CHAPTER 9: UNEVEN-AGED MANAGEMENT

Up to now, we have dealt only with even-aged management decisions. A regeneration harvest of an even-aged stand often means a clearcut, but it may also refer to a seed tree cut, a final shelterwood harvest, an overstory removal with residuals, or a variety of other types of harvests. By definition, there is a point in the life of an even-aged stand where the majority of the existing stand of trees is harvested in order to establish a new stand. The age where this occurs is called the rotation age. In even-aged systems, trees from an existing stand may be left uncut at the end of the rotation, as in a seed tree harvest or with reserve trees, but these are a relatively small portion (generally less than one fourth) of the original stand. In any case, the defining characteristic of an even-aged stand is that the majority of the trees are approximately the same size and age, and regeneration of a new stand happens more-or-less all at once. Even-aged management is unpopular with many people because of the total or near-total harvest of the stand that is required. For many people, clearcutting is deforestation.

Uneven-aged management is the primary alternative to even-aged management. Uneven-aged management cannot replace even-aged management in all situations, but it often is a viable alternative. It has been practiced in North America with many shade tolerant and even moderately shade intolerant hardwoods, and it has been successfully applied in managing loblolly, shortleaf and longleaf pines in the southeastern US. While uneven-aged management may seem very different from even-aged management, they actually have much in common. In fact, the two can be viewed as the same thing, just practiced at different scales. At the landscape level, a 1,000-acre forest composed of many 20 to 40-acre even-aged stands of different ages can be viewed as uneven-aged. If you imagine that the sizes of the stands are reduced in size, say to 5 to 10 acres, we might refer to the silvicultural management regime as "patch" clearcutting. As the sizes of these patches are reduced even further, say to 1 to 5 acres, we would eventually call the management regime uneven-aged management based on group selection. If the patches are the size of a single tree, we would call it uneven-aged management with individual tree selection. This chapter conceptualizes uneven-aged stands as having all age-classes mixed together as much as possible. If group selection is used, the larger the patches, the more the silviculture will resemble even-aged management and the less the concepts discussed in this chapter will apply. However, one of the themes of this chapter is that the concepts of uneven-aged management are not that different from even-aged management. If you see the common principles, then you can apply those principles to any kind of management on the continuum from large even-aged harvests to individual-tree selection.

Uneven-aged stands have, by definition, at least three age groups, or cohorts. In the theoretical model of an even-aged stand that we will work with in this chapter, all age groups (up to some maximum) are represented in an uneven-aged stand. In practice, however, uneven-aged stands often contain three or more distinct cohorts of trees that were established after disturbances removed parts of the stand and created large enough openings to allow tree regeneration to occur or to give already-established seedlings enough light and space to grow. Since only part of the

stand is removed in each harvest the volumes removed in each harvest are lower, but this is offset by harvesting more often. Thus, uneven-aged management is characterized by frequent harvests—typically every 10 to 30 years. The frequency of these harvests is one of the key decisions in uneven-aged management. While this decision is superficially similar to the rotation decision in even-aged management, it is really quite different. Additionally, in contrast to even-aged stands, harvesting decisions in uneven-aged stands are relatively complex. How much of the stand should be harvested? Which trees should be removed? The number of possible answers to these questions is far greater than the number of possible rotation ages for an even-aged stand. What's more, the answers to these questions also depend on how often you wish to harvest the stand. Like thinning decisions in even-aged management, uneven-aged management decisions tend to be interrelated.

This chapter considers the primary decisions that need to be made in uneven-aged management. Often the interrelated nature of these decisions will be ignored in order to keep the discussion simple. However you should always keep in mind that few decisions can be made independently in uneven-aged management.

1. <u>What Is Uneven-aged Management?</u>

It is useful to begin by clearly defining uneven-aged management.

> The defining characteristic of an uneven-aged stand is that it has three or more age classes, or cohorts, of trees at all times. Uneven-aged management is the process of making decisions to select management actions that best achieve ownership objectives while maintaining an uneven-aged structure.

