



# Solving applications of Linear Programming with Excel SOLVER

---

**APPLIED OPERATIONAL  
RESEARCH**

Susana Barreiro  
February 27

ISA - Applied Operational Research - 2024/2025



## Question 2.

Test - 25 May 2016

A farmer has the following resource endowments: 500 acres of land, 1500 hours of family labour and 120000 € of capital investment. She can use these resources to grow the following crops: corn, sorghum, wheat and soybeans. Any crop should not occupy more than 250 acres. Assume that the farmer works to maximize profit (revenue minus costs) from the production of these crops. She expects the following in terms of prices, crop yields, costs and labor requirements.

Crop	Price (€/bushel)	Yield (bushel/acre)	Cost (€/acre)	Labor Requirement (hours/acre)
Corn	2.75	120	250	3.25
Sorghum	2.65	100	200	3.00
Wheat	3.15	105	245	3.15
Soybeans	6.75	45	230	3.30

1. Formulate the problem as a linear program.
2. Solve the problem using the Excel Solver. What is the optimal solution to the problem?
3. Which constraints are binding? How do you interpret each binding constraint?
4. By how much should sorghum's current profit per acre increase in order for the optimal solution to change? Sorghum's current profit per acre decrease would change the optimal solution?
5. What is the shadow price for each constraint? How do you interpret this value?
6. What is the range of feasibility for the maximum number of hours of family labour with respect to the shadow price? How do you interpret this range?
7. How much the profit would change if one acre of soybeans were forced into the current optimal solution?



## Question 2.

Test - 25 May 2016

A farmer has the following resource endowments: 500 acres of land, 1500 hours of family labour and 120000 € of capital investment. She can use these resources to grow the following crops: corn, sorghum, wheat and soybeans. Any crop should not occupy more than 250 acres. Assume that the farmer works to maximize profit (revenue minus costs) from the production of these crops. She expects the following in terms of prices, crop yields, costs and labor requirements.

Crop	Price (€/bushel)	Yield (bushel/acre)	Cost (€/acre)	Labor Requirement (hours/acre)
Corn	2.75	120	250	3.25
Sorghum	2.65	100	200	3.00
Wheat	3.15	105	245	3.15
Soybeans	6.75	45	230	3.30

1. Formulate the problem as a linear program.

$$\text{Maximize } Z = 80x_1 + 65x_2 + 85.75x_3 + 73.75x_4$$

$$x_1 + x_2 + x_3 + x_4 \leq 500$$

$$x_1 \leq 250$$

$$x_2 \leq 250$$

$$x_3 \leq 250$$

$$x_4 \leq 250$$

$$3.25x_1 + 3.00x_2 + 3.15x_3 + 3.30x_4 \leq 1500$$

$$250x_1 + 200x_2 + 245x_3 + 230x_4 \leq 120000$$

$$x_1, x_2, x_3, x_4 \geq 0$$



## Question 2.

---

Test - 25 May 2016

A farmer has the following resource endowments: 500 acres of land, 1500 hours of family labour and 120000 € of capital investment. She can use these resources to grow the following crops: corn, sorghum, wheat and soybeans. Any crop should not occupy more than 250 acres. Assume that the farmer works to maximize profit (revenue minus costs) from the production of these crops. She expects the following in terms of prices, crop yields, costs and labor requirements.

Crop	Price (€/bushel)	Yield (bushel/acre)	Cost (€/acre)	Labor Requirement (hours/acre)
Corn	2.75	120	250	3.25
Sorghum	2.65	100	200	3.00
Wheat	3.15	105	245	3.15
Soybeans	6.75	45	230	3.30

1. Formulate the problem as a linear program.
2. Solve the problem using the Excel Solver. What is the optimal solution to the problem?

File Home Insert Page Layout Formulas Data Review View Developer Tell me what you want to do...

From Access From Web From Text Get External Data From Other Sources Existing Connections New Query From Table Recent Sources Show Queries From Table Refresh All Edit Links Connections Sort Filter Reapply Advanced Text to Columns Flash Fill Remove Duplicates Data Validation Consolidate Relationships Manage Data Model What-If Analysis Forecast Sheet Group Ungroup Subtotal Solver Data Analysis Analysis

P11

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
1																						
2							<b>Crop</b>	<b>Price</b>	<b>Yield</b>	<b>Cost</b>	<b>Labor</b>											
3							(€/bushel)	(bushel/ha)	(€/acre)	(hours/ha)												
4			ha corn	x1	Corn	2.75	120	250	3.25	<b>80</b>												
5			ha sorghum	x2	Sorghum	2.65	100	200	3	<b>65</b>												
6			ha wheat	x3	Wheat	3.15	105	245	3.15	<b>85.75</b>												
7			ha soybeans	x4	Soybeans	6.75	45	230	3.3	<b>73.75</b>												

Max Profit: (money make from selling what I produced per ha minus how much it costed me to produce in an ha)

11	Max profit:	0																				
12							<b>x1</b>	<b>x2</b>	<b>x3</b>	<b>x4</b>												
13	how many ha of each crop to plant to maximize profit:						<b>80</b>	<b>65</b>	<b>85.75</b>	<b>73.75</b>												
14							0	0	0	0												
15	subject to these constraints:						<b>x1</b>	<b>x2</b>	<b>x3</b>	<b>x4</b>	<b>LHS</b>		<b>RHS</b>									
16	total area	area 1					1	1	1	1	0	<=	1500									
17	area restriction for species x1	area 2					1				0	<=	250									
18	area restriction for species x2	area 3						1			0	<=	250									
19	area restriction for species x3	area 4							1		0	<=	250									
20	area restriction for species x4	area 5								1	0	<=	250									
21	hours of family labour	labour					3.25	3	3.15	3.3	0	<=	1500									
22	cost per ha	cost					250	200	245	230	0	<=	120000									

**Solver Parameters**

Set Objective:

To:  Max  Min  Value Of:

By Changing Variable Cells:

Subject to the Constraints:

Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method  
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

File Home Insert Page Layout Formulas Data Review View Developer Tell me what you want to do... Share

Get External Data: From Access, From Web, From Text, From Other Sources, Existing Connections

Get & Transform: Show Queries, From Table, Recent Sources, New Query

Connections: Refresh All, Properties, Edit Links

Sort & Filter: Sort, Filter, Clear, Reapply, Advanced

Data Tools: Flash Fill, Remove Duplicates, Data Validation, Manage Data Model, Consolidate, Relationships

Forecast: What-If Analysis, Forecast Sheet

Outline: Group, Ungroup, Subtotal

Analysis: Solver, Data Analysis

L16 fx

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
1																							
2							<b>Crop</b>	<b>Price</b>	<b>Yield</b>	<b>Cost</b>	<b>Labor</b>												
3							(€/bushel)	(bushel/ha)	(€/acre)	(hours/ha)													
4					ha corn	x1	Corn	2.75	120	250	3.25												
5					ha sorghum	x2	Sorghum	2.65	100	200	3												
6					ha wheat	x3	Wheat	3.15	105	245	3.15												
7					ha soybeans	x4	Soybeans	6.75	45	230	3.3												
8																							

Max Profit: (money make from selling what I produced per ha minus how much it costed me to produce in an ha)

Max profit:	0	x1	x2	x3	x4
		80	65	85.75	73.75

how many ha of each crop to plant to maximize profit:

	x1	x2	x3	x4
	0	0	0	0

subject to these constraints:

	x1	x2	x3	x4	LHS	RHS
total area area 1	1	1	1	1	0	<= 1500
area restriction for species x1 area 2	1				0	<= 250
area restriction for species x2 area 3		1			0	<= 250
area restriction for species x3 area 4			1		0	<= 250
area restriction for species x4 area 5				1	0	<= 250
hours of family labour labour	3.25	3	3.15	3.3	0	<= 1500
cost per ha cost	250	200	245	230	0	<= 120000

Add Constraint

\$L\$16:\$L\$22

	B	C	D	E	F	G	H	I	J	K	L	M	N
1											Labor		
2							Crop	Price	Yield	Cost	Requirement		
3								(€/bushel)	(bushel/ha)	(€/acre)	(hours/ha)		
4			ha corn	x1	Corn	2.75	120	250	3.25			80	
5			ha sorghum	x2	Sorghum	2.65	100	200	3			65	
6			ha wheat	x3	Wheat	3.15	105	245	3.15			85.75	

### Solver Parameters

Set Objective:

To:  Max  Min  Value Of:

By Changing Variable Cells:

Subject to the Constraints:

Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method: Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Buttons: Add, Change, Delete, Reset All, Load/Save, Options, Help, Solve, Close

### Solver Results

Solver found a solution. All Constraints and optimality conditions are satisfied.

