CHAPTER 15: IMPLEMENTING NON-TIMBER OBJECTIVES IN THE HARVEST SCHEDULING MODEL

Earlier chapters the formulation of basic cost-minimizing and profit-maximizing harvest scheduling models. The basic models just scratch the surface of the types of management objectives and activities that can be modeled with linear programming. This chapter explores a couple of these possibilities. The possibilities are infinite, and there is much more that we will not have time to discuss. The objective of this chapter is to give you some simple examples to stimulate your imagination.

The first model modification that will be considered requires the model to maintain minimum areas of older stands. A common – and well-justified – concern with managing forests using economic rotations is that such rotations will be too short to maintain certain values, such as wildlife habitat, aesthetics, and recreation. It is not too hard, however, to require the model to maintain a target amount of land in older age classes. These will be called *extended rotation areas*. Extended rotation areas can be stipulated in the model by adding a new set of constraints.

All of the activities represented in the models discussed so far involve scheduling harvests. Activities not directly related to timber management can also be specified, however. As an example, this chapter discusses setting aside acres that will be devoted to wildlife openings in the forest, where trees will be excluded and different food crops, such as oats, corn, berries, etc., could be planted. This will require the addition of some new variables, which will, in turn, require modifying the objective function and many of the model's constraints.

Finally, this chapter will discuss how stream-side management zones (SMZs) can be incorporated into the linear programming framework. In a real-world application, this can be a difficult exercise, primarily because of the difficulty of identifying the areas to be included in the SMZs. This task is best done with a geographical information system (GIS), which is beyond the scope of this text. Once the areas to be included in the SMZs have been determined, however, they are easy to implement within the linear programming model.

Within the example problem developed so far, these three model modifications will be included to meet the following specific requirements:

1. Extended Rotations – the model will be modified to require 1,500 acres from site class I to be maintained in stands at least 40 years old in all periods and to require 1,500 acres from site class II to be maintained in stands at least 30 years old in all periods. (Recall that the optimal rotation for site class I is 40 years and the optimal rotation for site class II is 30 years, so without this constraint, acres would seldom be allowed to reach these ages.)
2. Wildlife Openings – the model will be required to set aside 500 acres of wildlife openings. These areas will be clearcut in the first period and

maintained in browse and forage species for the remainder of the planning horizon.

3. Stream-side Management Zones (SMZs) – 8% of the area of the forest, evenly distributed among site classes, will be set aside as SMZs, with harvests precluded in these areas.

The basic four-period model developed in the last chapter will be used as a starting point for these modifications. A complete model incorporating all three modifications is presented in the final section of the chapter.

An important question is how much it costs, in terms of opportunity costs, to implement these types of constraints. Often, formulation changes like those discussed in this chapter lead to significant reductions in the amount of wood that can be harvested from a forest and/or the net revenue that can be earned. Whether or not these costs are justified by the benefits created is a difficult question that can only be answered by the landowner or landowners.

1. Implementing Extended Rotation Areas

As discussed in the introduction to this chapter, economic rotations tend to be short relative to biologically-oriented rotations. This means that if forests are managed solely under economic rotations, older stands will be rare. Populations of wildlife species that require or prefer older stands will decline. People who like to look at or recreate in stands of large trees will be unhappy. For these and other reasons, it is common to assign some areas to be maintained as older stands. Of course, it is important to recognize that older stands cannot always be preserved in this state. Stands will eventually be lost to fire, disease, wind or ice damage, or simply old age. A forest plan should recognize these losses and ensure that new generations of stands will be available to replace the extended rotation stands as they are lost. It is not necessarily advisable to identify areas to set aside for preservation once and for all. Rather, it may be more viable in the long run to require that a certain area of forest be at least a certain age in all periods and let the model determine where those areas should be and how they should be maintained.

Extended Rotation Constraints

Constraints can be added to the model that require that at all times there are at least 1,500 acres from site class I that are at least 40 years old and that there are at least 1,500 acres from site class II that are at least 30 years old. In other words, these constraints require at least 1,500 acres from each site class to be held at least one decade longer than the optimal rotation for that site class. (Of course, this is not particularly old. These low extended rotation ages have been selected to keep the problem size small while illustrating how the basic model can be modified to create areas with longer rotations.)

A separate constraint will be required for each period and site class (potentially eight constraints). For each constraint, the set of variables representing acres that can contribute to

Table 15.1. Age classes of acres assigned to each variable. Age class 1 means the acres will be 0 to10 years old at the beginning of the planning period, 2 means the acres will be 11 to 20 years old, etc. Zeros indicate that the acres will be harvested in that period. A small *a* is a variable equal to the initial age class of the stand.

Period	Variable							
	X_{sa10}	X_{sa13}	X_{sa14}	X_{sa20}	X_{sa24}	X_{sa30}	X_{sa40}	X_{sa00}
1	0	0	0	a	a	a	a	a
2	1	1	1	0	0	a+1	a+1	a+1
3	2	0	2	1	1	0	a+2	a+2
4	3	1	0	2	0	1	0	a+3

the extended rotation goals must be identified. Only acres that are old enough to meet the extended-rotation age requirement at the beginning of the period can be counted towards the extended-rotation acreage goal for that period. Also, acres scheduled to be harvested during the period cannot be counted since they will not provide extended rotation services for the entire decade. Table 15.1 helps identify which variables will meet these conditions. The table shows, for each period in the planning horizon, the age class of acres assigned to each variable. An age class of 1 in the table means that the acres assigned to that variable will be in the 0 to 10 year age class at the beginning of the corresponding period; an age class of 2 means that the stand will be in the 11 to 20 year age class in that period, and so on. An age class of 0 indicates that the stand will be harvested during that period. A small *a* indicates that the age class of acres assigned to that variable depends on their initial age class; specifically, *a* indicates that the age class in that period equals the initial age class; *a+1* indicates that the age class for that period will be one more than the initial age class, and so on.

In period 1, the oldest acres are those initially in the 21 to 30-year age-class. These acres aren't old enough to satisfy the extended rotation requirements for either site class. Thus, it would not be possible to meet the extended rotation requirement for either site class in the first period. Since it is not feasible to meet the requirements for extended rotation areas in the first period, extended rotation constraints for period 1 for either site class would make the problem infeasible. Thus, no extended rotation constraints should be included for the first period.

By period 2, however, acres that were initially in the 21 to 30-yr age class (initial age class 3) that were not cut in the first period will be in the 31 to 40 year age class (age class 4) for the second period – old enough to satisfy the extended-rotation criterion for site class II.

However, there still will be no stands old enough to satisfy the extended-rotation criterion for site class I. Those acres in site class II that are old enough to meet the extended-rotation criterion for the entire period can be counted toward the extended rotation goal as long as they are not harvested in period 2. The variables that satisfy these conditions are:

X_{2330}, X_{2340}, and X_{2300}.