Maintaining an uneven-aged structure should seldom be a management objective in itself. Rather, this structure is maintained because it is believed to be the best way to achieve some other ownership objective. These objectives might include maintaining constant forest cover, earning more frequent income from the stand, providing a specific type of wildlife habitat, maintaining a specific plant community, or demonstrating or studying uneven-aged management techniques. In order to maintain an uneven-aged structure, harvesting happens either through individual tree selection or by group selection. Individual tree selection creates relatively small gaps, so it is more applicable when the desired species are shade tolerant. Group selection would generally be applied when the desired species are less shade intolerant and need larger openings to successfully reproduce.

A key way of describing the status of an uneven-aged stand is by its diameter class distribution. The diameter class distribution is a histogram or table that shows the number of trees in the stand by diameter class (see Figure 9.1) and possibly species. Theoretically, the age-class distribution could be used, but diameter is highly correlated with age and is much easier to measure. In addition, diameter is more closely related to value than is age. The primary silvicultural objective of uneven-aged management is generally to achieve and maintain some target diameter class distribution. The target diameter class distribution should have the property that the stand

can be returned to the same diameter class distribution following each harvest. This is accomplished by removing the surplus trees that have accumulated in each diameter class since the last harvest. The regeneration of new trees must be sufficient to maintain adequate numbers of trees in each diameter class.

2. <u>Advantages and Disadvantages of Uneven-aged Management</u>

Uneven-aged management can be a useful technique for achieving particular objectives. At the same time, uneven-aged management will not be the best tool in other situations. Choosing to use or not use uneven-aged management to accomplish one's objectives is a management decision. It should be made with full consideration of the pros and cons of applying the technique.

Advantages of Uneven-aged Management

- Uneven-aged management tends to mimic the small-patch (gap-phase) disturbance patterns of natural late-successional forests.
- If the desired species mix for a stand includes shade-tolerant tree species, such as sugar maple, hemlock, or beech, uneven-aged management will generally be the best way to maintain that species mix.
- Uneven-aged stands have a diverse structure, with small, medium and large trees providing a multi-layered canopy. Similarly, opening sizes can be varied to create a diverse horizontal structure. This forest structure provides needed or preferred habitat for many plant and wildlife species.
- Many people do not like to see clearcuts. With uneven-aged management the stand is never clearcut. Thus, an uneven-aged management system may more socially or aesthetically beneficial than an even-aged management system.
- Uneven-aged stands provide continuous cover on a site, reducing problems with erosion and excessive run-off after heavy rains.
- Uneven-aged management is more compatible with many owners' financial constraints, as it provides more frequent cash flow, and, because it typically relies on natural regeneration, it has relatively low investment requirements.
- In theory, because the site is always occupied by trees, uneven-aged management could provide more growth than even-aged management. In practice, however, the densities required to maintain adequate regeneration are relatively low, so growth rates are typically lower with uneven-aged management than with even-aged management.

Disadvantages of Uneven-aged Management

- A key disadvantage of uneven-aged management is that it does not work well when shade-intolerant species are desired. Many of the most valuable timber species – including most pines, douglas-fir, aspen, black cherry, hickories, and, to a lesser degree, oaks – are relatively shade-intolerant.

- Many wildlife species prefer or require habitat provided by even-aged stands. The openings created by clearcuts and other even-aged harvesting methods and the more mature stages of even-aged stands are particularly important.
- Uneven-aged management is complex. A uneven-aged stand structure often is difficult to achieve and maintain. Some management costs are higher because more detailed information about the stand is required and because more frequent stand entries are involved.
- While not really a disadvantage of uneven-aged management, per se, because uneven-aged management requires relatively frequent (e.g., every 20 to 30 years, instead of every 60 to 100 years) pulses of regeneration, it can be difficult to practice in areas where heavy deer browsing limits the establishment of adequate regeneration of desirable species. This is particularly relevant in much of the eastern United States.
- While clearcuts admittedly have their own disadvantages, the frequent, lighter harvests required for uneven-aged management also have numerous problems. First, since less volume is removed per unit of area, average woods hauling distances and logging costs will be greater. Furthermore, more area has to be disturbed in order to produce an equivalent amount of wood. Third, it is difficult to avoid some damage to the residual stand. Finally, more frequent entries into the stand increase the potential for site degradation due to soil compaction and rutting.