Keep Solver Solution  Restore Original Values

Return to Solver Parameters Dialog  Outline Reports

Reports: Answer, Sensitivity, Limits

Buttons: OK, Cancel, Save Scenario...

Reports: Creates the type of report that you specify, and places each report on a separate sheet in the workbook

	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
					Crop	Price (€/bushel)	Yield (bushel/ha)	Cost (€/acre)	Labor Requirement (hours/ha)								
		ha corn	x1	Corn	2.75	120	250	3.25									
		ha sorghum	x2	Sorghum	2.65	100	200	3									
		ha wheat	x3	Wheat	3.15	105	245	3.15									
		ha soybeans	x4	Soybeans	6.75	45	230	3.3									

Profit: (money make from selling what I produced per ha minus how much it costed me to produce in an ha)

Max profit:	<b>38976</b>																
					x1	x2	x3	x4									
					80	65	85.75	73.75									
any ha of each crop to plant to maximize profit:					219.2	0	250	0	the amount of resource spent	the amount of resource available	the amount of resource unspent						

subject to these constraints:					x1	x2	x3	x4	LHS	RHS							
total area	area 1				1	1	1	1	469.230769	<=	1500			1031			
area restriction for species x1	area 2				1				219.230769	<=	250			31			
area restriction for species x2	area 3					1			0	<=	250			250			
area restriction for species x3	area 4						1		250	<=	250			0	resource fully used	binding constraint	
area restriction for species x4	area 5							1	0	<=	250			250			
hours of family labour	labour				3.25	3	3.15	3.3	1500	<=	1500			0	resource fully used	binding constraint	
cost per ha	cost				250	200	245	230	116057.692	<=	120000			3942			





## Question 2.

Test - 25 May 2016

A farmer has the following resource endowments: 500 acres of land, 1500 hours of family labour and 120000 € of capital investment. She can use these resources to grow the following crops: corn, sorghum, wheat and soybeans. Any crop should not occupy more than 250 acres. Assume that the farmer works to maximize profit (revenue minus costs) from the production of these crops. She expects the following in terms of prices, crop yields, costs and labor requirements.

Crop	Price (€/bushel)	Yield (bushel/acre)	Cost (€/acre)	Labor Requirement (hours/acre)
Corn	2.75	120	250	3.25
Sorghum	2.65	100	200	3.00
Wheat	3.15	105	245	3.15
Soybeans	6.75	45	230	3.30

1. Formulate the problem as a linear program.
2. Solve the problem using the Excel Solver. What is the optimal solution to the problem?

Once applied solver we find out the values of the decision variables ( $x_1, x_2, x_3, x_4$ ) that maximize the profit of the farmer and the corresponding profit.

The optimal solution tells the farmer that to maximize profit she should plant:

**219.2** ha of  $x_1$  (corn)    **0** ha of  $x_2$  (sorghum)    **250** ha of  $x_3$  (wheat)    **0** ha of  $x_4$  (soybeans)

$(x_1, x_2, x_3, x_4) = (219.2, 0, 250, 0)$  and that this would allow a profit of **38 976 €**



## Question 2.

---

Test - 25 May 2016

A farmer has the following resource endowments: 500 acres of land, 1500 hours of family labour and 120000 € of capital investment. She can use these resources to grow the following crops: corn, sorghum, wheat and soybeans. Any crop should not occupy more than 250 acres. Assume that the farmer works to maximize profit (revenue minus costs) from the production of these crops. She expects the following in terms of prices, crop yields, costs and labor requirements.

Crop	Price (€/bushel)	Yield (bushel/acre)	Cost (€/acre)	Labor Requirement (hours/acre)
Corn	2.75	120	250	3.25
Sorghum	2.65	100	200	3.00
Wheat	3.15	105	245	3.15
Soybeans	6.75	45	230	3.30

1. Formulate the problem as a linear program.
2. Solve the problem using the Excel Solver. What is the optimal solution to the problem?
3. Which constraints are binding? How do you interpret each binding constraint?

# The Answer Report

A	B	C	D	E	F	G
1	Microsoft Excel 16.0 Answer Report					
2	Worksheet: [Book2]Sheet1					
3	Report Created: 25/02/2025 12:17:53					
4	Result: Solver found a solution. All Constraints and optimality conditions are satisfied.					
5	<b>Solver Engine</b>					
6	Engine: Simplex LP					
7	Solution Time: 0.063 Seconds.					
8	Iterations: 2 Subproblems: 0					
9	<b>Solver Options</b>					
10	Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling					
11	Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative					
12						
13						
14	Objective Cell (Max)					
15	<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
16	\$E\$11	Max profit:	0	38975.96154		
17						
18						
19	Variable Cells					
20	<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>	<b>Integer</b>	
21	\$H\$13	how many ha of each crop to plant to maximize profit: x1	0	219.2307692	Contin	
22	\$I\$13	how many ha of each crop to plant to maximize profit: x2	0	0	Contin	
23	\$J\$13	how many ha of each crop to plant to maximize profit: x3	0	250	Contin	
24	\$K\$13	how many ha of each crop to plant to maximize profit: x4	0	0	Contin	
25						
26						
27	Constraints					
28	<b>Cell</b>	<b>Name</b>	<b>Cell Value</b>	<b>Formula</b>	<b>Status</b>	<b>Slack</b>
29	\$L\$16	area 1 LHS	469.2307692	\$L\$16<=\$N\$16	Not Binding	1030.76923
30	\$L\$17	area 2 LHS	219.2307692	\$L\$17<=\$N\$17	Not Binding	30.7692308
31	\$L\$18	area 3 LHS	0	\$L\$18<=\$N\$18	Not Binding	250
32	\$L\$19	area 4 LHS	250	\$L\$19<=\$N\$19	Binding	0
33	\$L\$20	area 5 LHS	0	\$L\$20<=\$N\$20	Not Binding	250
34	\$L\$21	labour LHS	1500	\$L\$21<=\$N\$21	Binding	0
35	\$L\$22	cost LHS	116057.6923	\$L\$22<=\$N\$22	Not Binding	3942.30769

The initial, optimal solutions (x1, x2, x3, x4), amount of (un)spent resources can be read across tables

The **Objective Cell** table tells us the starting value of the objective function (Z) when *Solver* was applied and the optimal value after *Solver*

The **Variables Cells** table shows the values of the decision variables (x1, x2, x3, x4) for the initial solution and the optimal solution

The **Constraints** table provides information regarding the restrictions applied to each of the decision variables and resources (*Formula*), providing indication on which are the limiting resources - the binding constraints - that will have a **positive shadow price** (the table does not the shadow price value)

# The Answer Report

Microsoft Excel 16.0 Answer Report  
 Worksheet: [Book2]Sheet1  
 Report Created: 25/02/2025 12:17:53  
 Result: Solver found a solution. All Constraints and optimality conditions are satisfied.  
 Solver Engine  
 Engine: Simplex LP  
 Solution Time: 0.063 Seconds.  
 Iterations: 2 Subproblems: 0  
 Solver Options  
 Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling  
 Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
\$E\$11	Max profit:	0	38975.96154

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$H\$13	how many ha of each crop to plant to maximize profit: x1	0	219.2307692	Contin
\$I\$13	how many ha of each crop to plant to maximize profit: x2	0	0	Contin
\$J\$13	how many ha of each crop to plant to maximize profit: x3	0	250	Contin
\$K\$13	how many ha of each crop to plant to maximize profit: x4	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$L\$16	area 1 LHS	469.2307692	\$L\$16<=\$N\$16	Not Binding	1030.76923
\$L\$17	area 2 LHS	219.2307692	\$L\$17<=\$N\$17	Not Binding	30.7692308
\$L\$18	area 3 LHS	0	\$L\$18<=\$N\$18	Not Binding	250
\$L\$19	area 4 LHS	250	\$L\$19<=\$N\$19	Binding	0
\$L\$20	area 5 LHS	0	\$L\$20<=\$N\$20	Not Binding	250
\$L\$21	labour LHS	1500	\$L\$21<=\$N\$21	Binding	0
\$L\$22	cost LHS	116057.6923	\$L\$22<=\$N\$22	Not Binding	3942.30769

The initial, optimal solutions (Z, x1, x2, x3, x4), amount of (un)spent resources can be read across tables

A **binding constraint** is one whose alteration causes a subsequent change in the optimal solution.