These are the variables identified in Table 15.1 as being in age-class $a+1$ in period 2, with the added requirement that a=3. Note that all of the acres assigned to these three variables must come from site class II, initial age class 3, and that there are 7,000 acres in this analysis area. Thus, it is feasible to assign at least 1,500 acres to these variables. The following constraint will ensure that there will be 1,500 acres from site class II that are over 30 years old in period 2:

$$X_{2330} + X_{2340} + X_{2300} \geq 1{,}500 \qquad \text{(Ext.-rotation constr. for site class II, pd. 2)}$$

In the third period, stands from initial age classes 2 or 3 will be old enough to meet the extended-rotation criterion for site class II if they are not scheduled for harvest in periods 1, 2, or 3. The variables satisfying these criteria are:

X_{2a40} and X_{2a00} for a = 2 or 3.

These are the variables identified in Table 15.1 as being in age-class $a+2$ in period 3, with the added requirement that $a \geq 2$. Thus, the constraint providing for 1,500 acres of extended rotation in site class II in period 3 is:

$$\sum_{a=2}^{3} [X_{2a40} + X_{2a00}] \geq 1{,}500 \qquad \text{(Ext.-rotation constr. for site class II, pd. 3)}$$

Another way to write this constraint is:

$$X_{2240} + X_{2200} + X_{2340} + X_{2300} \geq 1{,}500.$$

For site class I, the minimum age for extended rotation areas is higher. Thus, only acres that were initially in the oldest age class (i.e., a=3) that are not scheduled to be harvested in periods 1, 2 or 3 meet the extended-rotation requirements for site class I in period 3. The variables satisfying these requirements for site class I in period 3 will be:

X_{1340} and X_{1300}.

Note that the acres assigned to both of these variables must come from site class I, initial age class 3. As with site class II, it is wise to check whether there are enough acres in this analysis area to meet the extended rotation acreage target. Since there are 9,000 acres in this analysis area and only 1,500 acres are needed, it will be possible to meet the extended rotation acreage targets for site class I in period 3. The following constraint will ensure that 1,500 acres of extended rotation acres will be assigned from site class I in period 3:

$$X_{1340} + X_{1300} \geq 1{,}500 \qquad \text{(Ext.-rotation constr. for site class I, pd. 3)}$$

By the fourth period, acres from any initial age class – 1, 2, or 3 – will meet the age criterion for extended-rotation stands in site class II as long as they are not harvested at any time during the planning horizon. All of the variables satisfying the site class II extended-rotation criterion for period 4 will have the form X_{2a00}. The constraint can be written:

$$\sum_{a=1}^{3} X_{2a00} \geq 1{,}500 \qquad \text{(Ext.-rotation constr. for site class II, pd. 4)}$$

This constraint can also be written as:

$$X_{2100} + X_{2200} + X_{2300} \geq 1{,}500.$$

Since the extended-rotation age requirement is higher for site class I, acres in this site class would have to start out at least in age class 2 in order to qualify as extended rotation stands in period 4. Of course, only acres that are not cut during the planning horizon will qualify. Thus, the extended-rotation constraint for site class I in period 4 will be:

$$\sum_{a=2}^{3} X_{1a00} \geq 1{,}500 \qquad \text{(Ext.-rotation constr. for site class I, pd. 4)}$$

Or:

$$X_{1200} + X_{1300} \geq 1{,}500.$$

These five constraints will ensure that there will be at least 1,500 acres from site class I over 40 years old in periods 3 and 4 and 1,500 acres from site class II over 30 years old in periods 2, 3, and 4. They are summarized here:

$$X_{2330} + X_{2340} + X_{2300} \geq 1{,}500 \qquad \text{(Ext.-rotation constr. for site class II, pd. 2)}$$

$$X_{1340} + X_{1300} \geq 1{,}500 \qquad \text{(Ext.-rotation constr. for site class I, pd. 3)}$$

$$X_{2240} + X_{2200} + X_{2340} + X_{2300} \geq 1{,}500 \qquad \text{(Ext.-rotation constr. for site class II, pd. 3)}$$

$$X_{1200} + X_{1300} \geq 1{,}500. \qquad \text{(Ext.-rotation constr. for site class I, pd. 4)}$$

$$X_{2100} + X_{2200} + X_{2300} \geq 1{,}500. \qquad \text{(Ext.-rotation constr. for site class II, pd. 4)}$$

Modifying the Average Ending Age Constraint

While the above constraints are sufficient to provide the desired number of extended rotation acres in each site class, there is one more modification that should be made to the model. If at least 1,500 acres from each site class are maintained in these older age classes, the model will be able to over-harvest the remaining areas of the forest and still meet the original ending age constraints because the extended rotation acres will tend to pull up the average age of the forest. Thus, in order to ensure that the remainder of the forest is not over-harvested, it is also necessary to modify the average ending age constraints. This could be done by re-calculating the target ending age, taking account of the acres assigned to the extended rotation areas. The problem with this approach is that it is not possible to say ahead of time just how old the acres assigned to the extended rotation areas will be. For example, if the last constraint above – the extended rotation constraint for site class II in period 4 – is met with acres assigned to the variable X_{2300}, those acres will be 65 years old at the end of the 40-year planning horizon. On the other hand, if the constraint is met with acres assigned to the variable X_{2100}, the acres will only be 45 years old.

A good way to account for the impact of the extended rotation areas on the target ending age of the forest is to simply exclude acres applied to the extended rotation areas in the final period from the average ending age calculations. The remaining acres – the acres under regular management regimes – will then be required to meet the average ending age requirements on their own. The extended rotation acres, after all, have their own age requirements to meet and can be considered as a separate management area.

The problem with excluding some of the variables from the average ending age constraints is that the right-hand-side of the original constraints treated the total acreage in the forest or in a site class as a constant. If some variables are in the constraint and others are not, however, this assumption will no longer work. To exclude some acres from the average ending age constraints, therefore, it will be necessary to reconsider how the constraints were originally constructed. First, let's review the original constraints. With four periods, the average ending age constraints were:

$$\sum_{a=1}^{3}\left[Age_{sa00}^{40}X_{sa00}+\sum_{p_1=1}^{4}\left(Age_{sap_10}^{40}X_{sap_10}+\sum_{p_2=p_1+2}^{4}Age_{sap_1p_2}^{40}X_{sap_1p_2}\right)\right]$$
$$\geq \overline{Age}^{40}\times TotalArea_s \qquad s=1,2$$

Note the term $TotalArea_s$ on the right-hand side of these constraints. When all of the variables are included in the constraint, it is safe to consider the total area in a site class to be a constant. In order to exclude some variables from the constraint, however, this assumption cannot be made because we cannot say for sure how many acres will be excluded from the ending age calculation. Of course, the total area in a site class can be expressed as the sum of all of the X variables for that site class. That is:

$$TotalArea_s=\sum_{a=1}^{3}\left[X_{sa00}+\sum_{p_1=1}^{4}\left(X_{sap_10}+\sum_{p_2=p_1+2}^{4}X_{sap_1p_2}\right)\right]$$

