classes that are under-represented. Even if an age-class gap currently exists in an age class that does not provide a critical type of habitat, as time passes the gap in the age class will inevitably move into older age classes. At some time, the gap is bound to move into a more important age class. From a wildlife perspective, the biggest concern with regulation is that the harvest age is generally set too young. As we have seen, the optimal economic rotation can be quite short relative to the biological life cycles of natural forest communities. If an entire forest is regulated with the economic rotation age determining the oldest age class, there will be no acres in more mature age classes, which will present problems for wildlife species that require the habitat characteristics provided by older forests.

The Mean Annual Increment (MAI) and the Regulated Forest

As noted in Chapter 5, the mean annual increment (MAI) gives the average growth of a forest stand per acre per year. Multiplying the mean annual increment at the rotation age by the area of a stand gives the average total growth of the stand per year over the life of the stand. In a simple regulated forest with one forest type and site class, multiplying the MAI at the forest rotation age by the area of the forest gives the total growth of the forest. Because the regulated forest is in a steady state, the harvest from a regulated forest equals the growth of the forest. (If growth was not equal to the harvest, then the total inventory would be changing, and the forest would not be in a steady state.) Thus, in this simple forest, the product of the MAI at the forest rotation and the area of a regulated forest also gives the harvest volume for each year. This result is demonstrated mathematically in this section, but first some notation is required. Rather than introduce the notation as it is needed, all of the new notation used in this chapter is presented here for easy reference. Let:

A = the total area of the forest,
$A_{i,t}$ = the area in the i^{th} age class in period t,
$A_{i,Reg}$ = the area in the i^{th} age class in a regulated forest,
v_i = the volume per unit area in the i^{th} age class,
H_t = the total harvest volume in period t,
H_{Reg} = the total periodic harvest volume from the regulated forest (i.e., the volume harvested each period),
R = the rotation age (and the age of the oldest age class in the regulated forest),
MAI_R = the mean annual increment at the rotation age,
$I_{i,t}$ = the inventory volume in age class i in period t,
I_t = the total inventory volume in period t,
$I_{i,Reg}$ = the inventory volume in age class i in the regulated forest,
I_{Reg} = the total inventory volume in the regulated forest,
$G_{i,t}$ = the growth in age class i in period t,
G_t = the total growth in period t,
$G_{i,Reg}$ = the growth in age class i in the regulated forest,
G_{Reg} = the total growth in the regulated forest,
n = the number of years in a period (equal to the number of years in an age-class),
N_t = the number of age classes up to the oldest age class in period t, and
N_{Reg} = the number of age classes up to the oldest age class in the regulated forest.

Note that in this chapter t refers to the tth period, not the year (unless the period length is one year; i.e., $n = 1$). It will be useful to divide the time during which the forest is being regulated into blocks of time, called *periods*, whose length equals the number of years in each age class in the forest. For example, in Table 10.1 the age-class distribution is divided into 10-year age classes. In planning for the conversion of this forest to a regulated state, the conversion interval will be divided into 10-year periods. Five-year age classes could also have been used, in which case the conversion interval should be divided into 5-year periods. The conversion interval – the time when the forest is being regulated – is also commonly called the *planning horizon*. You will see this terminology in later chapters that discuss more modern forest management techniques based on linear programming.

It is easy, with the above notation, to demonstrate that the total harvest each year from the regulated forest is equal to the MAI at the rotation age times the total area of the forest. First, note that, by definition, the area in each age class in the regulated forest is equal. That is,

$$A_{1,\,Reg} = A_{2,\,Reg} = A_{3,\,Reg} = \ldots = A_{N,\,Reg}$$

Since the area in each age class is equal, the area in each age class is equal to the total area divided by the rotation age times the number of years in an age class (i.e., the period length):

$$A_{i,\mathrm{Reg}} = \frac{A}{R} \times n = \frac{A}{N_{\mathrm{Reg}}} \quad for\, i = 1 \ldots N_{\mathrm{Reg}}$$

Since the entire area in the oldest age class will be harvested in a period, the periodic harvest from the regulated forest equals the area in the oldest age class times the yield for that age class. The following equations use this logic to demonstrate that the annual harvest from a regulated forest equals the total area times the mean annual increment at the rotation age:

$$H_{Reg}^{Annual} = A_{R,\mathrm{Reg}} \times v_R = \frac{A}{R} \times v_R = A \times \frac{v_R}{R} = A \times MAI_R$$

Since a period equals *n* years, the periodic harvest, i.e., the volume harvested during a planning period, is equal the total area times the mean annual increment times the number of years in a period:

$$H_{Reg} = H_{Reg}^{Annual} \times n = A \times MAI_R \times n$$

If the area of the forest is fixed and the yield function is fixed, the only way to change the annual (or periodic) harvest from the regulated forest is to change the rotation age. Clearly, selecting the rotation age that maximizes the MAI will also maximize the annual harvest from the forest. For this reason, the age that maximizes the MAI is sometimes considered the best age for regulating a forest. However, the next section discusses the flaws in this view.

Selecting the Regulation Rotation

To select the "best" rotation for the forest, one must first ask what the objectives for the forest are. As with the majority of the situations discussed in this book, we assume here that the primary objective is to maximize the profitability of the forest. The rotation that maximizes the MAI (the culmination of the mean annual increment, or CMAI) is the rotation that maximizes the annual harvest from a regulated forest. Since this rotation maximizes the annual harvest, it also maximizes the annual income from the forest. Since the annual income from the forest is maximized, the net present value of the forest must also be maximized with this rotation. Right? Not so. This conclusion has confused many foresters; after all, why not select the rotation that results in the maximum annual harvest and the maximum annual revenues? How can it be that the rotation that maximizes the annual income from the forest is not the best rotation for a regulated forest? The reason is subtle, but important. The problem with the logic of using the CMAI rotation is that it does not consider the cost of achieving the regulated forest in the first place or the cost of maintaining that forest once it is achieved, it only looks at the additional revenues earned after the forest is regulated. In fact, the cost of either creating or maintaining a forest regulated on the CMAI is higher than the benefit from having such a forest. Because the CMAI rotation is generally longer than the rotation that maximizes the LEV, more revenue would have to be foregone in the short run to regulate the forest on the CMAI rotation than to regulate the forest on the rotation that maximizes the LEV. Even though regulating the forest based on the CMAI rotation results in a forest that produces the greatest annual revenue once the forest is regulated, it costs more to achieve such a forest than it does to achieve a forest that is regulated on the optimal economic rotation. Furthermore, the present value of this cost exceeds the present value of the additional revenue that is gained in the long run from having a forest regulated on the CMAI rotation instead of a forest that is regulated on the maximum LEV rotation. Even if the forest is already regulated using the CMAI rotation, the forest could be restructured to a forest regulated based on the maximum LEV rotation. More revenue would be realized during this restructuring period because it would be necessary to reduce the average age of the forest, and thus to reduce the inventory of the forest, so during this time the volume harvested would have to exceed the inventory volume of the forest. As with the cost of regulating the forest in the first place, the present value of this additional revenue from the conversion period would exceed the present value of the reduced revenues that would be produced once the forest was restructured based on the maximum LEV rotation. It can be shown that regulating the forest based on the maximum LEV rotation maximizes the net present value of both the conversion period and the period after the forest is regulated. Therefore, the best rotation for a regulated forest if we properly account for the combined costs of achieving (or maintaining) the regulated forest and the net benefits of the regulated forest once we achieve it is the rotation that maximizes the LEV.

