Solving applications of Linear Programming with Excel

Isabel Martins

Exercice 1 (complete)

ISA - Applied Operations Research - 2024/2025

Outline

Question 1. Land use planning

Question 1. Land use planning

The problem

A river is a major source of water of a certain city, and the council has an annual plan to expand the city's development in the area along the river. A total of 100 acres of land is projected to be needed for residential, business, and recreational use. According to the plan, at least 20 acres of land should be designed to for residential development, 30 acres will be used for business development, and a recreational park will be built on at least 10 acres. The initial investment cost for residential land is 8 million euros for the first 20 acres of land and 300000 euros for extra acre of land thereafter. The initial investment costs for business land and recreational land are 20 million and 12 million euros, respectively, while the costs for additional land are 500000 and 400000 euros per acre, respectively. An acre of residential land can yield a profit of 50000 euros per year, and the expected annual profits for business land and recreational land are 120000 and 150000 euros, respectively. On average, every acre of residential land will use 20 m^3 of water per month, and every acre of business land and recreational land consumes 40 m^3 and 25 m^3 of water per month, respectively. The annual budget is 80 million euros, and the regulation of water from the river is 40000 m^3 per year. The city council wants to find an annual plan with the maximum profit.

1. Formulate this problem as a LP model.

The decision variables are as follows:

1. Formulate this problem as a LP model.

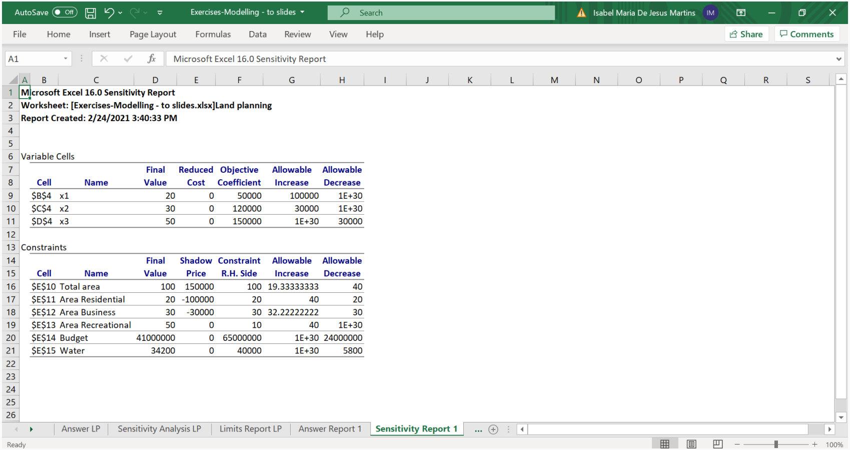
The decision variables are as follows:

- x_1 - area to residential development (acres)
- x_2 - area to business development (acres)
- x_3 - area to recreational development (acres)

1. Formulate this problem as a LP model.

$$
\begin{aligned}
\max\ Z = {} & 50000x_1 && +120000x_2 && +150000x_3 && && (1)\\
\text{subject to} \\
& x_1 && +x_2 && +x_3 && = 100 && (2)\\
& x_1 && && && \geq 20 && (3)\\
& && x_2 && && \geq 30 && (4)\\
& && && x_3 && \geq 10 && (5)\\
& 300000x_1 && +500000x_2 && +400000x_3 && \leq 65000000 && (6)\\
& 240x_1 && +480x_2 && +300x_3 && \leq 40000 && (7)\\
& x_1, && x_2, && x_3 && \geq 0 && (8)
\end{aligned}
$$

3. Report the shadow prices for each constraint and comment.



Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Exercises-Modelling - to slides.xlsx]Land planning
Report Created: 2/24/2021 3:40:33 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	x1	20	0	50000	100000	1E+30
C4	x2	30	0	120000	30000	1E+30
D4	x3	50	0	150000	1E+30	30000

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E10	Total area	100	150000	100	19.33333333	40
E11	Area Residential	20	-100000	20	40	20
E12	Area Business	30	-30000	30	32.22222222	30
E13	Area Recreational	50	0	10	40	1E+30
E14	Budget	41000000	0	65000000	1E+30	24000000
E15	Water	34200	0	40000	1E+30	5800

Figure 1: Excel - Sensitivity Report.