Thus, the right-hand side of the ending average age constraints can be written as:

$$\overline{Age}^{40}\times TotalArea_s=\overline{Age}_s^{40}\times\sum_{a=1}^{3}\left[X_{sa00}+\sum_{p_1=1}^{4}\left(X_{sap_10}+\sum_{p_2=p_1+2}^{4}X_{sap_1p_2}\right)\right]$$
$$=\sum_{a=1}^{3}\left[\overline{Age}_s^{40}X_{sa00}+\sum_{p_1=1}^{4}\left(\overline{Age}_s^{40}X_{sap_10}+\sum_{p_2=p_1+2}^{4}\overline{Age}_s^{40}X_{sap_1p_2}\right)\right]$$

Thus, the right-hand-side of the inequality is now a linear function of all of the variables representing acres from the site class, and the coefficient on each of these variables is the target average ending age. Since these terms involve variables, they must be moved back to the left-hand side of the constraint by subtracting them from both sides, leaving zero on the right-hand side. Combining the coefficients for each variable on the left-hand side results in new coefficients which are the difference between the original coefficients and the target average ending age. Thus, the resulting ending average age constraint will be:

$$\sum_{a=1}^{3}\Bigg[(Age_{sa00}^{40} - \overline{Age}_s^{40})X_{sa00} + \\ + \sum_{p_1=1}^{4}\left((Age_{sap_10}^{40} - \overline{Age}_s^{40})X_{sap_10} + \sum_{p_2=p_1+2}^{4}(Age_{sap_1p_2}^{40} - \overline{Age}_s^{40})X_{sap_1p_2}\right)\Bigg] \geq 0$$

By introducing the following new notation:

$$AgeDif_{sa00}^{40} = Age_{sa00}^{40} - \overline{Age}_s^{40}$$

This equation can be written more simply as:

$$\sum_{a=1}^{3}\left[AgeDif_{sa00}^{40}X_{sa00} + \sum_{p_1=1}^{4}\left(AgeDif_{sap_10}^{40}X_{sap_10} + \sum_{p_2=p_1+2}^{4} AgeDif_{sap_1p_2}^{40}X_{sap_1p_2}\right)\right] \geq 0$$

Note that the coefficients in this new version of the ending age constraint are simply the difference between the age at the end of the planning horizon of the acres assigned to the corresponding variable minus the target age at the end of the planning horizon. This version of the constraint says that the average deviation from the minimum average age, for all acres represented by the variables in the constraint, must be greater than zero. Constraints of this type have the equivalent effect as the original average ending age constraints. The advantage of this type of constraint is that one can include or exclude any subset of variables one wishes from this type of constraint. Thus, with this type of constraint it is possible to set a target average ending age for acres assigned to any subset of the variables without knowing ahead of time how many acres will be assigned to that subset.

With extended rotation constraints in the model it is necessary to exclude from the average ending age constraint the variables that will be counted toward the extended rotation acres in the last period. If you review the extended rotation constraints for the last period, you will see that all of the variables of the form X_{sa00}, except X_{1100}, should be excluded. This results in the following average ending age constraints for a model that includes the above extended rotation constraints:

$$AgeDif_{1100}^{40} X_{1100} + \sum_{a=1}^{3} \sum_{p_1=1}^{4} \left(AgeDif_{1ap_1 0}^{40} X_{1ap_1 0} + \sum_{p_2=p_1+2}^{4} AgeDif_{1ap_1 p_2}^{40} X_{1ap_1 p_2} \right) \geq 0$$

(for site class 1), and

$$\sum_{a=1}^{3} \sum_{p_1=1}^{4} \left(AgeDif_{2ap_1 0}^{40} X_{2ap_1 0} + \sum_{p_2=p_1+2}^{4} AgeDif_{2ap_1 p_2}^{40} X_{2ap_1 p_2} \right) \geq 0$$

(for site class 2). Note that the variables X_{1200}, X_{1300}, X_{2100}, X_{2200}, and X_{2300} were excluded from these constraints because acres assigned to these variables will be set aside to meet the extended rotation requirements in the last planning period. The remaining acres in each site class should meet the target average ending ages on their own. Note that the set of variables that will occur in the extended rotation constraints for the last period may be different from one problem to the next, so the specific set of variables to be excluded from the average ending age constraints may be different for different problems. In general, however, the the variables that have non-zero coefficients in the extended rotation constraint(s) for the final period should have their coefficients set to zero in the ending age constraint(s).

You should understand how the average ending age constraint was changed because this type of average ending age constraint will also be necessary in order to accommodate the addition of wildlife opening variables in the next section of this chapter. You should note that before the variables representing extended rotation areas for the last period were dropped from the revised constraint, it was functionally identical to the original constraint. The revised form – with the *AgeDif* coefficients – makes it easy to exclude areas assigned to certain prescriptions that should not be counted in the calculation of the average age of the forest at the end of the planning horizon.

2. Implementing Wildlife Openings

Most of the variables in the models so far have to do with when stands of timber should be harvested. The only other variables that have been used have been the harvest accounting variables. Many of the treatments that are applied in forest management – while often involving manipulating forest vegetation – are not primarily directed at producing timber. This section illustrates how new treatments that are not necessarily directly related to producing timber can be incorporated in a harvest scheduling model. As more of these types of management activities are added to the model it becomes less of a "harvest scheduling model" and more of a "forest management activity scheduling model," or, more simply, "management planning model."

The particular management practice that is considered here is the creation of wildlife openings. Specifically, these openings will be created to provide 500 acres of wildlife browse habitat for a variety of species. The openings will be cleared in the first period of the planning horizon. Once the openings have been created, they will be maintained over time by occasionally planting browse species such as oats, corn, and, perhaps, berries. Assume that the average cost of maintaining these wildlife openings is $10/ac per year.

Wildlife Opening Variables

As discussed earlier, to implement the creation of wildlife openings in the harvest scheduling model, a new set of variables must be introduced. Recall most of that the variables in a harvest scheduling model correspond to alternative treatments that could be applied to acres from a given analysis area. The creation of wildlife openings adds an additional treatment to which acres from each analysis areas could be assigned. This new treatment will be represented by a new set of variables, defined as follows:

W_{sa} = the number of acres from site class *s,* initial age class *a* assigned to be cleared in period 1 and maintained as wildlife openings for the remainder of the planning horizon.

Because introducing this management activity involves creating of a new set of variables, these variables could potentially affect the objective function and each type of constraint in the model. We will begin by adding them to the area constraints.

Adding the Wildlife Openings to the Area Constraints

Since acres from each analysis area can be assigned to the wildlife openings prescription, these new variables must be included in the area constraints, along with the other variables representing alternative prescriptions acres in the analysis area can be assigned to. The area constraints should be revised as follows:

$$X_{sa00} + \sum_{p=1}^{4}\left[X_{sap_1 0} + \sum_{p_2=p_1+2}^{4} X_{sap_1 p_2}\right] + W_{sa} \leq A_{sa} \quad \text{for } s = 1,2 \text{ and } a = 1,2,3$$

The wildlife opening prescription represents the ninth potential prescription for acres in each analysis area. The constraint says that the number of acres assigned to these nine prescriptions from each analysis area must be less than or equal to the area initially in the analysis area.