*This is actually great for business managers and programmers alike because once they obtain their optimal solution, they can always tweak it using the constraint to achieve the desired goal!*

Changing the value of a **non-binding constraint** effect on the optimal solution obtained.

*Non-binding constraints have shadow price of zero*

*Total amount of  
resources  
originally available*

1500  
250  
250  
250  
250  
1500  
120000

*spent + unspent =*



## Question 2.

Test - 25 May 2016

A farmer has the following resource endowments: 500 acres of land, 1500 hours of family labour and 120000 € of capital investment. She can use these resources to grow the following crops: corn, sorghum, wheat and soybeans. Any crop should not occupy more than 250 acres. Assume that the farmer works to maximize profit (revenue minus costs) from the production of these crops. She expects the following in terms of prices, crop yields, costs and labor requirements.

Crop	Price (€/bushel)	Yield (bushel/acre)	Cost (€/acre)	Labor Requirement (hours/acre)
Corn	2.75	120	250	3.25
Sorghum	2.65	100	200	3.00
Wheat	3.15	105	245	3.15
Soybeans	6.75	45	230	3.30

1. Formulate the problem as a linear program.
2. Solve the problem using the Excel Solver. What is the optimal solution to the problem?
3. Which constraints are binding? How do you interpret each binding constraint?

The binding constraints are those representing the resources that are fully used (**area to plant wheat** and **labour time**) and indicate that any change to these constraints will lead to a change in the optimal solution

$$x_3 \leq 250$$

$$3.25x_1 + 3.00x_2 + 3.15x_3 + 3.30x_4 \leq 1500$$



## Question 2.

Test - 25 May 2016

A farmer has the following resource endowments: 500 acres of land, 1500 hours of family labour and 120000 € of capital investment. She can use these resources to grow the following crops: corn, sorghum, wheat and soybeans. Any crop should not occupy more than 250 acres. Assume that the farmer works to maximize profit (revenue minus costs) from the production of these crops. She expects the following in terms of prices, crop yields, costs and labor requirements.

Crop	Price (€/bushel)	Yield (bushel/acre)	Cost (€/acre)	Labor Requirement (hours/acre)
Corn	2.75	120	250	3.25
Sorghum	2.65	100	200	3.00
Wheat	3.15	105	245	3.15
Soybeans	6.75	45	230	3.30

1. Formulate the problem as a linear program.
2. Solve the problem using the Excel Solver. What is the optimal solution to the problem?
3. Which constraints are binding? How do you interpret each binding constraint?
4. By how much should sorghum's current profit per acre increase in order for the optimal solution to change? Sorghum's current profit per acre decrease would change the optimal solution?

Now we need to go into the output reports generated by solver:

- Answer report
- Sensitivity report

# Sensitivity Report

Allows answering to questions such as:

1. If the objective function changes, how does the solution change?
2. If resources available change, how does the solution change?

Max profit:	38976						
how many ha of each crop to plant to maximize profit:	x1	x2	x3	x4	the amount of resource spent	the amount of resource available	
	80	65	85.75	73.75			
	219.2	0	250	0			
subject to these constraints:	x1	x2	x3	x4	LHS	RHS	
total area	area 1	1	1	1	1	469.230769	<= 1500
area restriction for species x1	area 2	1				219.230769	<= 250
area restriction for species x2	area 3		1			0	<= 250
area restriction for species x3	area 4			1		250	<= 250
area restriction for species x4	area 5				1	0	<= 250
hours of family labour	labour	3.25	3	3.15	3.3	1500	<= 1500
cost per ha	cost	250	200	245	230	116057.692	<= 120000

x1	x2	x3	x4
80	65	85.75	73.75
219.2	0	250	0

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$H\$13	how many ha of each crop to plant to maximize profit: x1	219.2307692	0	80	8.472222222	7.367424242
\$I\$13	how many ha of each crop to plant to maximize profit: x2	0	-8.846153846	65	8.846153846	1E+30
\$J\$13	how many ha of each crop to plant to maximize profit: x3	250	0	85.75	1E+30	8.211538462
\$K\$13	how many ha of each crop to plant to maximize profit: x4	0	-7.480769231	73.75	7.480769231	1E+30

Resources spent

Total amount of resources originally available

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$L\$16	area 1 LHS	469.2307692	0	1500	1E+30	1030.769231
\$L\$17	area 2 LHS	219.2307692	0	250	1E+30	30.76923077
\$L\$18	area 3 LHS	0	0	250	1E+30	250
\$L\$19	area 4 LHS	250	8.211538462	250	226.1904762	31.74603175
\$L\$20	area 5 LHS	0	0	250	1E+30	250
\$L\$21	labour LHS	1500	24.61538462	1500	51.25	712.5
\$L\$22	cost LHS	116057.6923	0	120000	1E+30	3942.307692

the amount of resource available
RHS
1500
250
250
250
250
250
1500
120000

# Sensitivity Report

Allows answering to questions such as:

1. If the objective function changes, how does the solution change?

$$\text{Maximize } Z = 80x_1 + 65x_2 + 85.75x_3 + 73.75x_4$$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$H\$13	how many ha of each crop to plant to maximize profit: x1	219.2307692	0	80	8.472222222	7.367424242
\$I\$13	how many ha of each crop to plant to maximize profit: x2	0	-8.846153846	65	8.846153846	1E+30
\$J\$13	how many ha of each crop to plant to maximize profit: x3	250	0	85.75	1E+30	8.211538462
\$K\$13	how many ha of each crop to plant to maximize profit: x4	0	-7.480769231	73.75	7.480769231	1E+30

First, let us analyze the **Variable Cells** part of the table:

- The Final Value = **Optimal Solution**, thus replacing the final values ( $x_1, x_2, x_3, x_4$ ) = **(219.23, 0, 250, 0)** in the objective function leads to:

$$\text{MAX } Z = 80 * 219.2 + 65 * 0 + 85.75 * 250 + 73.75 * 0 = 38\,976 \text{ €}$$

This tells the farmer that to maximize profit she should plant:

- 219.2 ha of x1 (corn)
- 0 ha of x2 (sorghum)
- 250 ha of x3 (wheat)
- 0 ha of x4 (soybeans)



# Sensitivity Report

Allows answering to questions such as:

1. If the objective function changes, how does the solution change?

$$\text{Maximize } Z = 80x_1 + 65x_2 + 85.75x_3 + 73.75x_4$$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease	interval	
\$H\$13	how many ha of each crop to plant to maximize profit: x1	219.2307692	0	80	8.472222222	7.367424242	72.6	88.5
\$I\$13	how many ha of each crop to plant to maximize profit: x2	0	-8.846153846	65	8.846153846	1E+30	INFINITY	73.8
\$J\$13	how many ha of each crop to plant to maximize profit: x3	250	0	85.75	1E+30	8.211538462	77.5	INFINITY
\$K\$13	how many ha of each crop to plant to maximize profit: x4	0	-7.480769231	73.75	7.480769231	1E+30	INFINITY	81.2

