INSTITUTO SUPERIOR DE AGRONOMIA

Applied Operations Research

**Exercises**

**Goal programming**

1) The Dewright Company problem includes all three possible types of goals: a lower, one-sided goal (long-term profit); a two-sided goal (employment level); and an upper, one-sided goal (capital investment). Letting the decision variables *x*1, *x*2, *x*3 be the production rates of products 1, 2, and 3, respectively, we see that these goals can be stated as

profit goal is a lower one-sided goal: 12 x1 + 9 x2 + 15 x3 ≥ 125

employment goal is a two-sided goal: 5 x1 + 3 x2 + 4 x3 = 40

investment goal is an upper one-sided goal: 5x 1 + 7 x2 + 8 x3 ≤ 55

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Factor** | **Unit contribution** | | | **Goal Unit** | | | **Penalty Weight** |
| **Product** | | |
| **1** | **2** | **3** |
| Long-term profit | 12 | 9 | 15 | ≥ | 125 | (millions of dollars) | 5(-) |
| Employment level | 5 | 3 | 4 | = | 40 | (hundreds of employees) | 2(+), 4(-) |
| Capital investment | 5 | 7 | 8 | ≤ | 55 | (millions of dollars) | 3(+) |

Given the penalty weights incurred by missing these goals shown in the rightmost column of the table, find the production rates of the 3 products that minimize the sum of weighted penalties.

2) Reconsider the original version of the Dewright Co. problem presented in exercise 1 and summarized in the table below. After further reflection about the solution obtained by the simplex method, management now is asking some what-if questions.

a)Management wonders what would happen if the penalty weights in the rightmost column of the table were to be changed to 7, 4, 1, and 3, respectively. Would you expect the optimal solution to change? Why?

b) Management is wondering what would happen if the total profit goal were to be increased to at least $140 million (without any change in the original penalty weights). Solve the revised model with this change.

c) Solve the revised model if both changes are made.

3) Solving the Keeping the river clean problem using goal programming. A pulp mill makes mechanical and chemical pulp. The owners would like to:

* Pollution goal: 1 ton of mechanical pulp produces 1 unit of biological oxygen demand – BOD, 1 ton of chemical pulp produces 1.5 units. The mill environmental department would like to stay close to 400 BOD;
* Revenue goal: Chemical pulp sells at 200€/ton, mechanical at 100€/ton and the mill aims to generate around 40000€ of revenue per day (don’t mind it the revenue is higher);
* Labour goal: both chemical and mechanical pulp require the labor of one worker for about 1 day, or 1 workday (wd), per ton produced and the mill human resources wish, if possible, to keep at least 300 people employed at the mill;
* The maximum capacity of the mill to make mechanical pulp is 300 tons/d and 200 tons/d for chemical pulp (the mechanical pulp line cannot be used to make chemical pulp, and vice-versa)

3.1) Formulate the goal programing problem writing the goals in the form of constraints

3.2) Specify the objective function so that the weighted sum of the deviations from all

the goals are minimized

3.2.1) reduce the objective function to the simplest expression, ie, variables are needed (e.g. in our example, if the concern is about falling short of the employment goal but not about exceeding it, leave the positive deviation out)

3.2.2) solve the problem using EXCEL solver considering that **all deviations are equally important**. Note that it is advisable to consider relative deviations since each goal has its own unit

3.2.3) ) solve the problem using EXCEL solver considering that **all deviations are equally important** but this time, instead of being conservative, be bolder and rise the revenue and labour goals to 60000 and 600, respectively

3.2.4) solve the problem using EXCEL solver considering that **all deviations are equally important** varying the targets to see if you manage to find null deviations to all goals

4) A project manager wants to find the quantities of 3 products. Producing 1 unit of:

i) product 1 requires 40 employees, 2 tons of raw material and will bring the company a profit of 5 hundred €;

ii) product 2 requires 30 employees, 4 tons of raw material and will bring the company a profit of 8 hundred €;

iii) product 3 requires 20 employees, 3 tons of raw material and will bring the company a profit of 4 hundred €.

The manager has 3 goals:

* The maximum number of employees that can be allocated to producing these 3 products is 100 employees
* There are 10 tons of raw material in the warehouse and he wants to consume no more no less than that
* The total profit is expected to be at least 30 hundred €

The manager suspects he might not be able to meet these 3 goals simultaneously therefore he sets some penalty weights to each of the goals:

* Each extra employee is associated to a penalty of 5
* Each ton below the goal is associated to a penalty of 8 (-) whereas each ton above the goal of 10 is associated to a penalty of 12 (+)
* If profit is less than 30 hundred €, each hundred € is associated to a penalty of 15

Formulate the problem as a linear programming problem and use excel solver (LP simplex) to find the combination of the 3 products that minimizes the penalties.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Factor** | **Unit contribution** | | | **Goal Unit** | | **Penalty Weight** |
| **Product** | | |
| **1** | **2** | **3** |
| Employees | 40 | 30 | 20 | ≤ | 100 (employees) | 5 |
| Raw material | 2 | 4 | 3 | = | 10 (tons) | 12(+), 8(-) |
| Profit | 5 | 8 | 4 | ≥ | 30 (hundred €) | 15 |

5) Davis is the owner of a resort hotel and to increase profits during the rest of the year, Davis wants to expand his convention business but, to do so, he needs to expand his conference facilities. Davis hired a marketing research firm to determine the number and sizes of conference rooms that would be required by the conventions he wants to attract.