Regulating a forest with the rotation that maximizes the MAI results in a forest with too much inventory and, therefore, inventory costs that are too high. The inventory cost is an opportunity cost and not an explicit cost. That is why trying to maximize only the annual return after the forest is regulated gives the wrong answer – the implicit inventory cost is not considered. Allowing the possibility of liquidating part of the inventory and allowing the forest to be re-regulated to any rotation explicitly includes the inventory costs. When the full costs are recognized, the optimal rotation for regulation is the one that maximizes the LEV. This is the only rotation for a regulated forest where you cannot improve the net present value of the forest by departing temporarily from the regulated state to adjust to a new rotation.

What if the objectives for managing the forest are not necessarily to maximize the profitability of the forest? Since there are a countless number of alternative objectives that we could consider, it is impossible to cover all of the possibilities here, so only a few alternatives will be considered. Other likely objectives might include wildlife habitat objectives or aesthetics. In both of these cases, it is likely that a major drawback of a forest regulated on the maximum LEV rotation will not contain enough older forest. Older forest can be provided by setting aside some areas that will not be harvested at all, by managing some areas on longer rotations, or by managing for a target regulated forest based on a longer rotation. In any of these cases, it should be noted that there is really nothing special about the rotation that maximizes the CMAI. Any number of different rotations could be used. Additionally, with the exception of using a target regulated forest structure based on a longer rotation, none of these objectives are easily handled with the techniques of forest regulation discussed in this chapter. They are, however, easily handled with the linear programming approaches discussed in later chapters, so we will defer further discussion of these objectives until then.

Example – Determining the best rotation for a regulated forest

Table 10.3 shows hypothetical yields at different ages for even-aged forest stands. The MAI is shown for rotation-ages between 20 and 70. The MAI is a useful calculation because of its relationship to the annual harvest from the regulated forest. LEVs have been calculated using the economic assumptions shown to the right of the table. Determine the optimal rotation for regulating a forest for which these yields and economic data apply.

Table 10.3. Hypothetical yield table with the MAI and the LEV for rotation ages 20 to 70.

Age	Volume (cd/ac)	MAI (cd/ac/yr)	LEV ($/ac)
10	5		
20	16	0.8	-32.09
30	32	1.067	67.45
40	55	1.375	109.13
50	75	1.5	74.29
60	90	1.5	15.34
70	100	1.429	-42.16

Economic assumptions:

- The stumpage price is $25/cd.
- The stand establishment cost is $200/ac.
- The real interest rate is 4%.
- All prices and costs are assumed to increase at the rate of inflation.

Answer: Since the LEV is maximized at the 40-year rotation, this is the appropriate rotation age for forest regulation. Below is an example LEV calculation (rotation age = 30):

$$LEV_{30} = \frac{Y_{30}P - E(1+r)^R}{(1+r)^R - 1} = \frac{32 \times 25 - 200(1.04)^{30}}{(1.04)^{30} - 1} = \$67.45$$

Note that the MAI in the example is maximized at a rotation age somewhere between 50 and 60. The annual harvest from the regulated forest would be 9% higher with a rotation 50 or 60 than with a rotation of 40. Even though regulating on a 50 or 60-year rotation would produce more volume and revenue in the long run, the 40-year rotation is the best rotation for the regulated forest.

All stand-level management options – like regeneration, release, and thinning decisions – in a regulated forest should be selected by identifying the prescription that maximizes the LEV for each forest type and site condition in the forest. However, these prescriptions may not be always be appropriate during the conversion to the regulated forest. Converting an unregulated forest to a regulated state will require the use of at least some prescriptions that do not maximize the LEV for that stand.

Long Term Sustained Yield (LTSY)

The harvest from a regulated forest is called the *Long-Term Sustained Yield* (*LTSY*). This is an important concept because it indicates the amount of wood that can be harvested from a forest on a sustained basis, given the choice of rotation and management intensity. The LTSY is therefore a critical piece of information about a forested area. Determining the LTSY is an important calculation that you should be able to do for an actual forest. Therefore, for this concept, we will break with the general assumption of the chapter that there is only one site class and discuss the calculation of the LTSY under the more realistic condition where there is more than one site quality class. Generalization of the concept to a forest with more than one forest type is a straightforward extension of the approach demonstrated here. To calculate the LTSY for a forest with one forest type and multiple site classes:

1. First, determine the rotation age to be used for each site class. This should be done by calculating the rotation that maximizes the LEV. Because the yield functions are different for different site classes, the optimal rotation may also differ for different site classes.

2. Next, calculate the MAI for each site class at the selected rotation.

3. The LTSY is equal to the sum over site classes of the MAI for that site class under the chosen rotation times the area in each site class. Let A_s equal the number of acres in site class s, and let R_s^* equal the optimal rotation for site class s. Now, the formula for the LTSY for a forest with S site classes is:

$$LTSY = \sum_{s=1}^{S} [MAI_{R_s^*} \times A_s]$$

Example: Calculating the LTSY

Table 10.4 gives the area, optimal rotation, and yield at the optimal rotation for three site classes in a hypothetical forest. Use this information to calculate the LTSY of this forest.

Table 10.4. Area, optimal rotation and yield at the optimal rotation for a hypothetical forest with three site classes.

Site Class	Area (acres)	Optimal Rotation (R^*)	Yield at R^* (cords/acre)
I	4200	35	41
II	5600	45	46
III	3500	50	48

Answer:

$$LTSY = \sum_{S=1}^{3} \left(\frac{Y_S(R_S^*)}{R_S^*} \right) \times A_S = \left(\frac{41}{35} \right) 4{,}200 + \left(\frac{46}{45} \right) 5{,}600 + \left(\frac{48}{50} \right) 3{,}500$$

= 14,004 cords/year

Methods of Forest Regulation

Forest regulation methods deal with how to get from an existing forest with an unbalanced age-class distribution to a state where the forest is regulated. The two basic methods for regulating a forest are *area control* and *volume control*. The two approaches are based on cutting a target area or volume, respectively, in each period. Some hybrid methods have also been developed that combine some aspects of area control and some aspects of volume control, but they are not discussed here. Once the forest is regulated, all the approaches are the same. The next two sections of this chapter describe these methods in detail.