Specifying the Target Area for Wildlife Openings

A new constraint, or set of constraints, will also be needed that specifies a minimum area that must be assigned to the new wildlife openings variables. In this example, the total number of acres assigned to wildlife openings must be at least 500 acres. The following constraint ensures this:

$$\sum_{s=1}^{2}\sum_{a=1}^{3} W_{sa} \geq 500$$

Because the opportunity cost of giving up acres is less for site class I than for site class II, the model will tend to assign acres from site class I to these variables, rather than acres from site class II. If you wish to ensure that a certain number of acres will be assigned from each site class, a separate constraint will be needed for each site class. These constraints would have the form:

$$\sum_{a=1}^{3} W_{sa} \geq 500 \quad s = 1, 2$$

Adding the Wildlife Opening Variables to the Objective Function

Any time new activities or prescriptions are added to the model, costs or revenues are likely to be associated with those activities. If so, the new variables should appear in the objective function with the appropriate discounted net revenue or discounted cost coefficients. Both costs and revenues are associated with the creation of the wildlife openings. The clearing of the forest to create the openings will generate revenue from the sale of the timber. Timber sales costs are also incurred. Note that stand establishment costs will not be incurred since these areas will not be regenerated with trees. In addition to the costs and revenues from the timber sale, annual maintenance expenditures will be required for the wildlife openings. Remember, corn, oats, or other browse crops will be planted in these openings. These annual maintenance costs will begin in year 5 (the mid-point of period 1) and continue through the end of the planning horizon.

The revised profit-maximization objective function will be:

$$Max\ Z = \sum_{s=1}^{2}\sum_{a=1}^{3}\left[\sum_{p_1=1}^{4}\left(c^{p}_{sap_1 0}\cdot X_{sap_1 0} + \sum_{p_2=p_1+2}^{4} c^{p}_{sap_1 p_2}\cdot X_{sap_1 p_2}\right) + c^{wp}_{sa} W_{sa}\right]$$

where $c^{p}_{sap_1p_2}$ = the discounted net revenue (profit) from assigning one acre from site class s, initial age class a to be harvested in periods p_1 and p_2 (where $p_i = 0$ implies no i^{th} harvest); i.e., assigning an acre to the variable $X_{sap_1p_2}$, and

c^{wp}_{sa} = the discounted net revenue from assigning an acre from site class s, initial age class a to be managed as a wildlife opening; i.e., assigning an acre to the prescription W_{sa}.

This can also be written, equivalently but less compactly, as:

$$Max\ Z = \sum_{s=1}^{2}\sum_{a=1}^{3}[c^{p}_{sa10}X_{sa10} + c^{p}_{sa13}X_{sa13} + c^{p}_{sa14}X_{sa14} + c^{p}_{sa20}X_{sa20} + $$
$$+ c^{p}_{sa24}X_{sa24} + c^{p}_{sa30}X_{sa30} + c^{p}_{sa40}X_{sa40} + c^{wp}_{sa}W_{sa}]$$

As noted above, the coefficients on the wildlife opening variables specify the discounted net revenue from assigning an acre from analysis area *sa* to be managed as a wildlife opening.

The formula for these coefficients will include two components: 1) the discounted net revenue from the harvest in period 1, and 2) the discounted annual maintenance cost for managing the area as a wildlife opening – in this case, an annual cost of \$10 (c_w) for 35 years, beginning in year 5. The first component will be very similar to the coefficient on the variable X_{sa10}, except that the stand establishment cost is omitted. The second component is just the present value of a finite annual series of 35 payments, discounted for an additional 5 years to account for the fact that the first cost is not incurred until year 5. The general formula for the objective function coefficients on the wildlife opening variables for a four-period profit-maximization model is:

$$c_{sa}^{wp} = \frac{(P - s_v)v_{sa10}^1 - s_f}{(1+r)^5} - \frac{c_w[(1+r)^{35} - 1]}{r(1+r)^{35}(1+r)^5}$$

where P = the wood price,
s_v = the variable (per cord) timber sale administration cost,
s_f = the fixed (per acre) timber sale administration cost,
v^1_{sa10} = the volume of wood that will be harvested in period 1 for each acre assigned to the variable W_{sa}. Note that this is the same as the volume harvested in period 1 for acres assigned to the variable X_{sa10}.
c_w = the annual, per-acre cost of maintaining the wildlife openings, and
r = the real interest rate.

For our example, the profit-maximization objective function coefficient for the variable W_{23} can be calculated as follows:

$$c_{23}^{wp} = \frac{(25 - .2) \times 27 - 15}{(1.04)^5} - \frac{10[(1.04)^{35} - 1]}{0.04(1.04)^{40}} = \$384.62$$

For a cost-minimization formulation, the objective function and the coefficients would be only slightly different. The general form of the cost-minimizing objective function is:

$$Min\ Z = \sum_{s=1}^{2}\sum_{a=1}^{3}\left[\sum_{p_1=1}^{4}\left(c_{sap_1 0}\cdot X_{sap_1 0} + \sum_{p_2=p_1+2}^{4} c_{sap_1p_2}\cdot X_{sap_1p_2}\right) + c_{sa}^{wc}W_{sa}\right]$$

where $c_{sap_1p_2}$ = the discounted cost of assigning one acre from site class s, initial age class a to be harvested in periods p_1 and p_2 (where p_i = 0 implies no i^{th} harvest); i.e., of assigning an acre to the variable $X_{sap_1p_2}$, and

c_{sa}^{wc} = the discounted cost of assigning an acre from site class s, initial age class a to be managed as a wildlife opening; i.e., of assigning an acre to the prescription W_{sa}.

This can also be written, equivalently but less compactly, as:

$$Min\ Z = \sum_{s=1}^{2}\sum_{a=1}^{3}[c_{sa10}X_{sa10} + c_{sa13}X_{sa13} + c_{sa14}X_{sa14} + c_{sa20}X_{sa20} + c_{sa24}X_{sa24} + c_{sa30}X_{sa30} + c_{sa40}X_{sa40} + c_{sa}^{wc}W_{sa}]$$

In this case, the coefficients on the wildlife opening variables need to give the discounted cost of assigning an acre from analysis area *sa* to be managed as a wildlife opening. The formula for these coefficients will be almost the same as the formula for the discounted net revenue coefficient. The only difference will be that the term for the revenue from the volume harvested in the first period will be omitted, and costs will be positive, instead of negative. The general formula for the objective function coefficients for a four-period cost-minimization model is:

$$c_{sa}^{wp} = \frac{s_v v_{sa10}^1 + s_f}{(1+r)^5} + \frac{c_w[(1+r)^{35} - 1]}{r(1+r)^{35}(1+r)^5}$$

where all of the symbols are as previously defined.