First, let us analyze the **Variable Cells** part of the table:

- The Final Value = **Optimal Solution**, thus replacing the final values  $(x_1, x_2, x_3, x_4) = (219.23, 0, 250, 0)$  in the objective function leads to:

$$\text{MAX } Z = 80 * 219.2 + 65 * 0 + 85.75 * 250 + 73.75 * 0 = 38\,976 \text{ €}$$

- The **allowable increase and decrease** show how much the **coeff. of the objective function** can change before the **optimal solution** has to be altered

If the **coeff of X1** increases up to **88.5** or drops down to **72.6** the optimal solution will hold =  $(219.23, 0, 250, 0)$

$$\text{MAX } Z = 88.5 * 219.2 + 65 * 0 + 85.75 * 250 + 73.75 * 0 = 40833.3 \text{ €}$$

$$\text{MAX } Z = 72.6 * 219.2 + 65 * 0 + 85.75 * 250 + 73.75 * 0 = 22827.7 \text{ €}$$

# Sensitivity Report

Allows answering to questions such as:

1. If the objective function changes, how does the solution change?

$$\text{Maximize } Z = 80x_1 + 65x_2 + 85.75x_3 + 73.75x_4$$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$H\$13	how many ha of each crop to plant to maximize profit: x1	219.2307692	0	80	8.472222222	7.367424242
\$I\$13	how many ha of each crop to plant to maximize profit: x2	0	-8.846153846	65	8.846153846	1E+30
\$J\$13	how many ha of each crop to plant to maximize profit: x3	250	0	85.75	1E+30	8.211538462
\$K\$13	how many ha of each crop to plant to maximize profit: x4	0	-7.480769231	73.75	7.480769231	1E+30

First, let us analyze the **Variable Cells** part of the table:

- The Final Value = **Optimal Solution**, thus replacing the final values ( $x_1, x_2, x_3, x_4$ ) = (219.23, 0, 250, 0) in the objective function leads to:

$$\text{MAX } Z = 80 * 219.2 + 65 * 0 + 85.75 * 250 + 73.75 * 0 = 38\,976 \text{ €}$$

The **Reduced Cost** measures the change in the objective function's value per unit increase in the decision variable's value.

The optimal solution tells the farmer:

"If you want to maximize profit **DO NOT PLANT sorghum and soybeans**"

**What would happen if she planted 1 ha of sorghum (x2)?**

**The profit would reduce to 38 967 € (38 976 - 8.85)**

**What would happen if she planted 1 ha of soybeans (x4)?**

**The profit would reduce to 38 968.5 € (38 976 - 7.48)**



## Question 2.

4. By how much should sorghum's current profit per acre increase in order for the optimal solution to change? Sorghum's current profit per acre decrease would change the optimal solution?

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease		
\$H\$13	how many ha of each crop to plant to maximize profit: x1	219.2307692	0	80	8.472222222	7.367424242	72.6	88.5
\$I\$13	how many ha of each crop to plant to maximize profit: x2	0	-8.846153846	65	8.846153846	1E+30	INFINITY	73.8

Sorghum profit  $\Leftrightarrow$  coeficiente of x2 (set to 65)

How much can it increase to change the optimal solution?

If it increases up to the current value + the the allowable increase (73.8) the optimal solution holds

if the new value  $> 65 + 8.846$  (the allowable increase) we'd be outsider the range (73.8) and the optimal solution would NOT hold

The profit can drop to zero that the solution will hold



## Question 2.

---

Test - 25 May 2016

A farmer has the following resource endowments: 500 acres of land, 1500 hours of family labour and 120000 € of capital investment. She can use these resources to grow the following crops: corn, sorghum, wheat and soybeans. Any crop should not occupy more than 250 acres. Assume that the farmer works to maximize profit (revenue minus costs) from the production of these crops. She expects the following in terms of prices, crop yields, costs and labor requirements.

Crop	Price (€/bushel)	Yield (bushel/acre)	Cost (€/acre)	Labor Requirement (hours/acre)
Corn	2.75	120	250	3.25
Sorghum	2.65	100	200	3.00
Wheat	3.15	105	245	3.15
Soybeans	6.75	45	230	3.30

1. Formulate the problem as a linear program.
2. Solve the problem using the Excel Solver. What is the optimal solution to the problem?
3. Which constraints are binding? How do you interpret each binding constraint?
4. By how much should sorghum's current profit per acre increase in order for the optimal solution to change? Sorghum's current profit per acre decrease would change the optimal solution?
5. What is the shadow price for each constraint? How do you interpret this value?

# Sensitivity Report

Allows answering to questions such as:

1. If the objective function changes, how does the solution change?
2. **If resources available change, how does the solution change?**

1	Microsoft Excel 16.0 Sensitivity Report
2	Worksheet: [Book2]Sheet1
3	Report Created: 25/02/2025 12:17:53
4	
5	
6	

Constraints		Resources spent	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	interval
\$L\$16	area 1 LHS	469.2307692	0	1500	1E+30	1030.769231	469.2 INFINITY
\$L\$17	area 2 LHS	219.2307692	0	250	1E+30	30.76923077	219.2 INFINITY
\$L\$18	area 3 LHS	0	0	250	1E+30	250	0.0 INFINITY
\$L\$19	area 4 LHS	250	8.211538462	250	226.1904762	31.74603175	<b>218.3</b> <b>476.2</b>
\$L\$20	area 5 LHS	0	0	250	1E+30	250	0.0 INFINITY
\$L\$21	labour LHS	1500	24.61538462	1500	51.25	712.5	<b>787.5</b> <b>1551.3</b>
\$L\$22	cost LHS	116057.6923	0	120000	1E+30	3942.307692	116057.7 INFINITY

The **Shadow Price** measures the change in the objective function's value per unit increase in the constraint's bound.

Resources fully spent lead to **shadow price > zero**

Changes to the RHS of the constraints (*Total amount of resources originally available*):

- if **within** the intervals set by the allowable decrease and increase **will not change the optimal solution** (but will affect the objective function value!)
- If **outside** the intervals set by the allowable decrease and increase **will change the optimal solution and the objective function value**

# Sensitivity Report

Allows answering to questions such as:

1. If the objective function changes, how does the solution change?
2. **If resources available change, how does the solution change?**

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	interval
\$L\$16 area 1 LHS		469.2307692	0	1500	1E+30	1030.769231	469.2 INFINITY
\$L\$17 area 2 LHS		219.2307692	0	250	1E+30	30.76923077	219.2 INFINITY
\$L\$18 area 3 LHS		0	0	250	1E+30	250	0.0 INFINITY
\$L\$19 area 4 LHS		250	8.211538462	250	226.1904762	31.74603175	<b>218.3</b> <b>476.2</b>
\$L\$20 area 5 LHS		0	0	250	1E+30	250	0.0 INFINITY
\$L\$21 labour LHS		1500	24.61538462	1500	51.25	712.5	<b>787.5</b> <b>1551.3</b>
\$L\$22 cost LHS		116057.6923	0	120000	1E+30	3942.307692	116057.7 INFINITY

Constraints with a **shadow price = zero** are **not worth changing** because it will not have any impact on the objective function value

Changes on the RHS of constraints **within** the **interval** with a **shadow price > zero** will have impact on the objective function value. How much?