2. <u>Area Control</u>

The simplest way to achieve a regulated forest is to harvest and regenerate the same number of acres each year (or period) as would be harvested if you had a regulated forest. This approach is called *area control*, and it will result in a regulated forest within at least one rotation. Note that, with the simplifying assumptions that we have made—that there is only one site class and that all acres are equally well stocked—the best way to select specific areas for harvest is to always harvest the oldest acres in the forest first.

The general steps for regulating a forest by area control can be summarized as follows:

1. Select the desired rotation for the regulated forest. This was discussed in the previous section. If the management objective for the forest is to maximize the net present value of the forest, given the constraint that it must be regulated, then the rotation age that maximizes the LEV should be selected for the target regulated forest.

2. Calculate the number of acres to be harvested each period by dividing the total number of acres in the forest by the number of age classes in the target regulated forest. The number of age classes in the regulated forest is the optimal rotation divided by the number of years in an age class ($N_{Reg}=R/n$). The target age-class distribution for the forest is one with an equal number of acres in each of these age classes. In area control, the number of acres in each age class in the target age-class distribution gives the number of acres to be harvested each period and is equal to the total area divided by the number of age classes.

3. Project the age-class distribution at the beginning of the next period by moving harvested acres to the youngest age class and unharvested acres up to the next age class.

4. Calculate the projected harvested volume by multiplying the per-acre yield for each harvested area by the area harvested.

5. Repeat steps 3 and 4 until the forest's projected age-class distribution is regulated.

Example: Identifying the Target Age-class Distribution

Use area control to identify the target age-class distribution for regulating a 750-acre forest with the following age-class distribution. Use the yield table and economic data in Table 10.3.

Table 10.5. Initial age-class distribution.

Age Class	Acres
0-10	250
11-20	250
21-30	0
31-40	250

Answer: From the earlier example, we already know that the optimal rotation is 40 years. Therefore, the regulated forest should have 4 age classes (N_{Reg}=*R/n* = 40/10 = 4) with 187.5 (750/4) acres in each age class. Table 10.6 shows the target age-class distribution for the regulated forest.

Table 10.6. Target age-class distribution.

Age Class	Acres
0-10	187.5
11-20	187.5
21-30	187.5
31-40	187.5

The next step is to project the age-class distribution forward, one decade at a time, until the forest is regulated – i.e., until the age-class distribution looks like the one in Table 10.6.

Projecting the Age-class Distribution

To project the age-class distribution forward using area control, the target number of harvested acres must be moved back to the initial age class (simulating the harvest and regeneration of those acres) and the remaining un-harvested acres must be moved up one age class (simulating the aging of those stands). The target number of acres for harvest was determined by dividing the total acreage by the number of age classes in the target forest. Harvested acres are selected by taking the oldest acres first. The steps are as follows:

1. See if the acreage in the oldest age class is enough to meet the harvest area target.

2.a. If there are enough acres in the oldest age class to meet the harvest target, subtract the target area from the area in that age class to determine the remaining uncut area.

2.b. If the number of acres in the oldest age class is less than the harvest area target, all the area in that age class will be removed. Subtract the area in the oldest age class from the target. Move up to the next age class and check whether the acreage in that age class will be enough to meet the remaining area target. Keep moving up to younger age classes until enough acreage has been identified to meet the harvest area target. Once the target has been met, calculate the number of acres that will be left in the last age class.

3. Move the total area cut into the youngest age class. Move uncut acre ages up to the next age class.

Example: Calculating the Age-class Distribution After the First Period

In the example that was started earlier, calculate the age-class distribution after two periods of area control; i.e., move the age-class distribution forward two decades using area control.

Answer: The harvest target is 187.5 acres. There are 250 acres in the oldest age class, so the entire harvest target of 187.5 acres can be harvested from that age class. The remaining 62.5 acres in the age class will be left uncut, and in 10 years they will be in the 41 to 50-year age class. The 187.5 acres that were harvested will be regenerated, so they will appear in the 0 to 10-year age class at the beginning of the next decade. The 250 acres initially in the 0 to 10-year age class will move to the 11 to 20-year age class, and the 250 acres initially in the 11 to 20-year age class will move to the 21 to 30-year age class. This is summarized in the following table:

Table 10.7. Summary of actions for first decade of area control.

<table>
<tr><th>Initial Acres</th><th>Action</th><th>Result</th></tr>
<tr><td>250 acres, ages 0 to 10</td><td>uncut</td><td>moved to 11 to 20 age class</td></tr>
<tr><td>250 acres, ages 11 to 20</td><td>uncut</td><td>moved to 21 to 30 age class</td></tr>
<tr><td rowspan="2">250 acres, ages 31 to 40</td><td>187.5 acres cut</td><td>moved to 0 to 10 age class</td></tr>
<tr><td>62.5 acres uncut</td><td>moved to 41 to 50 age class</td></tr>
</table>

The projected age-class distribution after 10 years is shown in Table 10.8.

Table 10.8. Example age-class distribution after 10 years.

Age Class	Acres
0-10	187.5
11-20	250.0
21-30	250.0
31-40	0.0
41-50	62.5

Now, we move the age class distribution ahead a second decade. Again, the harvest target is 187.5 acres. There are only 62.5 acres in the oldest age class, so the entire area in that age class will be harvested. This leaves 125 more acres to be harvested. There are no acres in the 31 to 40-year age class, so acres must be harvested from the 21 to 30-year age class. Of the 250 acres in the 21 to 30-year age class, 125 acres will be harvested and 125 acres will be left uncut. The uncut acres from the 21 to 30-year age class will move to the 31 to 40-year age class at the beginning of the next decade. The 250 acres initially in the 11 to 20-year age class will move to the 21 to 30-year age class, and the 187.5 acres in the 0 to 10-year age class will move to the 11 to 20-year age class. The 187.5 acres that were harvested will be regenerated, and they will appear in the 0 to 10-year age class at the beginning of the next decade. This is summarized in the following table:

Table 10.9. Summary of actions for second decade of area control.