For our example, the cost-minimization objective function coefficient for the variable W_{23} can be calculated as follows:

$$c_{23}^{wp} = \frac{0.2 \times 27 + 15}{(1.04)^5} + \frac{10[(1.04)^{35} - 1]}{0.04(1.04)^{40}} = \$170.18$$

Adding the Wildlife Opening Variables to the Harvest Accounting Constraint for Period 1

The clearing of the wildlife openings will contribute to the timber harvest in period 1. Thus, the harvest accounting constraint for period 1 will have to be modified as follows:

$$\sum_{s=1}^{2}\sum_{a=1}^{3}\left[v_{sa10}^1 X_{sa10} + v_{sa13}^1 X_{sa13} + v_{sa14}^1 X_{sa14} + v_{sa10}^1 W_{sa}\right] - H_1 = 0$$

If you compare this with the harvest accounting constraint used in the original four-period model, you will see that the only change is the addition of the wildlife opening variables. Note again that the volume of wood produced for each acre harvested will be the same as the volume per acre for the variable X_{sa10} for the corresponding analysis area. Note also that none of the other harvest accounting constraints will need to be changed.

Accommodating Wildlife Openings in the Average Ending Age Constraint

The acres assigned to wildlife opening prescriptions should not be counted in the average ending age constraint. The modified average ending age constraint discussed earlier in the discussion of extended rotation constraints should be used, and the wildlife opening variables should be excluded from that constraint (i.e., their coefficients should be zero). This will properly account for the acres allocated to wildlife openings in the application of the average ending age constraint.

3. Implementing Stream-side Management Zones (SMZs)

In practice, the difficult thing about implementing stream-side management zones (SMZs) in a harvest scheduling model is identifying the areas that will be in the zones and reducing the areas in each analysis area appropriately. On a large ownership this would normally be accomplished with a GIS system. For the example here, it will be assumed that the area assigned to SMZs has already been calculated and that 8% of the area in each analysis area will be reserved for SMZs. With these areas already calculated, all that needs to be done is to subtract the area to be allocated to SMZs from each of the analysis areas. This is accomplished in the linear programming formulation by subtracting the SMZ area for each analysis area from the right-hand side of the corresponding area constraint. Thus, if the original area in analysis area *sa* is A_{sa}, and if smz_{sa} is the proportion of the area in that analysis area is to be included in SMZs, then the new area, A_{sa}^* , will be:

$$A_{sa}^* = A_{sa} \times (1 - smz_{sa}).$$

The new set of area constraints will have the following form:

$$X_{sa00} + \sum_{p=1}^{4}\left[X_{sap_1 0} + \sum_{p_2 = p_1 + 2}^{4} X_{sap_1 p_2} \right] \geq A_{sa}^* \quad \text{for } s = 1,2 \text{ and } a = 1,2,3$$

This is the only modification that is necessary to implement the SMZs.

For this chapter’s example, the acreages outside SMZs for each analysis area are shown in Table 15.2.

Table 15.2. Initial acreage by site and age class after accounting for SMZs.

Age Classes	Acres by site class	
	Site I	Site II
0 to 10	2,760	7,360
11 to 20	5,520	3,680
21 to 30	8,280	6,440
Total	16,560	17,480

4. A Model with Extended Rotation Areas, Wildlife Openings, and SMZs

This section presents the complete model formulation incorporating extended rotation area targets, wildlife openings, and SMZs, as discussed in this chapter. The example model is based on the four-period profit-maximization model discussed in Chapter 14 with all of the necessary modifications to create the extended rotation areas, wildlife openings, and SMZs, as specified in the beginning of the chapter.

General Formulation

$$Max\ Z = \sum_{s=1}^{2}\sum_{a=1}^{3}\left[\sum_{p_1=1}^{4}\left(c_{sap_1 0}^{p}\cdot X_{sap_1 0} + \sum_{p_2=p_1+2}^{4} c_{sap_1 p_2}^{p}\cdot X_{sap_1 p_2}\right) + c_{sa}^{wp} W_{sa}\right]$$

Subject to:

$$X_{sa00} + \sum_{p=1}^{4}\left[X_{sap_1 0} + \sum_{p_2=p_1+2}^{4} X_{sap_1 p_2}\right] + W_{sa} \le A_{sa} \quad \text{for } s = 1,2 \text{ and } a = 1,2,3$$

$$\sum_{s=1}^{2}\sum_{a=1}^{3} W_{sa} \ge \overline{W}$$

$$X_{2330} + X_{2340} + X_{2300} \ge \overline{ER}_{22}$$

$$X_{1340} + X_{1300} \geq \overline{ER}_{13}$$

$$\sum_{a=2}^{3}[X_{2a40} + X_{2a00}] \ge \overline{ER}_{23}$$

$$\sum_{a=2}^{3} X_{1a00} \ge \overline{ER}_{14}$$

$$\sum_{s=1}^{3} X_{2a00} \ge \overline{ER}_{24}$$

$$\sum_{s=1}^{2}\sum_{a=1}^{3}\left[v_{sa10}^{1}X_{sa10} + v_{sa13}^{1}X_{sa13} + v_{sa14}^{1}X_{sa14} + v_{sa10}^{1}W_{sa}\right] - H_1 = 0$$

$$\sum_{s=1}^{2}\sum_{a=1}^{3}\left[v_{sa20}^{1}X_{sa20} + v_{sa24}^{1}X_{sa24}\right] - H_2 = 0$$

$$\sum_{s=1}^{2}\sum_{a=1}^{3}\left[v_{sa30}^{1}X_{sa30} + v_{sa13}^{2}X_{sa13}\right] - H_3 = 0$$

```
LP OPTIMUM FOUND AT STEP     33
       OBJECTIVE FUNCTION VALUE
       1)      .1191013E+08

 VARIABLE        VALUE          REDUCED COST
    X1110          .000000        124.680100
    X1113          .000000        144.128500
    X1114          .000000        124.680100
    X1120          .000000         34.489690
    X1124          .000000         65.899490
    X1130      1032.951000           .000000
    X1140      1967.049000           .000000
    X1210          .000000         59.081680
    X1213          .000000         78.530110
    X1214          .000000         59.081680
    X1220      4383.481000           .000000
    X1224          .000000         31.409830
    X1230      1616.519000           .000000
    X1240          .000000         61.095940
    X1310      2850.115000           .000000
    X1313          .000000         19.448340
    X1314      2183.274000           .000000
    X1320      3966.611000           .000000
    X1324          .000000         31.409710
    X1330          .000000         93.475160
    X1340          .000000        176.654000
    X2110          .000000         95.882030
    X2113          .000000        109.286700
    X2114          .000000         95.881990
    X2120          .000000         29.178310
    X2124          .000000         64.927150
    X2130      8000.000000           .000000
    X2140          .000000         33.356710
    X2210          .000000         55.267910
    X2213          .000000         68.672610
    X2214          .000000         55.267940
    X2220      4000.000000           .000000
    X2224          .000000         35.748860
    X2230          .000000         45.702750
    X2240          .000000        120.629400
    X2310      1316.667000           .000000
    X2313          .000000         13.404730
    X2314      5683.333000           .000000
    X2320          .000000         65.968570
    X2324          .000000        101.717500
    X2330          .000000        177.833500
    X2340          .000000        289.138900
    X1100          .000000         37.918370
    X1200          .000000        104.389500
    X1300          .000000        218.814100
    X2100          .000000         56.088310
    X2200          .000000        153.075300
    X2300          .000000        318.281600
       H1    289667.800000           .000000
       H2    318634.600000           .000000
       H3    286771.100000           .000000
       H4    258094.000000           .000000
```