Perhaps the most significant problem with uneven-aged management is not really a disadvantage of the technique itself; rather, it is due to the application of a more questionable management technique that is often practiced in the name of uneven-aged management. *High-grading* is the practice of removing only the most valuable trees from a stand, often leaving behind mostly damaged, diseased, or genetically inferior trees with little potential for future growth. Loggers and other timber buyers often dupe unsuspecting landowners into allowing their forestland to be high-graded by convincing them that they are doing "selection" harvesting.

3. The Key Decision Parameters in Uneven-aged Management

The primary decisions in even-aged management are 1) when to harvest the existing stand and regenerate a new stand (the rotation decision), 2) how to regenerate the new stand (the stand establishment decision), 3) thinning decisions (density management), and 4) whether, when, and how to conduct miscellaneous intermediate treatments such as competition control, prescribed burning, fertilization, etc. The three basic decision parameters in uneven-aged management are quite different. In particular, we will consider 1) selecting the target diameter class distribution, 2) individual tree harvesting decisions, and 3) choosing a cutting cycle. Due to space and time constraints, this chapter is primarily concerned with management decisions in situations where you already have an existing uneven-aged stand or something close to it. Little consideration will be given to how you might go about converting an existing even-aged stand to an uneven-aged stand.

Perhaps the most critical decision in uneven-aged management is specifying the target diameter class distribution. This distribution describes the desired number of trees in each successive diameter class. For simplicity, the target diameter class distribution is often specified in terms of a "Q factor," the basal area, and the maximum diameter. The *Q factor* refers to the ratio of the number of trees in adjacent diameter classes and determines the relative balance between smaller trees and larger trees. In order for the target diameter class distribution to be sustainable, there must always be a surplus of trees in each diameter class at the end of the cutting cycle so that this surplus can be harvested, leaving the stand back at the target diameter class distribution. This means that the ingrowth of smaller trees into a diameter class must equal or exceed the outgrowth and mortality from the diameter class. This would allow the stand to develop between cutting cycles without creating any gaps in the diameter distribution. This generally means that there should be more trees in smaller diameter classes than in larger diameter classes. The Q factor determines the relative balance between the number of trees in the smaller diameter classes compared with the number in larger classes. The stand density – generally measured by the stand basal area – is important because it influences stand volume and growth potential. In addition, the stand density also influences the amount of light reaching the forest floor and, hence, the amount of regeneration and the rate at which trees in the understory will grow. The maximum diameter is determined by the size at which trees are considered mature and are no longer growing fast enough to justify their use of growing space in the stand. While the concept of a rotation does not strictly apply to an uneven-aged stand, as we shall see, the decision of how long to grow an individual tree (or group of trees in the case of group selection) in an uneven-aged stand is fundamentally the same as the rotation decision in an even-aged stand. The length of time it takes for a tree to reach the maximum diameter class can be considered the rotation for an uneven-aged stand.