3. Report the shadow prices for each constraint and comment.

Shadow price

The **shadow price** of a constraint measures the impact on the optimal objective value with the (slight) increase of the RHS, remaining the other parameters the same.

3. Report the shadow prices for each constraint and comment.

Shadow price

The **shadow price** of a constraint measures the impact on the optimal objective value with the (slight) increase of the RHS, remaining the other parameters the same.

- The increase on the minimum area for recreational development, budget and regulation of water do not have impact on the optimal annual profit (shadow prices of non-binding constraints are equal to zero).

3. Report the shadow prices for each constraint and comment.

Shadow price

The **shadow price** of a constraint measures the impact on the optimal objective value with the (slight) increase of the RHS, remaining the other parameters the same.

- The increase on the minimum area for recreational development, budget and regulation of water do not have impact on the optimal annual profit (shadow prices of non-binding constraints are equal to zero).
- The increase on the total area has a positive impact of 150000 €/acre. The increase on the minimum areas for residential or business development have a negative impact of 100000 €/acre and 30000 €/acre, respectively.

4. Derive the range of feasibility for all RHS values.

Range of feasibility for a RHS value

Columns ”Allowable Increase” and ”Allowable Decrease” in Table Constraints give the maximum and minimum variation of each RHS value over which the shadow price (in column ”Shadow Price”) does not change.

4. Derive the range of feasibility for all RHS values.

- The shadow prices are valid for RHS increases by, at most, 19.33 acre (constraint (2)), 40 acre (constraint (3)), 32.22 acre (constraint (4)), 40 acre (constraint (5)) and $+\infty$ (constraints (6) and (7)).

4. Derive the range of feasibility for all RHS values.

- The shadow prices are valid for RHS increases by, at most, 19.33 acre (constraint (2)), 40 acre (constraint (3)), 32.22 acre (constraint (4)), 40 acre (constraint (5)) and $+\infty$ (constraints (6) and (7)).

  Thus,

  - if the total area increased from 100 acre to 119.33 acre, the optimal profit would increase $150000 \times 19.33 = 2900000$ €

4. Derive the range of feasibility for all RHS values.

- The shadow prices are valid for RHS increases by, at most, 19.33 acre (constraint (2)), 40 acre (constraint (3)), 32.22 acre (constraint (4)), 40 acre (constraint (5)) and $+\infty$ (constraints (6) and (7)).

  Thus,
  - if the total area increased from 100 acre to 119.33 acre, the optimal profit would increase $150000 \times 19.33 = 2900000$ €
  - if the minimum area for residential development increased from 20 acre to 60 acre, the optimal profit would decrease $100000 \times 40 = 4000000$ €

4. Derive the range of feasibility for all RHS values.

- The shadow prices are valid for RHS increases by, at most, 19.33 acre (constraint (2)), 40 acre (constraint (3)), 32.22 acre (constraint (4)), 40 acre (constraint (5)) and $+\infty$ (constraints (6) and (7)).

  Thus,

  - if the total area increased from 100 acre to 119.33 acre, the optimal profit would increase $150000 \times 19.33 = 2900000$ €
  - if the minimum area for residential development increased from 20 acre to 60 acre, the optimal profit would decrease $100000 \times 40 = 4000000$ €
  - if the minimum area for business development increased from 30 acre to 62.22 acre, the optimal profit would decrease $30000 \times 32.22 = 966666.67$ €

4. Derive the range of feasibility for all RHS values.

- The shadow prices are valid for RHS increases by, at most, 19.33 acre (constraint (2)), 40 acre (constraint (3)), 32.22 acre (constraint (4)), 40 acre (constraint (5)) and $+\infty$ (constraints (6) and (7)).

  Thus,

  - if the total area increased from 100 acre to 119.33 acre, the optimal profit would increase $150000 \times 19.33 = 2900000$ €
  - if the minimum area for residential development increased from 20 acre to 60 acre, the optimal profit would decrease $100000 \times 40 = 4000000$ €
  - if the minimum area for business development increased from 30 acre to 62.22 acre, the optimal profit would decrease $30000 \times 32.22 = 966666.67$ €
  - if the minimum area for recreational development increased from 10 acre to 50 acre, the optimal profit would not change

4. Derive the range of feasibility for all RHS values.

- The shadow prices are valid for RHS increases by, at most, 19.33 acre (constraint (2)), 40 acre (constraint (3)), 32.22 acre (constraint (4)), 40 acre (constraint (5)) and $+\infty$ (constraints (6) and (7)).