Figure 14.1. LINDO solution to the four-period profit-maximization harvest scheduling problem.

$$\sum_{s=1}^{2}\sum_{a=1}^{3}\left[v_{sa40}^{1}X_{sa40}+v_{sa14}^{2}X_{sa14}+v_{sa24}^{2}X_{sa24}\right]-H_{4}=0$$

0.9 H_p - H_{p+1} # 0 $\quad\quad$ p = 1, 2, 3
- 1.1 H_p + H_{p+1} # 0

$$\sum_{a=1}^{3}\sum_{p_1=1}^{4}\left(AgeDif_{2ap_{1}0}^{40}X_{2ap_{1}0}+\sum_{p_2=p_1+2}^{4}AgeDif_{2ap_{1}p_{2}}^{40}X_{2ap_{1}p_{2}}\right)\geq 0$$

$X_{sap_1p_2}$ $0 $\quad$ for s = 1, 2; a = 1, 2, 3; and p_1p_2 = 00, 10, 13, 14, 20, 24, 30, and 40,
H_p $ 0 $\quad$ for p = 1, 2, 3, 4; and
W_{sa} $0 $\quad$ for s = 1, 2; a = 1, 2, 3.

where,

$X_{sap_1p_2}$ = the number of acres from site class s, initial age class a, assigned to be harvested first in period p_1 and again in period p_2.
W_{sa} = the number of acres from site class s, initial age class a assigned to be cleared in period 1 and maintained as wildlife openings for the remainder of the planning horizon.
H_p = the volume harvested in decade p (in cords);
$c_{sap_1p_2}^{p}$ = the discounted net revenue (profit) from assigning one acre from site class s, initial age class a to be harvested in periods p_1 and p_2 (where $p_i = 0$ implies no i^{th} harvest); i.e., assigning an acre to the variable $X_{sap_1p_2}$, and
c_{sa}^{wp} = the discounted net revenue from assigning an acre from site class s, initial age class a to be managed as a wildlife opening; i.e., assigning an acre to the prescription W_{sa}.
A_{sa} = the total number of acres in site class s, initial age class a;
$\overline{W}$ = the minimum area that must be allocated to wildlife openings,
$\overline{ER}_{sp}$ = the minimum area that must be allocated to extended rotation areas from site class s in period p,
$v_{sap_1p_2}^{1}$ = the harvest volume per acre for the first harvest (in period p_1) from acres assigned to the variable $X_{sap_1p_2}$, and
$v_{sap_1p_2}^{2}$ = the harvest volume per acre for the second harvest (in period p_2) from acres assigned to the variable $X_{sap_1p_2}$.
Age_{sap}^{20} = the age in year 20 of acres assigned to the variable X_{sap};
$\overline{Age}^{20}$ = the target (minimum) average age of the forest in year 20; and

$$AgeDif_{sa00}^{40}=Age_{sa00}^{40}-\overline{Age}_{s}^{40}$$

Specific Formulation

$$\begin{aligned}
\text{Max } Z = & - 53.75404\, X_{1110} - 3.863499\, X_{1113} + 42.79726\, X_{1114} + 73.85018\, X_{1120} \\
& + 107.5544\, X_{1124} + 142.9195\, X_{1130} + 165.683\, X_{1140} + 109.3163\, X_{1210} \\
& + 159.2068\, X_{1213} + 205.8676\, X_{1214} + 211.5558\, X_{1220} + 245.26\, X_{1224} \\
& + 245.2514\, X_{1230} + 203.3913\, X_{1240} + 313.1542\, X_{1310} + 363.0448\, X_{1313} \\
& + 409.7055\, X_{1314} + 363.0319\, X_{1320} + 396.7362\, X_{1324} + 301.0688\, X_{1330} \\
& + 234.8148\, X_{1340} + 7.397344\, X_{2110} + 94.49947\, X_{2113} + 147.9416\, X_{2114} \\
& + 128.9324\, X_{2120} + 187.7755\, X_{2124} + 208.0398\, X_{2130} + 209.676\, X_{2140} \\
& + 190.8515\, X_{2210} + 277.9536\, X_{2213} + 331.3957\, X_{2214} + 307.9497\, X_{2220} \\
& + 366.7928\, X_{2224} + 310.3716\, X_{2230} + 266.2383\, X_{2240} + 455.8408\, X_{2310} \\
& + 542.9429\, X_{2313} + 596.385\, X_{2314} + 459.4259\, X_{2320} + 518.2689\, X_{2324} \\
& + 394.0977\, X_{2330} + 310.2312\, X_{2340} - 124.971\, W_{11} + 38.0995\, W_{12} \\
& + 241.9374\, W_{13} - 63.8195\, W_{21} + 119.6347\, W_{22} + 384.624\, W_{23}
\end{aligned}$$

subject to

$$\begin{aligned}
& X_{1110} + X_{1113} + X_{1114} + X_{1120} + X_{1124} + X_{1130} + X_{1140} + X_{1100} + W_{11} <= 2{,}760 \\
& X_{1210} + X_{1213} + X_{1214} + X_{1220} + X_{1224} + X_{1230} + X_{1240} + X_{1200} + W_{12} <= 5{,}520 \\
& X_{1310} + X_{1313} + X_{1314} + X_{1320} + X_{1324} + X_{1330} + X_{1340} + X_{1300} + W_{13} <= 8{,}280 \\
& X_{2110} + X_{2113} + X_{2114} + X_{2120} + X_{2124} + X_{2130} + X_{2140} + X_{2100} + W_{21} <= 7{,}360 \\
& X_{2210} + X_{2213} + X_{2214} + X_{2220} + X_{2224} + X_{2230} + X_{2240} + X_{2200} + W_{22} <= 3{,}680 \\
& X_{2310} + X_{2313} + X_{2314} + X_{2320} + X_{2324} + X_{2330} + X_{2340} + X_{2300} + W_{23} <= 6{,}440 \\
& W_{11} + W_{12} + W_{13} + W_{21} + W_{22} + W_{23} >= 500 \\
& X_{2330} + X_{2340} + X_{2300} >= 1{,}500 \\
& X_{1340} + X_{1300} >= 1{,}500 \\
& X_{2240} + X_{2340} + X_{2200} + X_{2300} >= 1{,}500 \\
& X_{1200} + X_{1300} >= 1{,}500 \\
& X_{2100} + X_{2200} + X_{2300} >= 1{,}500 \\
& 2\, X_{1110} + 2\, X_{1113} + 2\, X_{1114} + 10\, X_{1210} + 10\, X_{1213} + 10\, X_{1214} + 20\, X_{1310} + 20\, X_{1313} \\
& \quad + 20\, X_{1314} + 5\, X_{2110} + 5\, X_{2113} + 5\, X_{2114} + 14\, X_{2210} + 14\, X_{2213} + 14\, X_{2214} + 27\, X_{2310} \\
& \quad + 27\, X_{2313} + 27\, X_{2314} + 2\, W_{11} + 10\, W_{12} + 20\, W_{13} + 5\, W_{21} + 14\, W_{22} + 27\, W_{23} - H_1 = 0 \\
& 10\, X_{1120} + 10\, X_{1124} + 20\, X_{1220} + 20\, X_{1224} + 31\, X_{1320} + 31\, X_{1324} + 14\, X_{2120} + 14\, X_{2124} \\
& \quad + 27\, X_{2220} + 27\, X_{2224} + 38\, X_{2320} + 38\, X_{2324} - H_2 = 0 \\
& 10\, X_{1113} + 20\, X_{1130} + 10\, X_{1213} + 31\, X_{1230} + 10\, X_{1313} + 37\, X_{1330} + 14\, X_{2113} + 27\, X_{2130} \\
& \quad + 14\, X_{2213} + 38\, X_{2230} + 14\, X_{2313} + 47\, X_{2330} - H_3 = 0 \\
& 20\, X_{1114} + 10\, X_{1124} + 31\, X_{1140} + 20\, X_{1214} + 10\, X_{1224} + 37\, X_{1240} + 20\, X_{1314} + 10\, X_{1324} \\
& \quad + 42\, X_{1340} + 27\, X_{2114} + 14\, X_{2124} + 38\, X_{2140} + 27\, X_{2214} + 14\, X_{2224} + 47\, X_{2240} + 27\, X_{2314} \\
& \quad + 14\, X_{2324} + 54\, X_{2340} - H_4 = 0 \\
& 0.9\, H_1 - H_2 <= 0 \\
& - 1.1\, H_1 + H_2 <= 0 \\
& 0.9\, H_2 - H_3 <= 0 \\
& - 1.1\, H_2 + H_3 <= 0 \\
& 0.9\, H_3 - H_4 <= 0 \\
& - 1.1\, H_3 + H_4 <= 0
\end{aligned}$$