While the cutting cycle is superficially similar to the rotation decision, from a management point of view it is fundamentally quite different. In fact, if one views the typical length of time it takes a tree to reach the maximum diameter as the stand "rotation," then the ratio of this rotation over the cutting cycle determines the number of age-classes that will be found in the stand. For example, if it takes 80 years for a tree to reach the maximum diameter class and the cutting cycle is 20 years, then there should be four age classes in the stand. Thus, the cutting cycle cannot be longer than one third of the "rotation" or the stand will not be uneven-aged at all. Other than this general rule, the cutting cycle should be selected to avoid cutting too frequently, which results in harvests with too little volume and therefore higher harvest costs, and cutting too infrequently, which allows the stand to deviate too far from the target diameter class distribution. The choice of the target diameter class distribution will also depend on the choice of the cutting cycle. With shorter cutting cycles, the stand density can be kept very close to an optimum for growth and regeneration. With longer cutting cycles the stand density at the beginning of the cutting cycle will have to be relatively low in order to allow more time before the stand becomes too dense for regeneration to occur. Thus, with longer cutting cycles the stand will tend to be too sparse at the beginning of the cutting cycle and too dense at the end of the cutting cycle.
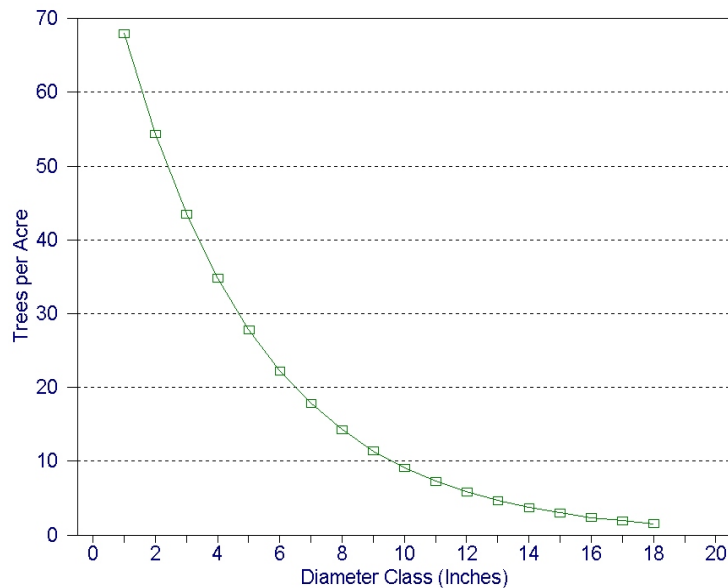
Figure 9.1. The "ideal" shape of the diameter distribution for an
uneven-aged stand.

Once the target diameter class distribution and cutting cycle have been determined, many decisions still remain regarding which trees to cut and which to leave. Within those diameter classes with surplus stocking, individual trees should be selected for harvest based on their potential to increase in value over the next cutting cycle. Undesirable species or trees that have defects or insect or disease problems should be removed first. Next, trees that have the least potential to produce quality sawlogs should be removed, as harvests of sawtimber-sized trees provide the bulk of the revenue in uneven-aged management. Vigorous trees will have healthy crowns, but trees with too many large limbs will be less valuable as they will produce knotty wood.

4. <u>The "Ideal" Diameter Class Distribution</u>

As discussed earlier, the target diameter distribution should be designed so that after one cutting cycle of growth the excess in each diameter class can be removed to return the stand to the target diameter class distribution. This means that enough trees must grow into each diameter class over the cutting cycle to at least replace the trees that leave the diameter class as a result of either growth or mortality. Excess trees in a diameter class can be removed by harvesting. In order to provide ample trees growing into any given diameter class, there must generally be more trees in the diameter classes immediately below that diameter class. This requirement suggests that an ideal diameter class distribution will have an inverse J-shape, such as that depicted in Figure 9.1.

A simplifying assumption that is widely used in uneven-aged management is that the ratio of the number of trees in any diameter class to the number of trees in the next larger diameter class should be constant. This ratio is called the "Q factor," where Q stands for "quotient." If *n(d)* is the number of trees in diameter class *d*, then the Q factor can be expressed as follows:

$$Q = \frac{n(d)}{n(d+1)}$$

To simplify our consideration of the target diameter class distribution, we will assume that this ratio must be constant for any pair of adjacent diameter classes. It is not necessarily the case that it is actually desirable – certainly no one has ever shown that it is optimal – to have a constant Q factor for all diameter classes in an uneven-aged stand. Rather, it is a simplifying assumption, made for convenience, that works reasonably well.

The Negative Exponential Diameter Class Distribution

It turns out that the negative exponential function is a relatively simple mathematical function that can be used to represent a diameter class distribution with a constant Q factor for all diameter classes. A negative exponential diameter class distribution function can be written as follows:

$$n(d) = ke^{-ad}$$

where *k* and *a* are parameters, and *e* is the base of the natural logarithm.