If the **area available to plant wheat (constraint 4) increases** in:

- 1 unit : 250 to 251 =>  $Z_{\text{new}} = 38\,976 \text{ €} + 1 * 8.21 = 38\,984$
- 2 units: 250 to 252 =>  $Z_{\text{new}} = 38\,976 \text{ €} + 2 * 8.21 = 38\,992$
- 226** units: 250 to **476** =>  $Z_{\text{new}} = 38\,976 \text{ €} + 226 * 8.21 = 40\,832$

The same exercise can be achieved considering decreases in the RHS of constraint 4 of 31 units

# Sensitivity Report

Allows answering to questions such as:

1. If the objective function changes, how does the solution change?
2. **If resources available change, how does the solution change?**

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	interval
\$L\$16 area 1 LHS		469.2307692	0	1500	1E+30	1030.769231	469.2 INFINITY
\$L\$17 area 2 LHS		219.2307692	0	250	1E+30	30.76923077	219.2 INFINITY
\$L\$18 area 3 LHS		0	0	250	1E+30	250	0.0 INFINITY
\$L\$19 area 4 LHS		250	8.211538462	250	226.1904762	31.74603175	<b>218.3</b> <b>476.2</b>
\$L\$20 area 5 LHS		0	0	250	1E+30	250	0.0 INFINITY
\$L\$21 labour LHS		1500	24.61538462	1500	51.25	712.5	<b>787.5</b> <b>1551.3</b>
\$L\$22 cost LHS		116057.6923	0	120000	1E+30	3942.307692	116057.7 INFINITY

Constraints with a **shadow price = zero** are **not worth changing** because it will not have any impact on the objective function value

Changes on the RHS of constraints **within** the **interval** with a **shadow price > zero** will have impact on the objective function value. How much?

If the **family labour time available for agriculture (constraint 6)** decreases in:

1 unit : 1500 to 1499 =>  $Z_{\text{new}} = 38\,976 \text{ €} - 1 * 24.62 = 38\,951$

2 units: 1500 to 1498 =>  $Z_{\text{new}} = 38\,976 \text{ €} - 2 * 24.62 = 38\,927$

**712.5** units: 1500 to **1449** =>  $Z_{\text{new}} = 38\,976 \text{ €} - 712.5 * 24.62 = 21\,438$

The same exercise can be achieved considering increases in the RHS of constraint 6 of 51 units

# Sensitivity Report

Allows answering to questions such as:

1. If the objective function changes, how does the solution change?
2. **If resources available change, how does the solution change?**

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	interval
\$L\$16 area 1 LHS		469.2307692	0	1500	1E+30	1030.769231	469.2 INFINITY
\$L\$17 area 2 LHS		219.2307692	0	250	1E+30	30.76923077	219.2 INFINITY
\$L\$18 area 3 LHS		0	0	250	1E+30	250	0.0 INFINITY
\$L\$19 area 4 LHS		250	8.211538462	250	226.1904762	31.74603175	<b>218.3</b> <b>476.2</b>
\$L\$20 area 5 LHS		0	0	250	1E+30	250	0.0 INFINITY
\$L\$21 labour LHS		1500	24.61538462	1500	51.25	712.5	<b>787.5</b> <b>1551.3</b>
\$L\$22 cost LHS		116057.6923	0	120000	1E+30	3942.307692	116057.7 INFINITY

Constraints with a **shadow price = zero** are **not worth changing** because it will not have any impact on the objective function value

The impact of changes on the RHS of constraints that fall **outside** the **interval** can only be determined by solving the problem again!





## Question 2.

Test - 25 May 2016

A farmer has the following resource endowments: 500 acres of land, 1500 hours of family labour and 120000 € of capital investment. She can use these resources to grow the following crops: corn, sorghum, wheat and soybeans. Any crop should not occupy more than 250 acres. Assume that the farmer works to maximize profit (revenue minus costs) from the production of these crops. She expects the following in terms of prices, crop yields, costs and labor requirements.

Crop	Price (€/bushel)	Yield (bushel/acre)	Cost (€/acre)	Labor Requirement (hours/acre)
Corn	2.75	120	250	3.25
Sorghum	2.65	100	200	3.00
Wheat	3.15	105	245	3.15
Soybeans	6.75	45	230	3.30

1. Formulate the problem as a linear program.
2. Solve the problem using the Excel Solver. What is the optimal solution to the problem?
3. Which constraints are binding? How do you interpret each binding constraint?
4. By how much should sorghum's current profit per acre increase in order for the optimal solution to change? Sorghum's current profit per acre decrease would change the optimal solution?
5. What is the shadow price for each constraint? How do you interpret this value?

The Shadow Price is the impact that one unitary change in the RHS of any given constraint will have in the objective function value. This is only true as long as changes on the RHS of constraints remain within the interval determined by the allowable increases and decreases



## Question 2.

Test - 25 May 2016

A farmer has the following resource endowments: 500 acres of land, 1500 hours of family labour and 120000 € of capital investment. She can use these resources to grow the following crops: corn, sorghum, wheat and soybeans. Any crop should not occupy more than 250 acres. Assume that the farmer works to maximize profit (revenue minus costs) from the production of these crops. She expects the following in terms of prices, crop yields, costs and labor requirements.

Crop	Price (€/bushel)	Yield (bushel/acre)	Cost (€/acre)	Labor Requirement (hours/acre)
Corn	2.75	120	250	3.25
Sorghum	2.65	100	200	3.00
Wheat	3.15	105	245	3.15
Soybeans	6.75	45	230	3.30

1. Formulate the problem as a linear program.
2. Solve the problem using the Excel Solver. What is the optimal solution to the problem?
3. Which constraints are binding? How do you interpret each binding constraint?
4. By how much should sorghum's current profit per acre increase in order for the optimal solution to change? Sorghum's current profit per acre decrease would change the optimal solution?
5. What is the shadow price for each constraint? How do you interpret this value?
6. What is the range of feasibility for the maximum number of hours of family labour with respect to the shadow price? How do you interpret this range?



## Question 2.

6. What is the range of feasibility for the maximum number of hours of family labour with respect to the shadow price? How do you interpret this range?

The range of feasibility with respect to the maximum number of hours sets the upper limit at 1551.3 considering increases in labour time that can reach a maximum of 51.25 hours with a immediate consequence in the objective function value. Thus, an increase of **51.25** : from 1500 to **1551.25** =>

$$Z_{\text{new}} = 38\,976 \text{ €} + 51.25 * 24.62 = 40\,238 \text{ €} \text{ (profit increases, original values } 38\,976 \text{ €)}$$

26	\$L\$20 area 5 LHS	0	0	250	1E+30	250	0.0	INFINITY
27	\$L\$21 labour LHS	1500	24.61538462	1500	51.25	712.5	<b>787.5</b>	<b>1551.3</b>
28	\$L\$22 cost LHS	116057.6923	0	120000	1E+30	3942.307692	116057.7	INFINITY

However, an increase of 52 hours would already fall outside the range of feasibility making impossible to use the shadow price to determine the impact on the objective function and forcing the problem to be solved again.



## Question 2.

Test - 25 May 2016

A farmer has the following resource endowments: 500 acres of land, 1500 hours of family labour and 120000 € of capital investment. She can use these resources to grow the following crops: corn, sorghum, wheat and soybeans. Any crop should not occupy more than 250 acres. Assume that the farmer works to maximize profit (revenue minus costs) from the production of these crops. She expects the following in terms of prices, crop yields, costs and labor requirements.

Crop	Price (€/bushel)	Yield (bushel/acre)	Cost (€/acre)	Labor Requirement (hours/acre)
Corn	2.75	120	250	3.25
Sorghum	2.65	100	200	3.00
Wheat	3.15	105	245	3.15
Soybeans	6.75	45	230	3.30

1. Formulate the problem as a linear program.
2. Solve the problem using the Excel Solver. What is the optimal solution to the problem?
3. Which constraints are binding? How do you interpret each binding constraint?
4. By how much should sorghum's current profit per acre increase in order for the optimal solution to change? Sorghum's current profit per acre decrease would change the optimal solution?
5. What is the shadow price for each constraint? How do you interpret this value?
6. What is the range of feasibility for the maximum number of hours of family labour with respect to the shadow price? How do you interpret this range?
7. How much the profit would change if one acre of soybeans were forced into the current optimal solution?