  Thus,

  - if the total area increased from 100 acre to 119.33 acre, the optimal profit would increase $150000 \times 19.33 = 2900000$ €
  - if the minimum area for residential development increased from 20 acre to 60 acre, the optimal profit would decrease $100000 \times 40 = 4000000$ €
  - if the minimum area for business development increased from 30 acre to 62.22 acre, the optimal profit would decrease $30000 \times 32.22 = 966666.67$ €
  - if the minimum area for recreational development increased from 10 acre to 50 acre, the optimal profit would not change
  - if the budget and the regulation of water increased infinitely, the optimal profit would not change.

4. Derive the range of feasibility for all RHS values.

- The shadow prices are valid for RHS decreases by, at most, 40 acre (constraint (2)), 20 acre (constraint (3)), 30 acre (constraint (4)), $-\infty$ (constraint (5)), 24000000 (constraints (6)) and 5800 (constraints (7)).

4. Derive the range of feasibility for all RHS values.

- The shadow prices are valid for RHS decreases by, at most, 40 acre (constraint (2)), 20 acre (constraint (3)), 30 acre (constraint (4)), $-\infty$ (constraint (5)), 24000000 (constraints (6)) and 5800 (constraints (7)).

  Thus,

  - if the total area decreased from 100 acre to 60 acre, the optimal profit would decrease $150000 \times 40 = 6000000$ €

4. Derive the range of feasibility for all RHS values.

- The shadow prices are valid for RHS decreases by, at most, 40 acre (constraint (2)), 20 acre (constraint (3)), 30 acre (constraint (4)), $-\infty$ (constraint (5)), 24000000 (constraints (6)) and 5800 (constraints (7)).

  Thus,

  - if the total area decreased from 100 acre to 60 acre, the optimal profit would decrease $150000 \times 40 = 6000000$ €
  - if the minimum area for residential development decreased from 20 acre to 0 acre, the optimal profit would increase $100000 \times 20 = 2000000$ €

4. Derive the range of feasibility for all RHS values.

- The shadow prices are valid for RHS decreases by, at most, 40 acre (constraint (2)), 20 acre (constraint (3)), 30 acre (constraint (4)), $-\infty$ (constraint (5)), 24000000 (constraints (6)) and 5800 (constraints (7)).

  Thus,

  - if the total area decreased from 100 acre to 60 acre, the optimal profit would decrease $150000 \times 40 = 6000000$ €
  - if the minimum area for residential development decreased from 20 acre to 0 acre, the optimal profit would increase $100000 \times 20 = 2000000$ €
  - if the minimum area for business development decreased from 30 acre to 0 acre, the optimal profit would increase $30000 \times 30 = 900000$ €

4. Derive the range of feasibility for all RHS values.

- The shadow prices are valid for RHS decreases by, at most, 40 acre (constraint (2)), 20 acre (constraint (3)), 30 acre (constraint (4)), $-\infty$ (constraint (5)), 24000000 (constraints (6)) and 5800 (constraints (7)).

  Thus,

  - if the total area decreased from 100 acre to 60 acre, the optimal profit would decrease $150000 \times 40 = 6000000$ €
  - if the minimum area for residential development decreased from 20 acre to 0 acre, the optimal profit would increase $100000 \times 20 = 2000000$ €
  - if the minimum area for business development decreased from 30 acre to 0 acre, the optimal profit would increase $30000 \times 30 = 900000$ €
  - if the minimum area for recreational development decreased infinitely (in fact to zero), the optimal profit would not change

4. Derive the range of feasibility for all RHS values.

- The shadow prices are valid for RHS decreases by, at most, 40 acre (constraint (2)), 20 acre (constraint (3)), 30 acre (constraint (4)), $-\infty$ (constraint (5)), 24000000 (constraints (6)) and 5800 (constraints (7)).