To confirm that the negative exponential function really does give a constant ratio between the number of trees in successive diameter classes, start with the definition of Q and plug in the negative exponential distribution function:

$$Q = \frac{n(d)}{n(d+1)} = \frac{ke^{-ad}}{ke^{-a(d+1)}}$$

Now, using the fact that $e^{a+b} = e^a \cdot e^b$ (check your calculus book if you don't remember this) we have:

$$Q = \frac{n(d)}{n(d+1)} = \frac{ke^{-ad}}{ke^{-a(d+1)}} = \frac{e^{-ad}}{e^{-a(d)}e^{-a(1)}} = \frac{1}{e^{-a}} = e^a$$

Since e^a is a constant, this proves that the ratio of any two diameter classes in the negative exponential distribution function is constant. The proof that the negative exponential function gives a diameter distribution with a constant Q factor also shows how to determine the Q factor for a negative exponential diameter distribution function, or, conversely, how to find the value of the parameter *a* corresponding to a particular value of Q. That is,

$$Q = e^a \quad \Leftrightarrow \quad a = \ln(Q)$$

This relationship allows you to identify the Q factor of any negative exponential diameter class distribution function. Conversely, if you know what Q factor you want you can easily solve for the *a* parameter in the negative exponential diameter class distribution function by taking the natural log of the desired Q factor.

> **Example:** Calculating the Q factor from *a*.
>
> Determine the Q factor for the following diameter distribution function:
>
> $$n(d) = 65e^{-0.1823d}$$
>
> **Answer:** $Q = e^{0.1823} = 1.20$

> **Example:** Calculating *a* from the Q factor.
>
> What value of the *a* parameter gives a negative exponential diameter distribution function with a Q factor of 1.3?
>
> **Answer:** $a = \ln(Q) = \ln(1.3) = 0.26236$

Note that this relationship assumes that you are using 1-inch diameter classes. If you want to use 2-inch diameter classes, it is useful to know that the Q factor for 2-inch diameter classes is the square of the Q factor for the same distribution with 1-inch classes.

> **Example:** Converting Q factors for different diameter-class widths.
>
> If the Q factor with 1-inch diameter classes is 1.25, what is the equivalent Q factor for 2-inch diameter classes?
>
> **Answer:** $Q_{2"} = [Q_{1"}]^2 = 1.25^2 = 1.56$

Knowing the value of the *a* parameter in the negative exponential diameter distribution function is equivalent to knowing the Q factor. The other parameter in the negative exponential function is *k*. How should you interpret the *k* parameter? The *k* parameter gives the point where the diameter class distribution crosses the y-axis; i.e., the *k* parameter is the value of the function for *d* = 0 (dbh = 0):

$$n(0) = ke^{-a0} = ke^0 = k$$

Thus, *k* is the intercept of the function. (Similarly, -*a* can be interpreted as the slope of the function.) The parameter *k* can be interpreted as a measure of the amount of regeneration that is needed at any point in time in order to maintain the diameter class distribution.

Once you have specific values of *k* and *a*, you have completely identified a specific negative exponential function. However, for a diameter class distribution, one additional piece of information is required: the maximum diameter class. As discussed earlier, the choice of the maximum diameter class is analogous to determining the optimal rotation for an individual tree.

This is discussed in more detail in the next section. For now, we will assume that the maximum diameter class has already been determined.

Because the negative exponential diameter class distribution function has only two parameters (not including the maximum diameter class), only two pieces of information about the diameter distribution are needed to identify the specific form of the function. For example, this could consist of knowing Q and the number of trees in the initial diameter class (d=1), or knowing the number of trees in two diameter classes. The following examples demonstrate how to identify the values of k and a – and thus identify the specific form of the negative exponential function – from any two pieces of information about the diameter class distribution (other than the maximum diameter class).

Example: Determining the parameters of the negative exponential from Q and $n(1)$

Determine the specific form of the negative exponential function with a Q factor of 1.1 and 180 trees in the 1-inch diameter class ($n(1)$ =180).