## Question 2.

7. How much the profit would change if one acre of soybeans were forced into the current optimal solution?

Imposing the plantation of 1 ha of soybeans will have a negative impact of 7.48 € in the final profit value. Thus, the increase of  $x_4$  from 0 to **1 ha**

$$Z_{\text{new}} = 38\,976 \text{ €} - 7.48 = 38\,968.5 \text{ €} \text{ (profit decreases, original values } 38\,976 \text{ €)}$$

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$H\$13	how many ha of each crop to plant to maximize profit: x1	219.2307692	0	80	8.472222222	7.367424242
\$I\$13	how many ha of each crop to plant to maximize profit: x2	0	-8.846153846	65	8.846153846	1E+30
\$J\$13	how many ha of each crop to plant to maximize profit: x3	250	0	85.75	1E+30	8.211538462
\$K\$13	how many ha of each crop to plant to maximize profit: x4	0	-7.480769231	73.75	7.480769231	1E+30



## Question 7.

---

### Exam - 17 June 2022

A certain university maintains a powerful mainframe computer for research use. There are four operators (two graduate students and two undergraduate students) to operate and maintain the computer, as well as to perform some programming services, from Wednesday to Friday (in the other days the computer is to be operated by staff outside the university). They all have different wage rates because of differences in their experience with computers and in their programming ability. Table 1 shows their wage rates, along with the maximum number of hours that each can work each day. Each operator is guaranteed a certain minimum number of hours per week that will maintain an adequate knowledge of the operation. This level is set arbitrarily at 7 hours per week for the graduate students (A and B) and 8 hours per week for the undergraduate students (C and D).

Operators	Wage Rate (€/hour)	Maximum Hours of Availability		
		Wednesday	Thursday	Friday
A	10.00	6	0	6
B	10.10	6	6	0
C	9.90	0	4	4
D	9.80	5	0	5

The computer facility is to be open for operation from 8 A.M. to 6 P.M. every day. The director of the computer facility wishes to determine the **number of hours she should assign to each operator on each day** in order to minimize cost.

1. Formulate a linear programming model for this problem.
2. Determine a feasible solution.

File Home Insert Page Layout Formulas Data Review View Developer Tell me what you want to do... Share

Get External Data: From Access, From Web, From Text, From Other Sources, Existing Connections

Get & Transform: Show Queries, From Table, Recent Sources, New Query

Connections: Refresh All, Edit Links, Connections, Properties

Sort & Filter: Sort, Filter, Clear, Reapply, Advanced

Data Tools: Flash Fill, Remove Duplicates, Data Validation, Text to Columns, Consolidate, Relationships, Manage Data Model

Forecast: What-If Analysis, Forecast Sheet

Outline: Group, Ungroup, Subtotal

Analysis: Solver, Data Analysis

=SUMPRODUCT(\$F\$13:\$M\$13,F15:M15)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
7			x5	nr working hours for worker C on Thursday												
8			x6	nr working hours for worker C on Friday												
9			x7	nr working hours for worker D on Wednesday												
10			x8	nr working hours for worker D on Friday												
11																
12																
13						10	10	10.1	10.1	9.9	9.9	9.8	9.8			
14			Min C			x1	x2	x3	x4	x5	x6	x7	x8			
15			0			0	0	0	0	0	0	0	0			
16																
17																
18				day	W	F	W	T	T	F	W	F				
19				worker	A	A	B	B	C	C	D	D				
20				nr hours	x1	x2	x3	x4	x5	x6	x7	x8	LHS	RHS		
21			total hours required on Wednesday		1		1				1		0	=	10	
22			total hours required on Thursday					1	1				0	=	10	
23			total hours required on Friday			1				1	1		0	=	10	
24			minimum hours required for worker A		1	1							0	>=	7	
25			minimum hours required for worker B				1	1					0	>=	7	
26			minimum hours required for worker C						1	1			0	>=	8	
27			minimum hours required for worker D								1	1	0	>=	8	
28			hours availability by worker A on Wednesday		1								0	<=	6	
29			hours availability by worker A on Friday			1							0	<=	6	
30			hours availability by worker B on Wednesday				1						0	<=	6	
31			hours availability by worker B on Thursday					1					0	<=	6	
32			hours availability by worker C on Thursday						1				0	<=	4	
33			hours availability by worker C on Friday							1			0	<=	4	
34			hours availability by worker D on Wednesday								1		0	<=	5	
35			hours availability by worker D on Friday									1	0	<=	5	

### Solver Parameters

Set Objective:

To:  Max  Min  Value Of:

By Changing Variable Cells:

Subject to the Constraints:

Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method  
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

File Home Insert Page Layout Formulas Data Review View Developer Tell me what you want to do...

Get External Data: From Access, From Web, From Text, From Other Sources, Existing Connections, New Query, Recent Sources

Get & Transform: Show Queries, From Table

Connections: Refresh All, Edit Links

Sort & Filter: Sort, Filter, Clear, Reapply, Advanced

Data Tools: Flash Fill, Remove Duplicates, Data Validation, Text to Columns, Consolidate, Relationships, Manage Data Model

Forecast: What-If Analysis, Forecast Sheet

Outline: Group, Ungroup, Subtotal

Analysis: Solver, Data Analysis

D15 =SUMPRODUCT(\$F\$13:\$M\$13,F15:M15)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y			
7			x5	nr working hours for worker C on Thursday																								
8			x6	nr working hours for worker C on Friday																								
9			x7	nr working hours for worker D on Wednesday																								
10			x8	nr working hours for worker D on Friday																								
11																												
12																												
13						10	10	10.1	10.1	9.9	9.9	9.8	9.8															
14			Min C			x1	x2	x3	x4	x5	x6	x7	x8															
15			0			0	0	0	0	0	0	0	0															
16																												
17																												
18					day	W	F	W	T	T	F	W	F															
19					worker	A	A	B	B	C	C	D	D															
20					nr hours	x1	x2	x3	x4	x5	x6	x7	x8	LHS	RHS													
21			total hours required on Wednesday			1		1				1		0	=	10												
22			total hours required on Thursday						1	1				0	=	10												
23			total hours required on Friday				1				1	1		0	=	10												
24			minimum hours required for worker A			1	1							0	>=	7												
25			minimum hours required for worker B					1	1					0	>=	7												
26			minimum hours required for worker C							1	1			0	>=	8												
27			minimum hours required for worker D									1	1	0	>=	8												
28			hours availability by worker A on Wednesday			1								0	<=	6												
29			hours availability by worker A on Friday				1							0	<=	6												
30			hours availability by worker B on Wednesday					1						0	<=	6												
31			hours availability by worker B on Thursday						1					0	<=	6												
32			hours availability by worker C on Thursday							1				0	<=	4												
33			hours availability by worker C on Friday								1			0	<=	4												
34			hours availability by worker D on Wednesday									1		0	<=	5												
35			hours availability by worker D on Friday										1	0	<=	5												

**Solver Parameters**

Set Objective:

To:  Max  Min  Value Of:

By Changing Variable Cells:

Subject to the Constraints:

Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method: Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.



File Home Insert Page Layout Formulas Data Review View Developer Tell me what you want to do...