Initial Acres	Action	Result
187.5 acres, ages 0 to 10	uncut	moved to 11 to 20 age class
250 acres, ages 11 to 20	uncut	moved to 21 to 30 age class
250 acres, ages 21 to 30	125 acres cut	moved to 0 to 10 age class
	125 acres uncut	moved to 41 to 50 age class
62.5 acres, ages 41 to 50	cut	moved to 0 to 10 age class

The projected age-class distribution after two decades of regulation is shown in Table 10.10, along with the initial age-class distribution and the age-class distribution after 10 years.

Table 10.10. Example age-class distribution over 20 years of area control.

Age Class	Initial age class distribution	Age class distribution after 10 years (acres)	Age class distribution after 20 years (acres)
0-10	250	187.5	187.5
11-20	250	250.0	187.5
21-30	0	250.0	250
31-40	250	0.0	125
41-50	0	62.5	0

If the regulation process in the example is continued for one more decade, the forest will be regulated. Table 10.11 shows the progression of the age-class distribution from through the third decade, when the forest is regulated. Table 10.11 also shows the average harvest volume for each decade and the average net revenue earned. These calculations are discussed below.

Projecting the Harvest Volume

The volume harvested in a given period is determined by multiplying the acreage cut from each age class during that period by the harvest volume per acre for the corresponding age class. Since the acres in a given age class may vary in age from the minimum age for the age class to the maximum age, there may be some question regarding the appropriate age at which to calculate the yield. One way to think about this is to assume that the average age of stands in an age class at the beginning of the period will be the age at the midpoint of the age class. However, on average, the stands will be harvested at the midpoint of the period. Therefore, on average, they will grow for one half of a period before being harvested. Their average age at harvest will therefore be the age at the upper end of the age class.

Consider, for example, acres in the 31 to 40-year age class. At the beginning of the period, they will be 35 years old, on average. However, the whole area will not be harvested at the beginning of the period; the harvest will be spread out over the 10-year period, so there will be an average delay of 5 years before a stand is harvested. Thus, at harvest the average age of the stands in this age class will be 40 years—i.e., the upper range of the age class. The volume per acre for harvested acres should therefore be based on the yield at the age at the upper end of the age class. In this example, acres cut from the 30 to 40 year age class should be assumed to be 40 years old when cut.

Table 10.11. Projected Age-Class Distribution, Annual Cut and Net Revenue under Area Control.

Age Class	Acres in each age class at the beginning of each decade			
	0	10	20	30
0-10	250	187.5	187.5	187.5
11-20	250	250	187.5	187.5
21-30	0	250	250	187.5
31-40	250	0	125	187.5
41-50	0	62.5	0	0
51-60	0	0	0	0
Total Acres	750	750	750	750
Average Annual Cut (Cords)	1,031.25	868.75	887.5	1,031.25
Average Annual Net Revenue ($)	22,031	17,969	18,438	22,031

Example: Calculating the Harvest Volume

Calculate the volume harvested in the first two decades of the example.

Answer: To calculate the harvest volume for the first decade, note that 187.5 acres will be harvested at age 40. From the yield table (Table 10.3), the yield at age 40 is 55 cords per acre. Thus, the total volume harvested will be 10,312.5 cords (187.5×55). On average, 1,031.25 cords will be harvested per year.

To calculate the harvest volume for the second decade, note that 62.5 acres will be harvested at age 50, and 125 acres are projected to be harvested at age 30. The yield at age 50 is 75 cd/ac, and the yield at age 30 is 32 cd/ac. Thus, the total volume harvested is 8,687.5 cords (62.5×75 + 125×32). On average, 868.75 cords will be harvested per year.

Table 10.11 shows the projected harvest volume for each decade of the example. Note the wide fluctuations in the volume harvested in each decade. The harvest in the second and third decades are about 15% less than the harvests in the first and fourth decades. Recall that the primary objective of regulating the forest in the first place was to extract a relatively stable flow of products from the forest. Unfortunately, with area control there is no control of the volume harvested during the conversion interval, and, ironically, harvests may fluctuate dramatically. This is the primary drawback of the area control method and the reason why volume control methods were developed. Before moving on to volume control methods, however, we will determine the cash flow from the forest.

Calculating the Forest Net Revenue

The net revenue for the forest is just the gross revenue—the volume harvested times the price—minus the costs of managing the forest. In our example the only cost is the cost of regenerating the harvested acres. Thus,

Net Revenue = Harvest Volume × Price - Acres Regenerated × Regeneration Cost.

Example: Calculating the Forest Net Revenue

Calculate the annual net revenue for the first two decades in the example problem.

Answer: The annual harvest volume for the first period is 1,031.25 cords; the price is $25/cd; and 18.75 acres must be regenerated each year at a cost of $200/ac. Thus, the net revenue is:

1,031.25 cd/yr × $25/cd - 18.75 ac/yr × $200/ac = $22,031.25/yr

The annual harvest volume for the second period is 868.75 cords; the price is $25/cd; and 18.75 acres must be regenerated each year at a cost of $200/ac. Thus, the net revenue is:

868.75 cd/yr × $25/cd - 18.75 ac/yr × $200/ac = $17,968.75/yr

These values are shown in the bottom row of Table 10.11.

3. Volume Control

Earlier, it was noted that with area control the harvest may fluctuate dramatically during the conversion period. With area control, the number of acres to be harvested is set first, and the volume that is harvested is just the amount that comes with those acres – i.e., there is no control of the volume harvested. With *volume control*, the volume to be harvested is determined first. Then, the number of acres needed to produce the required volume is determined. As with area control, the oldest acres are always cut first. By controlling the volume harvested, a more even harvest can be obtained during the conversion period. However, there is no control with this approach over the number of acres harvested in each period, and volume control will not generally result in a regulated forest within one rotation. Rather, the age-class distribution approaches a regulated state gradually over several rotations.

The steps for regulating a forest by volume control can be summarized as follows:

1. Select the desired rotation for the regulated forest.

2. Calculate the volume to be harvested each year during the first period using one of the volume control formulas.

3. Starting with the oldest age class, calculate the number of acres that need to be harvested from each age class in order to produce the target harvest volume.

4. Project the age-class distribution forward one period by moving harvested acres to the youngest age class in the next period and unharvested acres up to the next age class.

5. Repeat steps 2 through 4 until the forest's projected age-class distribution is regulated.

Unlike area control where determining the number of acres to cut is trivial, there are many ways to determine the volume to harvest in volume control. There are at least five formulas that have been widely applied. We will focus on one of the simplest and oldest of these formulas in this section: Hundeshagen's formula. A later section describes some of the alternate formulas for calculating the volume to harvest under volume control.