Answer: since Q is known, a can be determined using the relationship $a = \ln(Q)$:

$$a = \ln (Q) = \ln (1.1) = 0.09531$$

Now, the information that $n(1) = 180$ can be substituted into the negative exponential function with this value of a:

$$n(1) = 180 = ke^{-0.09531(1)}$$

This relationship can then be solved for k:

$$k = 180\,e^{0.09531} = 180 \times 1.1 = 198$$

Now, the specific form of the diameter distribution function is known:

$$n(d) = 198\,e^{-0.09531(d)}$$

Note that in solving for k in the example, we ended up multiplying the number of trees in the 1-inch class by 1.1 – the value of Q. This could have been anticipated by recognizing that Q gives the ratio of the number of trees in one diameter class over the number of trees in the next diameter class. Thus,

$$Q = \frac{n(0)}{n(1)} = \frac{k}{n(1)}$$

Solving this for k, we get $k = Q\ n(1)$.

Example: Determining the parameters of the negative exponential from two points.

Determine the specific form of the negative exponential function that gives 70 trees in the 3-inch diameter class (i.e., *n(3)* = 70) and 31 trees in the 6-inch diameter class (i.e., *n(6)* = 31).

Answer: this type of problem is more difficult than the previous example because it is not obvious where to start. To do this problem, you have to recognize that the relationship between succeeding diameter classes is determined by Q and use the fact that *n(4)* is determined by *n(3)*, and *n(5)* is, in turn, determined by *n(4)*, and so on. Consider the following three equations:

$$n(3) = Q \times n(4)$$

$$n(4) = Q \times n(5)$$

$$n(5) = Q \times n(6)$$

These equations can be combined recursively to give *n(6)* as a function of *n(3)* and Q. We can then solve that relationship for Q as a function of *n(3)* and *n(6)*. Combining the first and second equations gives:

$$n(3) = Q \cdot n(4) = Q[Q \cdot n(5)] = Q^2 n(5)$$

Now, plugging in the third equation gives:

$$n(3) = Q^2 n(5) = Q^2 [Q \cdot n(6)] = Q^3 n(6)$$

Solving for Q,

$$Q = \sqrt[3]{\frac{n(3)}{n(6)}}$$

Now, the specific value of Q for this example can be identified:

$$Q = \sqrt[3]{\frac{70}{31}} = 1.3119$$

Next, determine the value of *a*:

$$a = \ln(Q) = \ln(1.3119) = 0.2715$$

Plug the values of one of the known points into the equation

$$n(3) = 70 = k e^{-0.2715(3)}$$

. . . and solve for *k*:

$$k = 70\, e^{0.2715(3)} = 70 \cdot 2.2581 = 158.1$$

Now, the specific form of the diameter distribution function is known:

$$n(d) = 158.1\, e^{-0.2715\,(d)}$$

Note that the relationship used to identify Q in the above example can be written more generally as:

$$Q = \sqrt[j]{\frac{n(d)}{n(d+j)}}$$

where j is the difference in inches between the larger known diameter class and the smaller known diameter class.

Once you have a functional expression for the target diameter class distribution, it is not hard to calculate the basal area of a stand with this diameter class distribution. The diameter class distribution function gives you the number of trees in each diameter class. For each diameter class, you can calculate the basal area of each tree in the diameter class, and using this information, you can calculate the total basal area of the trees in the diameter class. Summing these values over all the diameter classes in the stand gives you the total basal area of the stand. The formula for the basal area is therefore:

$$BA = \sum_{d=1}^{d_{max}} n(d)\,\pi \left(\frac{d}{2 \cdot 12}\right)^2 = \sum_{d=1}^{d_{max}} 0.005454 \cdot k\,\pi\,e^{-ad}\,d^2$$

Note that the equation above accounts for the fact that diameters are usually measured in inches (at least in the U.S.) while basal area is measured in ft^2/acre (again, in the U.S.).

> **Example:** Calculating the basal area of a stand from the diameter distribution function
>
> Calculate the basal area of a stand with the diameter class distribution in the previous example. Assume that the maximum diameter class is 16 inches. Thus, the diameter class distribution is:
>
> $$n(d) = 158.1\, e^{-0.2715\,d} \quad for\ d \le 16,\, 0\ otherwise$$
>
> **Answer:** The basal area is 71 ft^2/acre. The calculations are done in the Table 9.1 below.