From Access From Web From Text From Other Sources Existing Connections New Query From Table Recent Sources Show Queries From Table Refresh All Edit Links Connections Sort Filter Reapply Advanced Flash Fill Remove Duplicates Data Validation Consolidate Relationships Manage Data Model What-If Analysis Forecast Sheet Group Ungroup Subtotal Solver Data Analysis

D15 =SUMPRODUCT(\$F\$13:\$M\$13,F15:M15)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y			
7			x5	nr working hours for worker C on Thursday																								
8			x6	nr working hours for worker C on Friday																								
9			x7	nr working hours for worker D on Wednesday																								
10			x8	nr working hours for worker D on Friday																								
11																												
12																												
13						10	10	10.1	10.1	9.9	9.9	9.8	9.8															
14			Min C			x1	x2	x3	x4	x5	x6	x7	x8															
15			0			0	0	0	0	0	0	0	0															
16																												
17																												
18					day	W	F	W	T	T	F	W	F															
19					worker	A	A	B	B	C	C	D	D															
20					nr hours	x1	x2	x3	x4	x5	x6	x7	x8	LHS	RHS													
21			total hours required on Wednesday			1		1				1		0	=	10												
22			total hours required on Thursday						1	1				0	=	10												
23			total hours required on Friday				1				1	1		0	=	10												
24			minimum hours required for worker A			1	1							0	>=	7												
25			minimum hours required for worker B					1	1					0	>=	7												
26			minimum hours required for worker C							1	1			0	>=	8												
27			minimum hours required for worker D									1	1	0	>=	8												
28			hours availability by worker A on Wednesday			1								0	<=	6												
29			hours availability by worker A on Friday				1							0	<=	6												
30			hours availability by worker B on Wednesday					1						0	<=	6												
31			hours availability by worker B on Thursday						1					0	<=	6												
32			hours availability by worker C on Thursday							1	1			0	<=	4												
33			hours availability by worker C on Friday								1			0	<=	4												
34			hours availability by worker D on Wednesday									1		0	<=	5												
35			hours availability by worker D on Friday									1		0	<=	5												

**Solver Parameters**

Set Objective:

To:  Max  Min  Value Of:

By Changing Variable Cells:

Subject to the Constraints:

- 
- 

Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method: Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

File Home Insert Page Layout Formulas Data Review View Developer Tell me what you want to do...

Get External Data: From Access, From Web, From Text, From Other Sources, Existing Connections

Get & Transform: Show Queries, From Table, Recent Sources, New Query

Connections: Refresh All, Properties, Edit Links

Sort & Filter: Sort, Filter, Clear, Reapply, Advanced

Data Tools: Flash Fill, Remove Duplicates, Data Validation, Manage Data Model, Consolidate, Relationships

Forecast: What-If Analysis, Forecast Sheet

Outline: Group, Ungroup, Subtotal

Analysis: Solver, Data Analysis

$=SUMPRODUCT(\$F\$13:\$M\$13,F15:M15)$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
7		x5	nr working hours for worker C on Thursday												
8		x6	nr working hours for worker C on Friday												
9		x7	nr working hours for worker D on Wednesday												
10		x8	nr working hours for worker D on Friday												
21			total hours r												
22			total hour												
23			total h												
24			minimum hours												
25			minimum hours												
26			minimum hours												
27			minimum hours												
28			hours availability by w												
29			hours availability												
30			hours availability by w												
31			hours availability by w												
32			hours availability by w												
33			hours availability												
34			hours availability by w												
35			hours availability by worker D on Friday									1	0		

**Solver Results**

Solver found a solution. All Constraints and optimality conditions are satisfied.

Keep Solver Solution  
 Restore Original Values

Return to Solver Parameters Dialog  
 Outline Reports

**Reports**  
 Answer  
 Sensitivity  
 Limits

OK Cancel Save Scenario...

**Solver Parameters**

Set Objective:  $\$D\$15$

To:  Max  Min  Value Of: 0

By Changing Variable Cells:  $\$F\$15:\$M\$15$

Subject to the Constraints:

- $\$N\$21:\$N\$23 = \$P\$21:\$P\$23$
- $\$N\$24:\$N\$27 \geq \$P\$24:\$P\$27$
- $\$N\$28:\$N\$35 \leq \$P\$28:\$P\$35$

Make Unconstrained Variables Non-Negative

Select a Solving Method: **Simplex LP**

Solving Method: Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Help Solve Close

**RHS**

=	10
=	10
=	10
>=	7
>=	7
>=	8
>=	8
<=	6
<=	6
<=	6
<=	6
<=	4
<=	4
<=	5
<=	5

**Solver found a solution. All Constraints and optimality conditions are satisfied.**

When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.

File Home Insert Page Layout Formulas Data Review View Developer Tell me what you want to do...

From Access From Web From Text From Other Sources Existing Connections New Query From Table Recent Sources Show Queries Refresh All Edit Links Connections Properties Edit Links Sort Filter Clear Reapply Advanced Text to Columns Flash Fill Remove Duplicates Data Validation Consolidate Relationships Manage Data Model What-If Analysis Forecast Sheet Group Outline Solver Data Analysis

T26

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y				
4			x3	nr working hours for worker B on Wednesday																									
5			x4	nr working hours for worker B on Thursday																									
6			x5	nr working hours for worker C on Thursday																									
7			x6	nr working hours for worker C on Friday																									
8			x7	nr working hours for worker D on Wednesday																									
9			x8	nr working hours for worker D on Friday																									
10																													
11						10	10	10.1	10.1	9.9	9.9	9.8	9.8																
12			Min C			x1	x2	x3	x4	x5	x6	x7	x8																
13			298.3			6	1	1	6	4	4	3	5																
14																													
15				day		W	F	W	T	T	F	W	F																
16				worker		A	A	B	B	C	C	D	D																
17				nr hours		x1	x2	x3	x4	x5	x6	x7	x8	LHS		RHS													
18			total hours required on Wednesday			1		1				1		10	=	10													
19			total hours required on Thursday						1	1				10	=	10													
20			total hours required on Friday				1				1	1		10	=	10													
21			minimum hours required for worker A			1	1							7	>=	7													
22			minimum hours required for worker B					1	1					7	>=	7													
23			minimum hours required for worker C							1	1			8	>=	8													
24			minimum hours required for worker D									1	1	8	>=	8													
25			hours availability by worker A on Wednesday			1								6	<=	6													
26			hours availability by worker A on Friday				1							1	<=	6													
27			hours availability by worker B on Wednesday					1						1	<=	6													
28			hours availability by worker B on Thursday						1					6	<=	6													
29			hours availability by worker C on Thursday							1				4	<=	4													
30			hours availability by worker C on Friday								1			4	<=	4													
31			hours availability by worker D on Wednesday									1		3	<=	5													
32			hours availability by worker D on Friday										1	5	<=	5													

See the excel file :  
Question7\_SOLVER.xlsx



## Question 8.

---

Exam - 30 June 2022

A company produces three types of forest products, A, B and C. Each ton of product A, B and C generates 50, 40 and 60 units of toxic waste and generate a profit of 10, 5 and 10 euros, respectively. The company has the capacity to produce up to 15 thousand tons of products per month. Commitments already made oblige the company to deliver to a certain customer 5 thousand tons of product A each month. One intends to determine the **monthly production plan** that generates the minimum amount of toxic waste with a monthly profit of at least 100 thousand euros and a monthly production of never less than 80% of the maximum production capacity of the company.

1. Formulate a linear programming model for this problem.
2. Consider that 12 thousand tons of products are produced each month, generating a profit of 100 thousand euros and that the monthly production of product A is 8 thousand tons. Indicate the quantities of products B and C produced and the amount of toxic waste generated.

File Home Insert Page Layout Formulas Data Review View Developer Tell me what you want to do... Share

Clipboard: Paste, Cut, Copy, Format Painter

Font: Calibri, 11, Bold, Italic, Underline, Text Color, Background Color

Alignment: Wrap Text, Merge & Center

Number: General, Percentage, Decimals

Styles: Normal, Bad, Good, Neutral, Calculation, Check Cell

Cells: Insert, Delete, Format

Editing: AutoSum, Fill, Clear, Sort & Filter, Find & Select

17

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
1																								
2		x1	tons of prod A produced per month																					
3		x2	tons of prod A produced per month																					
4		x3	tons of prod A produced per month																					
5																								
6						50	40	60																
7			Min Z			x1	x2	x3																
8			0			0	0	0																
9																								
10																								
11						x1	x2	x3	LHS		RHS													
12			max prod capacity			1	1	1	0	<=	15000													
13			delivery commitments			1			0	>=	5000													
14			min production			1	1	1	0	>=	12000													
15			profit			10	5	10	0	>=	10000													
16																								
17																								
18																								
19																								
20																								
21																								
22																								
23																								
24																								
25																								
26																								
27																								
28																								
29																								

File Home Insert Page Layout Formulas Data Review View Developer Tell me what you want to do... Share