Hundeshagen's Formula

Hundeshagen's formula is based on the notion that you should cut more when you have more inventory than you want and you should cut less when you have less inventory than you want. The inventory volume in the regulated forest is the target inventory volume. If the inventory volume in the current forest is greater than the amount in the target regulated forest then the harvest should be greater than the harvest that would occur with a regulated forest. If the inventory volume in the current forest is less than the inventory in the target regulated forest then the harvest should be less than the harvest that would occur with a regulated forest. The formula is:

$$H_t = \frac{H_{Reg}}{I_{Reg}} \times I_t = \frac{I_t}{I_{Reg}} \times H_{Reg}$$

where H_t = the volume harvested in the current period,
H_{reg} = the volume harvested from a regulated forest,
I_t = the inventory volume in the current forest, and
I_{reg} = the inventory volume in a regulated forest.

There are two ways to interpret Hundeshagen's formula: 1) that the harvest should be a fixed proportion of the inventory volume (the same proportion that would be harvested from a regulated forest), or 2) that the deviation of the harvest in the current period from the harvest from the regulated forest is proportional to the ratio of the current inventory volume to the inventory volume of the regulated forest. Note that the volume harvested from the regulated forest is the same as the LTSY, so we could have replaced H_{reg} with LTSY in the above formulas.

Calculating the Volume to Harvest

In order to use Hundeshagen's formula to determine the volume to harvest, we must calculate three quantities: 1) the inventory volume of the regulated forest (I_{Reg}), 2) the volume harvested from the regulated forest (H_{Reg} = LTSY), and 3) the inventory volume of the current forest (I_t).

The inventory volume in a forest is calculated by multiplying the number of acres in each age class by the average volume per acre in that age class. Unlike the harvest volume, when calculating the inventory volume, the volume at the midpoint age of each age class should be used. This is because the inventory volume is the volume at a point in time, for example at the beginning of a planning period. As we noted earlier, at the beginning of the period the average age of the stands in a given age class is equal to the midpoint age of the age class. Thus, when calculating the inventory volume in the 31 to 40-year age class, we would use the volume at age 35. The volume to be harvested from the regulated forest is simply the LTSY of the forest using the rotation on which the forest is to be regulated, which is equal to the MAI at this rotation times the total area in the forest. Once we have calculated these three quantities, we can use Hundeshagen's formula to calculate the volume to be harvested in the current decade.

Example: Calculating the Harvest Target for the First Decade

Assume the example forest described in the earlier examples is to be regulated using volume control. Calculate the harvest target for the first decade using Hundeshagen's formula.

Answer: To calculate the harvest target for the first decade, first calculate the inventory that will exist in the regulated forest. This is the sum of the acres in each age class times the average inventory volume per acre in that age class. First, it will be useful to calculate a yield table for the midpoints of the age classes. We will assume that the yield at the midpoints of the age classes is the average of the yields at the minimum and maximum age of each age class. Table 10.12 shows the yields given in Table 10.3 and the inventory yields at the midpoints of the age classes.

Table 10.12. Inventory yields for the example problem.

Age	Volume (cd/ac)	Age	Volume (cd/ac)
		5	2.5
10	5	15	10.5
20	16	25	24.0
30	32	35	43.5
40	55	45	65.0
50	75	55	82.5
60	90	65	95.0
70	100	75	100.0

In the target regulated forest, there will be 187.5 acres in the four age classes between 0 and 40. Thus, we multiply 187.5 times 2.5 to get the inventory in the first age class; we multiply 187.5 times 10.5 to get the inventory in the 11 to 20-year age class; we multiply 187.5 times 24 to get the inventory in the 21 to 30-year age class; and we multiply 187.5 times 43.5 to get the inventory in the 31 to 40-year age class. The total inventory is the sum of the inventory volumes in all of the age classes. Table 10.13 summarizes the inventory calculations for the regulated forest.

Table 10.13. Calculating the inventory volume in the target forest.

Age Class	Area (acres)	Inventory Volume per Acre (cds/ac)	Inventory Volume (cds)
0-10	187.5	2.5	468.75
11-20	187.5	10.5	1,968.75
21-30	187.5	24.0	4,500.00
31-40	187.5	43.5	8,156.25
Total	750.0	Not applicable	15,093.75

In the current forest, there are 250 acres in each age classes between 0 and 40, except the 21 to 30-year age class, which has no acres. Thus, we multiply 250 times 2.5 to get the inventory in the first age class; we multiply 250 times 10.5 to get the inventory in the 11 to 20-year age class; and we multiply 250 times

43.5 to get the inventory in the 31 to 40-year age class. Table 10.14 summarizes the inventory calculations for the current forest.

Table 10.14. Calculating the inventory volume in the current forest.

Age Class	Area (acres)	Inventory Volume per Acre (cds/ac)	Inventory Volume (cds)
0-10	250	2.5	625
11-20	250	10.5	2,625
21-30	0	24.0	0
31-40	250	43.5	10,875
Total	750	Not applicable	14,125

The MAI for this forest, with a 40-year rotation, is 1.375 cd/ac·yr (55 cd/ac ÷ 40 yr). There are 750 acres in the forest, so the LTSY for the forest will be 1,031.25 cords. We can now calculate the harvest target for the first year under volume control with Hundeshagen's formula:

$$H_t = \frac{I_t}{I_{\text{Re}g}} \times H_{Reg} = \frac{14{,}125}{15{,}093.75} \times 1{,}031.25 = 965.06\, cd / yr$$

The harvest target for the first decade will be ten times the annual harvest target—i.e., 9,565 cords.

Projecting the Age-class Distribution

Once you have calculated a harvest volume target, the next step is to identify the areas to be cut in order to produce this volume of wood. As always, the oldest age class will be the first cut. The steps in determining the acreage to cut are outlined below.

1. First, divide the harvest target by the volume per acre for that age class to determine how many acres would need to be cut from that age class to meet the harvest target. Then check whether there is enough acreage in the oldest age class to produce enough wood to meet the volume target.

2a. If the acreage that would need to be harvested is less than the acreage that is available in that age class, subtract the area that needs to be cut from the total acreage to determine the area that will not be cut.

2b. If the acreage that is needed is more than the acreage that is available in that age class, subtract the volume that you will get by harvesting all of the area in that age class from the volume target and move up to the next oldest age class to obtain the remaining unmet harvest.

3. Continue moving up to younger age classes until the target harvest volume is met.

Once the acreages that need to be cut from each age class have been determined, the total cut area is moved into the youngest age class, and all uncut areas are moved up to the next older age class. This process is just the same as projecting the age-class distribution under area control. After the new age-class distribution has been projected, another decade of volume control can be projected by repeating the same sequence of steps, starting with the calculation of the volume to be harvested.