  Thus,

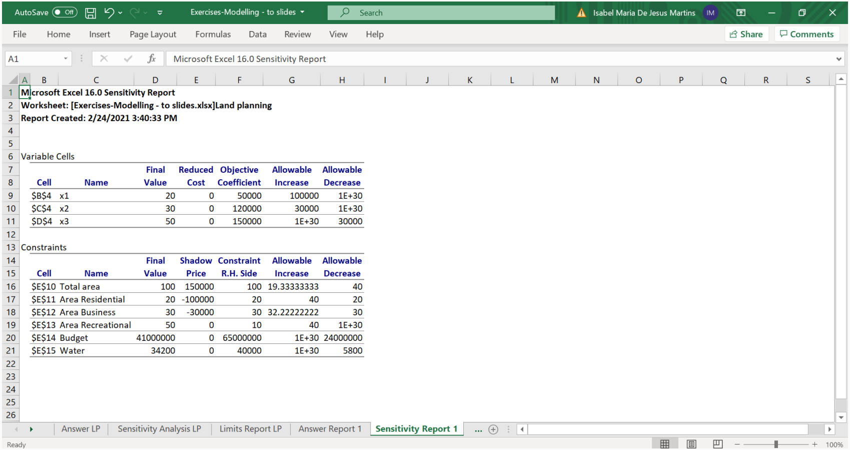
  - if the total area decreased from 100 acre to 60 acre, the optimal profit would decrease $150000 \times 40 = 6000000$ €
  - if the minimum area for residential development decreased from 20 acre to 0 acre, the optimal profit would increase $100000 \times 20 = 2000000$ €
  - if the minimum area for business development decreased from 30 acre to 0 acre, the optimal profit would increase $30000 \times 30 = 900000$ €
  - if the minimum area for recreational development decreased infinitely (in fact to zero), the optimal profit would not change
  - if the budget decreased from 80000000 € to 56000000 €, the optimal profit would not change

5. Derive the range of optimality for all objective function coeficients and comment.

Range of optimality for an objective coeficient value

Columns ”Allowable Increase” and ”Allowable Decrease” give the amount by which each objective function coefficient (in column ”Objective Coefficient”) can be increased or decreased, respectively, without changing the optimal activity levels.

5. Derive the range of optimality for all objective function coefficients.



Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Exercises-Modelling - to slides.xlsx]Land planning
Report Created: 2/24/2021 3:40:33 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	x1	20	0	50000	100000	1E+30
C4	x2	30	0	120000	30000	1E+30
D4	x3	50	0	150000	1E+30	30000

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E10	Total area	100	150000	100	19.33333333	40
E11	Area Residential	20	-100000	20	40	20
E12	Area Business	30	-30000	30	32.22222222	30
E13	Area Recreational	50	0	10	40	1E+30
E14	Budget	41000000	0	65000000	1E+30	24000000
E15	Water	34200	0	40000	1E+30	5800

Figure 2: Excel - Sensitivity Report.

5. Derive the range of optimality for all objective function coeficients and comment.

As long as the values of the objective function coefficients on x_1 (annual residential profit per acre), x_2 (annual business profit per acre) and x_3 (annual recreational profit per acre) are, respectively, in the following intervals, one at a time and remaining the other parameters unchanged, the optimal solution will be the same ($x_1^* = 20$ $x_2^* = 30$ and $x_3^* = 50$):

i) $]-\infty, 150000]$ (the allowable decrease is $+\infty$ and the allowable increase is 100000)

ii) $]-\infty, 150000]$ (the allowable decrease is $+\infty$ and the allowable increase is 30000)

iii) $[120000, +\infty[$ (the allowable decrease is 30000 and the allowable increase is $+\infty$).

Sensitivity Report

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Exercises-Modelling.xlsx]Land planning - alternative
Report Created: 2/27/2021 10:17:29 AM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	y1	0	-100000	50000	100000	1E+30
C4	y2	0	-30000	120000	30000	1E+30
D4	y3	40	0	150000	1E+30	30000

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E11	Budget	16000000	0	40000000	1E+30	24000000
E12	Water	12000	0	17800	1E+30	5800
E10	Total area	40	150000	40	19,33333333	40

Answer Report - alternative | **Sensitivity Report - alternativ** | Limits Report - alternative | Land planning - al ...

Bom estudo!