Clipboard: Paste, Cut, Copy, Format Painter

Font: Calibri, 11, Bold, Italic, Underline, Text Color, Background Color

Alignment: Wrap Text, Merge & Center

Number: General, Percentage, Decimals

Styles: Normal, Bad, Good, Neutral, Calculation, Check Cell

Cells: Insert, Delete, Format

Editing: AutoSum, Fill, Clear, Sort & Filter, Find & Select

120

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
1																								
2		x1	tons of prod A produced per month																					
3		x2	tons of prod A produced per month																					
4		x3	tons of prod A produced per month																					
5																								
6						50	40	60																
7			Min Z			x1	x2	x3																
8			530000			5000	7000	0																
9																								
10																								
11						x1	x2	x3	LHS		RHS													
12			max prod capacity			1	1	1	12000 <=		15000													
13			delivery commitments			1			5000 >=		5000													
14			min production			1	1	1	12000 >=		12000													
15			profit			10	5	10	85000 >=		10000													
16																								
17																								
18																								
19																								
20																								
21																								
22																								
23																								
24																								
25																								
26																								
27																								
28																								
29																								

See the excel file :  
Question8\_SOLVER.xlsx



## Question 9.

---

Test - 29 March 2023

A company transports a certain product from the warehouses A and B to the sales centers P and Q. The amounts of product in A and B are 35 and 50 tons, respectively. The demands for P and Q are 40 and 30 tons, respectively. The cost, in euros, of transporting 1 ton of product from each warehouse to each sale center is indicated in the following table.

	P	Q
A	12	20
B	15	17

The company wants to establish a supply plan that meets the minimum cost. Formulate the problem in linear programming.

File Home Insert Page Layout Formulas Data Review View Developer Tell me what you want to do... Share

Clipboard: Cut, Copy, Paste, Format Painter

Font: Calibri, 11, Bold, Italic, Underline, Text Color, Background Color

Alignment: Wrap Text, Merge & Center

Number: General, Percentage, Decimals

Styles: Normal, Bad, Good, Neutral, Calculation, Check Cell

Cells: Insert, Delete, Format

Editing: AutoSum, Fill, Clear, Sort & Filter, Find & Select

G10

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	
1												amount of												
2										sales		prod												
3										P	Q	stored												
4	xAP	nr of tons of prod transported from warehouse A to sales P						warehouse	A	12	20	35												
5	xAQ	nr of tons of prod transported from warehouse A to sales Q							B	15	17	50												
6	xBP	nr of tons of prod transported from warehouse B to sales P						demand for product at		40	30													
7	xBQ	nr of tons of prod transported from warehouse B to sales Q																						
8									12	20	15	17												
9									xAP	xAQ	xBP	xBQ												
10						Min C			35	0	5	30												
11						1005																		
12																								
13									xAP	xAQ	xBP	xBQ	LHS		RHS									
14								stock constraints at warehouse A	1	1			35	<=	35	0	<i>binding constraint</i>							
15								stock constraints at warehouse B			1	1	35	<=	50	15								
16								demand constraints of shop P	1		1		40	=	40	0	<i>binding constraint</i>							
17								demand constraints of shop Q		1		1	30	=	30	0	<i>binding constraint</i>							
18																								
19																								
20								<b>optimal solution</b>	All the 35 tons of product at warehouse A are transported to sales P															
21									no product is transported from warehouse A to sales Q															
22									5 tons of prod are transported from warehouse B to sales P															
23									35 tons of prod are transported from warehouse B to sales Q															
24									resulting in a minimum cost of 1005															
25									<a href="#">(xAP, xAQ, xBP, xBQ) (35, 0, 5, 30)</a>															
26																								
27																								
28																								
29																								
30																								

Positive Shadow Price: The objective function value will increase

Negative Shadow Price: The objective function value will decrease

shadow price

if one additional ton of product is transported, it will have a cost of 1005-3=1002

if I increase the demand in shop P in 1 ton (from 30 to 31) since no product is transported from warehouse A to shop Q and the transportation cost of B to Shop Q is 17, the cost would raise to 1005+17

See the excel file :  
Question9\_SOLVER.xlsx





## Question 11.

---

### Exam - 16 June 2023

A student on a tight budget wishes to plan a daily diet which will minimize his food expenditure while maintaining minimum nutritional requirements, according to the Recommended Daily Allowance (RDA). The student wants to plan his menu from the following goods: hamburgers, hot dogs, salad, chicken, pizza, carrots and cookies. The dietary information and cost of these foods is as follows:

Nutrient	Hamburger	Hot dog	Salad	Chicken	Pizza	Carrots	Cookies	RDA
Calories (cal/kg)	4400	4200	1000	1400	5000	600	5200	5000 cal
Calcium (mg/kg)	200	400	800	600	950	800	300	160 mg
Protein (mg/kg)	100	140	40	90	70	50	20	50 mg
Iron (mg/kg)	50	30	60	20	10	30	40	30 mg
Cost (€/kg)	5.0	4.0	3.5	6.0	10.0	4.5	7.0	

Furthermore, assume that the student wants to eat 125 grams of cookies each day and will eat at most 250 grams of carrots per day.

Formulate the problem as a linear program and indicate a feasible daily diet with the corresponding cost.

Question11\_Solver.xlsx - Excel

File Home Insert Page Layout Formulas Data Review View Developer Tell me what you want to do... Share

Clipboard: Cut, Copy, Format Painter

Font: Calibri, 11, Bold, Italic, Underline, Text Color, Background Color

Alignment: Wrap Text, Merge & Center

Number: General, Percentage, Decimals

Styles: Normal, Bad, Good, Neutral, Calculation, Check Cell

Cells: Insert, Delete, Format

Editing: AutoSum, Fill, Clear, Sort & Filter, Find & Select

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	
1															Reduced Cost											
2	Nutrient		Hamburger	Hotdog	Salad	Chicken	Pizza	Carrots	Cookies	RDA																
3	Calories	(cal/kg)	4400	4200	1000	1400	5000	600	5200	5000 cal	x1	Hamburger			0.809524		if the student decided to change his diet, changing to hamburgers would result in a smaller cost increase than if he went for any other option going for;									
4	Calcium	(mg/kg)	200	400	800	600	950	800	300	160 mg	x2	Hotdog			0											
5	Protein	(mg/kg)	100	140	40	90	70	50	20	50 mg	x3	Salad			2.547619											
6	Iron	(mg/kg)	50	30	60	20	10	30	40	30 mg	x4	Chicken			4.666667											
7	Cost	(e/kg)	5	4	3.5	6	10	4.5	7		x5	Pizza			5.238095	Hamburger	instesad of hotdog would increase the cost:						5.827381			
8											x6	Carrots			3.928571	Salad	instesad of hotdog would increase the cost:						7.565476			
9	assume that the student wants to eat 125 grams of cookies each day and will eat at most 250 grams of carrots per day.											x7	Cookies													
10																										
11				5	4	3.5	6	10	4.5	7																
12		Min C		x1	x2	x3	x4	x5	x6	x7																
13		5.017857		0	1.035714	0	0	0	0	0.125																
14												diet composition	dietary daily requirements													
15				x1	x2	x3	x4	x5	x6	x7	LHS		RHS													
16		calories requirements		4400	4200	1000	1400	5000	600	5200	5000	>=	5000		0	daily requirement for calories is met										
17		calcium requirements		200	400	800	600	950	800	300	452	>=	160		-291.786	calcium requirements above the daily need										
18		protein requirements		100	140	40	90	70	50	20	148	>=	50		-97.5	protein requirements above the daily need										
19		iron requirements		50	30	60	20	10	30	40	36	>=	30		-6.07143	iron requirements above the daily need										
20		daily consumption of cookies									1	>=	0.125		0	daily consumption of cookies is met										
21		daily consumption of carrots									1	<=	0.25		0.25	luckily no carrots are eaten										
22																										
23																										
24																										
25																										
26																										
27																										
28																										
29																										
30																										

**Optimum solution:**  
to meet the student requirements at a minimum cost he can simply eat 1.03 kg of hotdogs and 125 g of cookies

See the excel file :  
Question11\_SOLVER.xlsx