Example: Projecting the Age-class Distribution under Volume Control

Project the age-class distribution of the example to the end of the second decade of the conversion period.

Answer: Based on the harvest target we calculated earlier, enough acres must be harvested in the first period to produce 9,565 cords. There are 250 acres in the 31 to 40-year age class that will produce 55 cd/ac. We will need to harvest 173.9 (9,565/55) of these acres to meet our harvest target. The remaining 76.1 acres in the age class will not be cut. The age-class distribution after one decade of volume control with Hundeshagen's formula is shown in Table 10.15.

Table 10.15. Example age-class distribution after 10 years of volume control.

Age Class	Acres
0-10	173.9
11-20	250.0
21-30	250.0
31-40	0.0
41-50	76.1

To project the age-class distribution for the example forest after two decades of volume control with Hundeshagen's formula, we first need to calculate the harvest volume target using Hundeshagen's formula. As you may recall, Hundeshagen's formula requires three pieces of information: 1) the inventory volume of the regulated forest (I_{Reg}), 2) the volume harvested from the regulated forest (H_{Reg} = LTSY), and 3) the inventory volume of the current forest (I_t). The first two of these numbers do not need to be recalculated:

- The inventory volume in the regulated forest is 15,093.8 cords.
- The volume harvested annually from the regulated forest is 1,031.25 cords.

The inventory volume in the forest at the beginning of the second decade is calculated in Table 10.16. Applying Hundeshagen's formula now gives us the annual harvest volume target for the second decade:

$$H_t = \frac{I_t}{I_{Reg}} \times H_{Reg} = \frac{13{,}908.4}{15{,}093.75} \times 1{,}031.25 = 950.3\, cd / yr$$

Table 10.16. Calculating the forest inventory volume at the beginning of decade 2.

Age Class	Acres	Inventory Volume/Acre	Inventory Volume
0-10	173.9	2.5	438.66
11-20	250	10.5	2625.0
21-30	250	24.0	6000.0
31-40	0	43.5	0
41-50	76.1	65.0	4844.72
Total	750.0	Not	13,908.4

The total harvest target for the second decade is therefore 9,503 cords. Now, calculate the acres from each age class that need to be harvested in order to provide this harvest volume. The 76.1 acres in the 41 to 50-year age class, at 75 cd/ac, will produce 5,707.5 cords. This leaves an unmet harvest target of 3,795.5 cords. The acres in the 21 to 30-year age class will yield 32 cd/ac. Thus, 118.6 of those acres will have to be harvested, leaving 131.4 acres from that age class uncut. The projected age-class distribution for several decades under volume control is shown in Table 10.17.

Note in Table 10.17 how much more stable the harvest volume is from decade to decade under volume control. The price of this stability, however, is that the forest does not become regulated, even after 40 years. Recall that the forest was regulated after only 30 years with area control. Nevertheless, stability of harvests is, after all, the primary reason to regulate a forest in the

Table 10.17. Projected age-class distribution, inventory, annual cut, and net revenue under volume control using Hundeshagen's formula.

Age Class	Acres in each age class at the beginning of each decade				
	0	10	20	30	40
0-10	250	175.5	196.8	204.8	181.3
11-20	250	250.0	175.5	196.8	204.8
21-30	0	250.0	250.0	175.5	196.8
31-40	250	0.0	127.7	173.0	167.1
41-50	0	74.5	0	0	0
51-60	0	0	0	0	0
Total Acres	750	750	750	750	750
Inventory	14,125	13,908	13,891	14,313	14,597
Average Annual Cut (Cords)	965.1	950.3	949.1	977.9	997.3
Average Annual Net Revenue ($)	20,617	19,821	19,631	20,822	21,102

first place, and area control often fails miserably in this regard during the conversion interval. Also, the age-class distribution of the forest is much more balanced after 40 years than it was at the beginning of the planning horizon. How perfect does the regulation have to be in order to satisfy the goals of regulation?

4. Calculating the Inventory Growth

Many volume control formulas base the harvest target on the inventory growth of the forest (G_t). In general, it is useful to know what the total growth is in your forest to be able to compare the volume harvested against the volume that is grown each year. While some would argue that you should never cut more that your forest is growing, I disagree with that perspective. I would argue that the amount you harvest from a forest should be based on achieving your target forest condition, and in some cases it will be necessary to cut more than you grow in order to do this. At the same time, it is clear that in the long run you cannot sustainably harvest more volume than you grow. Therefore, it is useful to know how much volume your forest is growing each year. We will call this quantity the inventory growth to distinguish it from other measures of growth.

To calculate the inventory growth, estimate the growth per acre for each age class using the yield table. Then multiply the growth per acre for each age class times the number of acres in the age class and sum over age classes. The formula is:

$$G_t = \sum_{i=1}^{N} g_i A_i$$

where G_t = the net annual growth of the inventory;
g_i = the growth per acre for acres in age class i;
A_i = the area in the i^{th} age class, and
N = the oldest age class in the forest.

The growth per acre for a given age class is given by the periodic annual increment (PAI) for that age class. The PAI for age class i is given by the following formula:

$$g_i = PAI_i = \frac{v_i - v_{i-1}}{n}$$

where PAI_i = the periodic annual increment per acre per year for acres in age class i;
v_i = the volume in age class i;
v_{i-1} = the volume in age class $i-1$; and
n = the width of an age class in years (and the number of years in a period).

Example

Calculate the inventory growth for the example forest used in the earlier examples.

Answer: first, calculate the growth per acre for each age class. These calculations are demonstrated in Table 10.18 on the next page.

Now, multiply the growth per acre for each age class by the number of acres in each age class and sum over age classes:

$$G_t = g_1 \times A_1 + g_2 \times A_2 + g_3 \times A_3 + g_4 \times A_4$$

$$= 0.5 \times 250 + 1.1 \times 250 + 1.6 \times 0 + 2.3 \times 250 = 975 \text{ cd/yr}$$

Table 10.18. Growth (PAI) calculation for the example yield function.

Age	Yield (cd/ac)	Growth Calculation	PAI (cd/ac·yr)
10	5	$\frac{(5-0)}{10}$	0.5
20	16	$\frac{(16-5)}{10}$	1.1
30	32	$\frac{(32-16)}{10}$	1.6
40	55	$\frac{(55-32)}{10}$	2.3
50	75	$\frac{(75-55)}{10}$	2.0
60	90	$\frac{(90-75)}{10}$	1.5
70	100	$\frac{(100-90)}{10}$	1.0

Example

Calculate the inventory growth for the regulated forest from the examples in this chapter.