$$14.5\,X_{1110} - 5.5\,X_{1113} - 15.5\,X_{1114} + 4.5\,X_{1120} - 15.5\,X_{1124} - 5.5\,X_{1130} - 15.5\,X_{1140} + 14.5\,X_{1210} - 5.5\,X_{1213} - 15.5\,X_{1214} + 4.5\,X_{1220} - 15.5\,X_{1224} - 5.5\,X_{1230} - 15.5\,X_{1240} + 14.5\,X_{1310} - 5.5\,X_{1313} - 15.5\,X_{1314} + 4.5\,X_{1320} - 15.5\,X_{1324} - 5.5\,X_{1330} - 15.5\,X_{1340} + 24.5\,X_{1100} >= 0$$

$$19.5\,X_{2110} - 0.5\,X_{2113} - 10.5\,X_{2114} + 9.5\,X_{2120} - 10.5\,X_{2124} - 0.5\,X_{2130} - 10.5\,X_{2140} + 19.5\,X_{2210} - 0.5\,X_{2213} - 10.5\,X_{2214} + 9.5\,X_{2220} - 10.5\,X_{2224} - 0.5\,X_{2230} - 10.5\,X_{2240} + 19.5\,X_{2310} - 0.5\,X_{2313} - 10.5\,X_{2314} + 9.5\,X_{2320} - 10.5\,X_{2324} - 0.5\,X_{2330} - 10.5\,X_{2340} >= 0$$

$X_{sap_1p_2}$ \$0 for s = 1, 2; a = 1, 2, 3; and p_1p_2 = 00, 10, 13, 14, 20, 24, 30, and 40,
H_p \$ 0 for p = 1, 2, 3, 4; and
W_{sa} \$0 for s = 1, 2; a = 1, 2, 3.

5. The Solution

The LINDO output for the modified four-period profit-maximization model is presented in Figures 15.1 and 15.2. Note the new wildlife variables reported in the output. The 500 acres of wildlife openings were all taken out of site class I, initial age class 3. These acres were allocated from site class I because it is cheaper to take the poorer quality acres out of production than to take the better site class II acres out. Note the reduced cost value for the variable W23. This value indicates that the opportunity cost of allocating acres from site class II to wildlife openings (versus using acres from site class I) is \$46.43 per acre. Acres are allocated from the oldest age class because more volume will be obtained by harvesting these older acres. Consider the reduced cost value for the variable W12. This value indicates that the opportunity cost of clearing these younger acres for wildlife openings (as opposed to clearing older acres) would be \$59.10 per acre.

Figure 15.2 shows the slack and surplus values and the dual prices for each constraint. In order to make sense of this output, it is necessary to identify the constraint corresponding to each row. Rows 2 through 6 correspond to the area constraints. None of these constraints show any slack. This means that all of the acres from each analysis area have been allocated to one of the nine prescriptions considered. The dual prices for these constraints indicate the value of having one more acre in the corresponding analysis area. Thus, an additional acre in site class II, initial age class 1 would be worth \$200.23 in this scenario. Row 8 corresponds to the wildlife opening constraint. The dual price associated with that constraint indicates that an additional acre of wildlife openings will cost \$120.02. Rows 9 through 13 correspond to the extended rotation constraints. Their dual prices indicate the cost of increasing the corresponding extended rotation age target by an additional acre. Rows 14 to 17 correspond to the harvest volume accounting constraints. The interpretation of the dual prices corresponding to these constraints is not important, as these constraints do not actually constrain the solution to the problem. Rows 18 through 23 correspond to the harvest fluctuation constraints. Most of these constraints have positive slack values, indicating that they are not binding. (Of course, at most, only half of them could possibly be binding.) The first harvest fluctuation constraint that is binding limits the increase in the harvest level between periods 1 and 2 to no more than 10 percent. The second binding harvest fluctuation constraint limits the decrease between periods 3 and 4 to no more than 10 percent. The dual prices on these constraints indicate the savings that could be realized by shifting one unit of volume from the period with the least volume harvested to the period with the most harvested volume. Thus, $0.31 could be saved by shifting one cord of production from period 1 to period 2. Rows 24 and 25 correspond to the ending age constraints for sites I and II, respectively. The units of these constraints are years times acres. Thus, the dual prices corresponding to these constraints indicate the potential gain from allowing one acre to be one year older.

Tables 15.3 through 15.8 organize and present the results for this example problem. It is interesting to compare these results with the results from the model presented at the end of Chapter 14, since these models are both based on the same forest and the same economic data. Furthermore, both are profit maximization models with four periods. The difference between the two models, of course, is the added constraints and variables implementing extended rotations, wildlife openings, and SMZs.

Table 15.3 shows the harvest schedule for this model. The primary difference between this harvest schedule and the one in Table 15.4 at the end of Chapter 14 is that fewer acres are scheduled for harvest – about 20% fewer acres in the first two periods. Note that the 1,500 acres set aside as extended rotation areas in site class 2 for the first three periods are harvested in the fourth period.