The results of this study indicated that Davis’s facilities should include at least 5 small conference rooms (400 square foot), 10 medium conference rooms (750 square foot), and 15 large (conference rooms 1,050 square foot).

Additionally, the marketing research firm indicated that if the expansion consisted of a total of 25,000 square feet, Davis would have the largest convention center among his competitors—which would be desirable for advertising purposes.

While discussing his expansion plans with an architect, Davis learned that he can expect to pay $18,000 for each small conference room in the expansion, $33,000 for each medium conference room, and $45,150 for each large conference room.

Davis wants to limit his expenditures on the convention center expansion to approximately $1,000,000.

Determine the number of rooms to be built and the deviations to size expansion and budget limits considering:

a) all goals are equally important (non-preemptive goal programming)

b) the primary-goal is not exceeding the expansion size limit of 25,000 sq ft (streamlined approach preemptive goal programming)

c) the primary-goal is building at least 15 large conference rooms (streamlined approach preemptive goal programming)

**Multi-objective Linear Programming – a priori methods**

Blackstone Mining Company operates 2 coal mines Wythe and Giles producing 3 types of coal: high, medium and low level.

The manager is **anticipating a demand increase** for coal in the coming year. Projections indicate a 48 ton increase in the demand for high-grade coal, a 28-ton increase in the demand for medium-coal and a 100 ton increase in the demand for low-grade coal.

To handle these demand increases **extra shifts of workers** to the mines must be scheduled.

The amount of coal that can be produced in a month’s time at each mine by a shift of workers is summarizes in the table below:

|  |  |  |  |
| --- | --- | --- | --- |
| Coal production by type of coal | Wythe Mine | Giles Mine | Expected demand increase |
| High-grade | 12 | 4 | 48 |
| Medium-grade | 4 | 4 | 28 |
| Low-grade | 10 | 20 | 100 |
| cost | 40000 | 32000 | - |
| toxic water | 800 | 1250 | - |
| life-threatening accidents | 0.2 | 0.45 | - |

Each extra shift has an extra **cost** of 40000/month at Wythe and 32000/month at Giles. One extra shift can be scheduled per month at each mine.

The extraction methods lead to the production of **toxic water**. Running an extra shift leads to the production of 800 and 1250 gallons of toxic water at Wythe and Giles, respectively.

Despite safety guidelines are followed, 0.2 and 0.45 **life-threatening accidents** are expected to occur at Wythe and Giles, respectively.

a) Define the managers’ problem

b) Identify the decision variables

c) Write the objective function(s)

d) Define the constraints

1. Formulate the problem in Excel for solver application considering the use of the **Global criterion method (MiniMax method)** in order to determine the number of extra shifts at each of the mines that minimizes costs, toxic waste production and life threatening accidents considering the following weights:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Objective function** | **Wi’** | **Wi’’** | **Wi’’’** | **Wi’’’’** |
| Production costs | 1 | 10 | 1 | 1 |
| Pollution | 1 | 1 | 10 | 1 |
| Safety | 1 | 1 | 1 | 10 |

1. Formulate the problem in Excel for solver application considering the use of **The Absolute Priority Method (preemptive method)** to determine the number of extra shifts at each of the mines that minimizes costs, toxic waste production and life threatening accidents considering the following priorities:
   1. 1st production costs, 2nd pollution and 3rd safety
   2. 1st pollution, 2nd production costs; and 3rd safety
   3. 1st safety, 2nd production costs and 3rd pollution
2. Formulate the problem in Excel for solver application considering the use of **The Weighting Method** to determine the number of extra shifts at each of the mines that minimizes costs, toxic waste production and life threatening accidents considering the following weights to be applied to the global function

|  |  |  |  |
| --- | --- | --- | --- |
| **Objective function** | **Wi’** | **Wi’’** | **Wi’’’** |
| Production costs | 1/3 | 2.5/3 | 2.7/3 |
| Pollution | 1/3 | 0.25/3 | 0.15/3 |
| Safety | 1/3 | 0.25/3 | 0.15/3 |

**Multi-objective Linear Programming – a posteriori methods**

Take the following MOLP problem:

Max Z1 = 3 x1 - 2 x2

Max Z2 = - x1 + 2 x2

Subject to the following constraints:

|  |  |  |
| --- | --- | --- |
| 4 x1 + 8 x2 ≥ 8  3 x1 – 6 x2 ≤ 6  4 x1 - 2 x2 ≤ 14  1 x1 ≤ 6 |  | -1 x1 + 3 x2  ≤ 15  -2 x1 + 4 x2 ≤ 18  -6 x1 + 3 x2 ≤ 9  x1 , x2 ≥ 0 |

* + 1. Plot the feasible region in decision space for this problem.
    2. Plot the feasible region in objective space indicating for each corner point if it is a non-inferior or a dominated solution
    3. Use the weighted method to generate an approximation of the non-inferior set through 11 iterations (assign weights in a systematic way)
    4. Use the weighted method to generate an approximation of the non-inferior set through 6 iterations (assign weights in a systematic way)
    5. Discuss the advantages and disadvantages of a high or low number of iterations