Answer: There will be 187.5 acres in age classes 1 to 4. Again, multiply the growth per acre for each age class by the number of acres in each age class and sum over age classes:

$$G_t = g_1 \times A_1 + g_2 \times A_2 + g_3 \times A_3 + g_4 \times A_4$$
$$= 0.5 \times 187.5 + 1.1 \times 187.5 + 1.6 \times 187.5 + 2.3 \times 187.5 = 1{,}031.25 \text{ cd/yr}$$

Note that this is the same as the annual harvest from the regulated forest (H_{Reg} and also the LTSY). *Why?*

5. <u>Other Volume Control Formulas</u>

It was noted earlier that there is more than one way to calculate the harvest target with volume control. All of the volume target formulas that have been used are purely heuristic – i.e., they are based on intuition, rather than an analytical approach. The variety of formulas that have been proposed is actually quite remarkable. Each has advantages and disadvantages. Some may work well in some conditions, while others will work better in other situations. Four more formulas are presented here just so you are aware of the alternatives. Perhaps more important, each formula reflects unique insights about setting an appropriate harvest target for a forest.

von Mantel's Method

This method assumes that the inventory volume of a regulated forest can be approximated by the area of a triangle whose base is the forest area (A) and whose height is the volume per acre at rotation age (v_R). (The formula for the area of a triangle is ½ times the length of the base of the triangle times the height of the triangle.) von Mantel's assumption can be interpreted as assuming that the growth of a stand is constant. With this assumption, the inventory of the regulated forest is:

$$I_{Reg} = \frac{A \times v_R}{2}$$

Also, as we showed earlier, the periodic harvest from a regulated forest is the area in the rotation age class (A_i) times the MAI at the rotation age:

$$H_{Reg} = A \times \frac{v_R}{R}$$

Substituting these two expressions into Hundeshagen's formula gives:

$$H_t = \frac{H_{Reg} \times I_t}{I_{Reg}} = \frac{\frac{A v_R}{R} \times I_t}{\frac{A v_R}{2}} = \frac{2 I_t}{R}$$

Thus, all you need to know to calculate the harvest volume target for the current period is the rotation age and your current inventory volume! For example, you don't need to know anything about yield-age relationships.

The Austrian Formula

The Austrian formula assumes that the cut should equal the growth, plus or minus an adjustment factor. The adjustment will be positive if the forest is overstocked and negative if the forest is understocked.

$$H_t = G_t + \frac{I_t - I_{Reg}}{a}$$

where H_t = the periodic harvest;
G_t = the periodic net growth;
I_t = the current inventory volume;

I_{Reg} = the inventory volume in a regulated forest; and
a = the adjustment period.

Alternatively, *1/a* can be viewed as the proportion of the excess inventory to remove each period. Note that using this formula requires an estimate of the inventory growth for the forest. A method for calculating the inventory growth is discussed in the next section.

Modified Austrian Formula

This formula modifies the Austrian formula by cutting the average of the current growth and the expected growth from a regulated forest, plus or minus an adjustment factor. (Recall that the basic Austrian formula cuts the current growth plus or minus an adjustment factor.) The formula for the Modified Austrian volume control method is:

$$H_t = \frac{G_t + G_{Reg}}{2} + \frac{I_t - I_{Reg}}{a}$$

where H_t = the periodic harvest;
G_t = the periodic net growth;
G_{Reg} = the periodic net growth for a regulated forest;
I_t = the current inventory volume;
I_{Reg} = the inventory volume in a regulated forest; and
a = the adjustment period.

Again, *1/a* can be viewed as the proportion of the excess inventory to remove each period.

Hanzlik's Formula

Hanzlik's formula was developed for the Pacific Northwest, where the primary problem was how fast to cut old-growth. The idea is to cut $1/R^{th}$ of our "excess inventory"—i.e., the volume of merchantable timber in stands older than the rotation age, plus the growth.

$$H_t = \frac{I_m}{R} + G_t$$

where H_t = the harvest target for period *t*;
I_m = the inventory volume in the mature age classes—i.e. those age classes that are older than the rotation age;
R = the desired rotation age;
G_t = the periodic net growth.

6. Study Questions for Regulation

1. What is the difference between a tactical and a strategic decision? List some forest management decisions that are tactical and some that are strategic.

2. What is a regulated forest?

3. Why are models useful? What are the desirable characteristics of a model?

4. How does recognizing the complexity of actual forests complicate the basic definition of a regulated forest?

5. Why regulate a forest? What are the benefits? What are the costs?

6. Why is the rotation that maximizes the MAI sometimes considered the best rotation for a regulated forest?

7. Explain why using the rotation that maximizes the MAI maximizes the annual harvest from a regulated forest.

8. The rotation that maximizes the MAI will also maximize the LTSY. Why not regulate the forest on the rotation that maximizes the MAI?

9. How can it be that maximizing the annual income from a regulated forest does not maximize the net present value of the forest?

10. In what sense can you have too much inventory volume in a forest?

11. Under what conditions is it appropriate for the harvest from a forest to exceed the forest's growth?

12. What are the advantages and disadvantages of area control versus volume control?

13. Summarize the process of regulating a forest with area control.

14. Summarize the process of regulating a forest with volume control.

15. Explain the rationale for each of the following volume control formulas:
    - Hundeshagen's formula
    - von Mantel's formula
    - the Austrian formula

- the modified Austrian formula
- Hanzlik's formula

16. Why is the growth in a regulated forest equal to the LTSY?

7. Exercises

1. Calculate the long-term sustained yield for the forest described in Table 10.19.

Table 10.19. Acres, optimal rotation and yield by site class for long-term sustained yield calculations.

Site Class	Acres	Optimal Rotation	Yield (cords/ac)	MAI (cds/ac/yr)	LTSY (cords)
I	2,400	70	75		
II	1,600	80	89		
III	1,200	100	88		
Total	5,200				

2. You have just been hired to manage a 1,150 acre forest. The owner of the forest wants you to regulate the forest using area control. The current age class distribution of the forest is shown in Table 10.20. The predicted yield by age class is given in Table 10.21.

a. Assume that the owner wants the forest regulated using the rotation with the highest LEV. Calculate the MAI and LEV for age classes 15 through 35 to find the best rotation for regulation. Assume that real stumpage prices will be $35/cord for the foreseeable future, that stand establishment costs will be $180 per acre, and use a real interest rate of 4%. What is the rotation with the highest LEV?

Table 10.20. Age-class distribution.

Stand	Age (years)	Area (acres)
A	0-5	375
B	6-10	275
C	16-20	100
D	21-25	250
E	25-30	150

Table 10.21. Yields, MAI and LEV.