So, in review, the negative exponential function has the property of having a constant Q factor – i.e., the ratio of the number of trees in any given diameter class over the next larger diameter class is constant. This makes it particularly useful for modeling a target age-class distribution for an uneven-aged stand. The negative exponential function has two parameters: a and k. The parameter a is directly related to the Q factor, which also determines the slope of the diameter class distribution. The parameter k is the intercept of the function with the y-axis – i.e., it is the

Table 9.1. Harvest volumes for uneven-aged management cutting cycle and residual basal area example.

Diameter Class	Number of Trees	Basal Area
1	120.5	0.657
2	91.9	2.004
3	70.0	3.437
4	53.4	4.657
5	40.7	5.547
6	31.0	6.088
7	23.6	6.317
8	18.0	6.289
9	13.7	6.067
10	10.5	5.709
11	8.0	5.265
12	6.1	4.776
13	4.6	4.273
14	3.5	3.777
15	2.7	3.305
16	2.1	2.866
Total	**500.3**	**71.034**

number of trees one would have in the 0-inch diameter class (if there were such a class). You only need two pieces of information about a negative exponential diameter class distribution to be able to identify a specific form of the function. Identifying a specific form of the function requires that you know the values of the parameters a and k. The two pieces of information could be knowing Q and one point on the curve, or it could mean knowing two points on the curve. The third parameter you need to know to completely specify a target diameter distribution is the maximum diameter class.

5. Selecting the Maximum Diameter of the Diameter-Class Distribution

As mentioned earlier, selecting the maximum diameter for trees in an uneven-aged stand is essentially the same as the rotation decision in even-aged stands. A widely-used principle used in determining the best time to cut a tree in an uneven-aged stand is the financial maturity rule first proposed by a forest economist named William Duerr back in the 1950s. We start by considering the financial maturity rule and then show how this is similar to the marginal analysis of the rotation for even-aged stands that was discussed in Chapter 6.

Financial Maturity -- Single Tree (William Duerr)

William Duerr's *financial maturity* principle is based on the idea that the rate at which the value of a tree increases should at least equal the alternate rate of return (ARR). It is called financial maturity because the tree can be viewed as a capital asset that is "mature" when it no longer earns a sufficient financial rate of return on its value. As trees age the rate of increase in their value inevitably slows. Not only is the tree's growth rate slowing down, but the value of the tree itself is increasing, so the numerator of the annual percentage rate of growth formula is getting smaller at the same time that the denominator is getting larger. When the rate of return earned on the value of the tree becomes less than the rate of return that could be earned in an alternative investment, the tree should be harvested.

The following steps are used to apply the financial maturity rule to an individual tree:

1) determine the guiding rate of return (i.e., the ARR),
2) calculate the current stumpage value of the tree,
3) estimate the stumpage value of the tree at the next point in time when the tree could be cut,
4) compare the rate of value increase over the interval between this cutting and the next possible cutting with the guiding rate of return to judge whether the tree is financially mature and should be cut. The tree is financially mature when the projected rate of value increase is smaller than the guiding rate of return.

Example

Consider a tree whose stumpage value now (SV_0) is $80. We estimate that the stumpage value of the tree after 8 years (SV_8) will be $110. If the ARR is 4%, is the tree financially mature?

Answer: use the following equation to calculate the annual rate of value increase for the tree (r_{sv}):

$$r_{sv} = \left(\sqrt[n]{\frac{SV_n}{SV_0}} \right) - 1 = \left(\sqrt[8]{\frac{110}{80}} \right) - 1 = 0.04061 = 4..061\% > 4\%$$

Since the annual rate of value increase is greater than the alternate rate of return, the tree is not yet financially mature, so we should keep it.

The problem with the financial maturity criterion is that it only accounts for the opportunity cost of not reinvesting the value of the tree. But there are other costs that should be considered. It might be useful to compare the financial maturity rule with the approach we took to thinking about the optimal time to cut an even-aged stand. To do this, we will express the rule in terms of the marginal benefit of waiting to harvest the tree and the marginal cost. In this context, the financial maturity rule can be stated equivalently as follows:

> If the expected value of the growth of the tree is greater than the opportunity cost of not harvesting the tree and reinvesting its value at the ARR, then keep the tree, otherwise cut the tree.