Table 15.4 summarizes the acres harvested, the volume harvested and the costs and revenues over the four periods for this model. As mentioned earlier, about 20% less area is harvested in the first two periods with this model, as compared with the model in Chapter 14. The harvest volume is about 22% lower for the first two periods with this model. In this model, the harvest continues to rise between periods 2 and 3, whereas the harvest declined between periods 2 and 3 in the model in Chapter 14. Furthermore, although the harvest drops in this model between periods 3 and 4, the drop is not as sharp as it was with the model in Chapter 14. The harvest in the fourth period in the constrained model of this chapter is only about 12% lower than the fourth period harvest volume in the less constrained model from Chapter 14. Revenues and costs are, of course, proportionately lower in this model.

```
 LP OPTIMUM FOUND AT STEP     52

        OBJECTIVE FUNCTION VALUE

        1)      9610140.

  VARIABLE        VALUE          REDUCED COST
     X1110          .000000        124.713400
     X1113          .000000        144.122200
     X1114          .000000        124.713400
     X1120          .000000         34.499190
     X1124          .000000         65.898600
     X1130      2760.000000           .000000
     X1140          .000000           .071927
     X1210          .000000         59.095370
     X1213          .000000         78.504200
     X1214          .000000         59.095370
     X1220      4263.737000           .000000
     X1224          .000000         31.399430
     X1230      1256.263000           .000000
     X1240          .000000         61.184390
     X1310      1942.135000           .000000
     X1313          .000000         19.408740
     X1314      2238.936000           .000000
     X1320      2098.929000           .000000
     X1324          .000000         31.399310
     X1330          .000000         93.484400
     X1340          .000000         28.656750
     X2110          .000000         95.921940
     X2113          .000000        109.273300
     X2114          .000000         95.921900
     X2120          .000000         29.190770
     X2124          .000000         64.927130
     X2130      5860.000000           .000000
     X2140          .000000         33.441800
     X2210          .000000         55.285300
     X2213          .000000         68.636630
     X2214          .000000         55.285330
     X2220      3680.000000           .000000
     X2224          .000000         35.736370
     X2230          .000000         45.701980
     X2240          .000000          9.303281
     X2310      1186.333000           .000000
     X2313          .000000         13.351370
     X2314      3753.667000           .000000
     X2320          .000000         65.971380
     X2324          .000000        101.707800
     X2330          .000000           .000000
     X2340      1500.000000           .000000
       W11          .000000        124.713600
       W12          .000000         59.095360
       W13       500.000000           .000000
       W21          .000000        142.350100
       W22          .000000        101.713400
       W23          .000000         46.428120
     X1100          .000000         37.976780
     X1200          .000000           .000000
     X1300      1500.000000           .000000
     X2100      1500.000000           .000000
     X2200          .000000         34.411240
     X2300          .000000         70.646630
        H1    227001.400000           .000000
        H2    249701.600000           .000000
        H3    252364.100000           .000000
        H4    227127.700000           .000000
```

Figure 15.1. LINDO solution to the modified four-period profit-maximization model.

```
ROW   SLACK OR SURPLUS     DUAL PRICES
 2)          .000000       120.433800
 3)          .000000       220.579600
 4)          .000000       368.688900
 5)          .000000       200.231900
 6)          .000000       346.079500
 7)          .000000       560.160300
 8)          .000000      -120.017900
 9)          .000000      -177.845400
10)          .000000      -148.109300
11)          .000000      -111.436400
12)          .000000      -220.579600
13)          .000000      -200.231900
14)          .000000         -.336678
15)          .000000          .306071
16)          .000000          .198744
17)          .000000         -.220827
18)     45400.290000          .000000
19)          .000000          .306071
20)     27632.740000          .000000
21)     22307.580000          .000000
22)          .000000          .220827
23)     50472.840000          .000000
24)          .000000        -3.365595
25)          .000000        -4.883552
```

Figure 15.2. LINDO slack, surplus, and dual price report for the example problem.

Table 15.3. Harvest Schedule — Acres

Planning Period	Age at Harvest	Site 1	Site 2	Total
1	30	4,681.1	4,940.0	9,621.1
	Total	4,681.1	4,940.0	9,621.1
2	30	4,263.7	3,680.0	7,943.7
	40	2,098.9	0.0	2,098.9
	Total	6,362.7	3,680.0	10,042.7
3	30	2,760.0	5,860.0	8,620.0
	40	1,256.3	0.0	1,256.3
	Total	4,016.3	5,860.0	9,876.3
4	30	2,238.9	3,753.7	5,992.6
	60	0.0	1,500.0	1,500.0
	Total	2,238.9	5,253.7	7,492.6

Table 15.4. Summary table.

Item	Period 1	Period 2	Period 3	Period 4
Acres	9,621	10,043	9,876	7,793
Volume	227,001	249,702	252,364	227,128
Planting	912,107	1,004,267	987,626	749,260
Timber Sales	189,716	200,580	198,617	157,815
Wildlife Openings	22,259	49,341	49,341	49,341
Revenue	5,675,035	6,242,540	6,309,103	5,678,193
Net Revenue	4,550,952	4,988,352	5,073,519	4,721,777
Disc. NR	3,740,551	2,769,855	1,903,162	1,196,571

Tables 15.5 through 15.8 show the age class distribution of the forest over time. The overall areas have been reduced to reflect the area set aside in SMZs and wildlife openings. The fact that the model is setting aside areas for extended rotations is clearly evident from these tables. Also, note that the age-class distribution of the forest is only moderately regulated by the end of the planning horizon. However, there are no obvious imbalances in the projected age-class distribution that we should be concerned about.

Table 15.5. Age-class distribution at the end of period 1.

Age Classes	Acres by site class		
	Site I	Site II	Total
0 to 10	4,181.1	4,940.0	9,121.1
11 to 20	2,760.0	7,360.0	10,120.0
21 to 30	5,520.0	3,680.0	9,200.0
31 to 40	3,598.9	1,500.0	5,098.9
Total	16,060.0	17,480.0	33,540.0

Table 15.6. Age-class distribution at the end of period 2.

Age Classes	Acres by site class		
	Site I	Site II	Total
0 to 10	6,362.7	3,680.0	10,042.7
11 to 20	4,181.1	4,940.0	9,121.1
21 to 30	2,760.0	7,360.0	10,120.0
31 to 40	1,256.3	0.0	1,256.3
41 to 50	1,500.0	1,500.0	3,000.0
Total	16,060.0	17,480.0	33,540.0

Table 15.7. Age-class distribution at the end of period 3.

Age Classes	Acres by site class		
	Site I	Site II	Total
0 to 10	4,016.3	5,860.0	9,876.3
11 to 20	6,362.7	3,680.0	10,042.7
21 to 30	4,181.1	4,940.0	9,121.1
31 to 40	0.0	1,500.0	1,500.0
41 to 50	0.0	0.0	0.0
51 to 60	1,500.0	1,500.0	3,000.0
Total	16,060.0	17,480.0	33,540.0

Table 15.8. Age-class distribution at the end of period 4.

Age Classes	Acres by site class		
	Site I	Site II	Total
0 to 10	2,238.9	5,253.7	7,492.6
11 to 20	4,016.3	5,860.0	9,876.3
21 to 30	6,362.7	3,680.0	10,042.7
31 to 40	1,942.1	1,186.3	3,128.5
41 to 50	0.0	1,500.0	1,500.0
51 to 60	0.0	0.0	0.0
31 to 40	1,500.0	0.0	1,500.0
Total	16,060.0	17,480.0	33,540.0