Age (years)	Volume per acre (cords)	MAI	LEV
5	1		
10	5		
15	14		
20	23		
25	31		
30	38		
35	43		

b. Once you have determined the best rotation to use, complete Table 10.22 on the next page by projecting the age class distribution that you will have at the beginning of each of the next six 5-yr periods if you regulate the forest using area control. Also calculate the average volume cut and the average net revenue (in future dollars) for the 5-yr periods beginning with years 0, 5, 10, 15, etc.

c. How many acres will be cut each year?

d. What will be the MAI and the LTSY from the regulated forest?

3. In reviewing your results from Problem 3, you may have noticed that annual harvests and revenues fluctuate a lot. You know that volume control methods of regulation can be used to even these flows out over time. You decide to demonstrate this to the forest owner.

a. From Problem 3, identify the annual cut that will be achieved after the forest is fully regulated (cords per year).

b. Fill in the first column of Table 10.23, showing the initial age class distribution. Do not try to fill in the average annual cut or the average annual net revenue rows yet.

c. Next, calculate the volume of wood in the regulated forest by filling in the first column of Table 10.24. Calculate the volume in each age class by multiplying the number of acres in the age class for a regulated forest by the volume of wood per acre for stands in the given age class. Calculate the total volume of the forest when it is regulated by summing the volumes in each age class. Also, note the annual cut for a regulated forest (from part 2a) in the cell at the bottom of the first column.

d. Now, fill in the second column of Table 10.22. First, calculate the volume of wood in each age class for the initial inventory. Total the column to determine the current forest inventory volume. In the cell below this total, calculate the ratio of the inventory volume in the current forest (V_i) over the inventory volume in the regulated forest (V_R). Multiply this ratio times the annual harvest from the regulated forest to determine the volume that should be harvested annually during the first 5-yr period. Note this amount in the cell at the bottom of the second column.

e. Starting with the oldest age class, determine the area that must be harvested over the first 5-yr period from each age class in order to harvest annually the volume you calculated in part d (the number in the bottom cell of the second column). Then, determine the resulting age class distribution for the forest at the beginning of the next 5-yr period. As in Problem 2, show the average annual harvest and average annual net revenues for each 5-yr period.

f. Now, repeat the steps in parts c through e until you have projected the state of the forest for five 5-yr periods. Is the forest regulated at the end of five 5-yr periods?

g. Using a spreadsheet graph the average net revenue over five 5-yr periods under area control and volume control. Remember, the graph should have a title, the axes should be properly labeled and the series properly identified.

Table 10.22. Projected Age Class Distribution, Annual Cut and Net Revenue under Area Control.

Age Class	Acres in each age class at the beginning of each 5-yr period					
	0	5	10	15	20	25
0-5						
6-10						
11-15						
16-20						
21-25						
26-30						
31-35						
Total Acres						
Average Annual Cut (Cords)[1]						
Average Annual Net Revenue ($)[2]						

1. "Average annual cut" gives the average annual cut over the 5-yr period beginning with the year at the top of the column.
2. "Average annual net revenue" gives the average annual net revenue (gross revenues minus costs) over the 5-yr period beginning with the year at the top of the column.

Table 10.23. Projected Age Class Distribution, Annual Cut and Net Revenue under Volume Control (Hundeshagen's Formula).

Age Class	Acres in each age class at the beginning of each 5-yr period					
	0	5	10	15	20	25
0-5						
6-10						
11-15						
16-20						
21-25						
26-30						
31-35						
Total Acres						
Avg. Ann. Cut (Cords)						
Avg. Ann. Net Rev. ($)						

Table 10.24. Projected Age Class Volumes, Total Inventory Volume, Ratio of Existing Inventory Volume (V_i) to Regulated Inventory Volume (V_R), and Annual Harvest Volume under Volume Control (Hundeshagen's Formula).

Age Class	Regulated Forest Volumes	Volume in each age class at the beginning of each 5-yr period				
		0	5	10	15	20
0-5						
6-10						
11-15						
16-20						
21-25						
26-30						
31-35						
Total Volume						
V_i/V_R						
Annual Harv. Vol.						

4. Table 10.25 shows the initial age-class distribution for a forest. Table 10.26 gives yield data for the forest.

a. Calculate the MAI, the PAI, and the LEV for rotation ages 20, 25, 30, and 35 to complete Table 10.26. Assume that real stumpage prices will be $30/cord for the foreseeable future, that stand establishment costs will be $200 per acre, and use a real interest rate of 4%.

Table 10.25. Initial age-class distribution for problem 4.

Age Class (years)	Area (acres)
0-5	105
6-10	0
11-15	0
16-20	35
21-25	190
26-30	250
31-35	30

Table 10.26. Yield, PAI, MAI, and LEV for problem 4.

Age (years)	Yield per acre (cords)	PAI (cd/ ac·yr)	MAI (cd/ ac·yr)	LEV ($/acre)
5	0			
10	4			
15	13			
20	21			
25	31			
30	42			
35	50			
40	56			

b. In Table 10.27, show the age-class distribution, the inventory volume, and the inventory growth of the forest after it is regulated using the optimal economic rotation from part a.

c. Use Tables 10.28 through 10.30 to show how the forest age class distribution will change over the next 20 years if the forest is regulated using the modified Austrian formula. Set the parameter *a* in the formula to 30.

Table 10.27. Area, inventory, and growth of the regulated forest.

Age Class (years)	Area (acres)	Inventory Volume (cords)	Inventory Growth (cords/yr)
0-5			
6-10			
11-15			
16-20			
21-25			
26-30			
31-35			
Total			

Table 10.28. Projected age class distribution, annual cut and net revenue under volume control (Modified Austrian Formula).

Age Class	Acres in each age class at the beginning of each 5-yr period				
	0	5	10	15	20
0-5					
6-10					
11-15					
16-20					
21-25					
26-30					
31-35					
36-40					
Total Acres					
Avg. Ann. Cut (Cords)					
Avg. Ann. Net Rev. ($)					

Table 10.29. Projected inventory volume by age class and harvest volume under volume control (Modified Austrian Formula).

Age Class	Volume in each age class at the beginning of each 5-yr period			
	0	5	10	15
0-5				
6-10				
11-15				
16-20				
21-25				
26-30				
31-35				
36-40				
Total Volume				
Annual Harv. Vol.				

Table 10.30. Projected inventory growth by age class under volume control (Modified Austrian Formula).

Age Class	Inventory growth by age class at the beginning of each 5-yr period			
	0	5	10	15
0-5				
6-10				
11-15				
16-20				
21-25				
26-30				
31-35				
36-40				
Total Growth				