The expected value of the growth of the tree is just the difference between its expected future value and its current value. This is the marginal benefit of allowing the tree to continue growing. We will call this the *tree value growth*:

Tree value growth = SV_n - SV_0

The opportunity cost of not harvesting the tree and reinvesting its value will be called the *growing stock holding cost*, or *stock holding cost*, for short. As you shall see, this is similar to the inventory cost discussed in Chapter 6 in the discussion of the optimal even-aged rotation. This cost is equal to the interest we would have earned on the value of the tree if we cut it down now. It can be calculated as follows:

$$Stock\ holding\ cost = SV_0(1+r)^n - SV_0 = SV_0[(1+r)^n - 1]$$

Example

Calculate the stock holding cost and the tree value growth in the previous example.

Answer: the stock holding cost is:

Stock holding cost = $\$80\ [(1.04)^8 - 1] = \$29.49$

The tree value growth is:

Tree value growth = $\$110 - \$80 = \$30$

The tree value growth is greater than the stock holding cost, so the financial maturity rule says to leave the tree. According to this rule, the net benefit of leaving the tree is $0.51 ($30 - $29.49). (*Is this a present value or a future value? How can you tell?*)

As discussed above, something is missing from this rule. It needs to be generalized to include the opportunity cost of using the growing space occupied by the tree. This is because as long as the current tree is taking up the space it occupies, the growth of the next tree that will fill that space is being inhibited. This opportunity cost will be called the *land holding cost*.

Including the land holding cost

The land holding cost is simply opportunity cost of allowing a tree to continue to use the growing space that it occupies. Thus, keeping the tree on the site incurs a cost by delaying the start of the next crop tree that will occupy the site. This cost is quite difficult to measure, but a reasonable approach is to estimate the value of a likely crop tree for that site at various rotation ages and calculate a LEV for the space occupied by the tree. The opportunity cost of delaying the start of that next tree can then be estimated by the amount of interest that would have been earned by an

amount equal to this tree-level LEV if it were invested at the ARR for n years, when the current tree might otherwise be harvested. Thus, we will estimate the land holding cost with the following formula:

$$\text{Land Holding Cost} = LEV(1+r)^n - LEV = LEV[(1+r)^n - 1]$$

Note that this formula can be rearranged as follows:

$$\text{Land Holding Cost} = LEV[(1+r)^n - 1] = \frac{(rLEV)[(1+r)^n - 1]}{r}$$

This equation says that the land holding cost is equal to the future value of an annual payment of $r \cdot LEV$ for n years. The value of $r \cdot LEV$ can be interpreted as an annual rent for the growing space occupied by the tree.

The new rule for deciding whether or not to harvest a tree is:

> If the expected value growth of the tree is greater than the sum of the stock holding cost and the land holding cost, then keep the tree, otherwise cut the tree.

We can now return to the earlier example to see if accounting for the land holding cost changes the harvesting decision.

Example

In the previous example, the opportunity cost of leaving the tree on the site and delaying the start of a new crop tree on the site was ignored. Assume that a new crop tree will begin growing in the current tree's space when the current tree is gone. Assume that the value of this crop tree at age 60 is expected to be $60; at age 70 it is expected to be $105; and at age 80, the tree's expected value is $145. Assume that there are no management costs.

Answer: With no costs, the LEVs for the tree at each rotation will be:

$$LEV_{60} = \frac{60}{(1.04)^{60} - 1} = \$6.30$$

$$LEV_{70} = \frac{105}{(1.04)^{70} - 1} = \$7.21$$

$$LEV_{80} = \frac{145}{(1.04)^{80} - 1} = \$6.58$$

The 70-year rotation is best, so that is the LEV that should be used in calculating the land holding cost.

$$\text{Land holding cost} = LEV [(1+r)^n - 1] = \$7.21 [(1.04)^8 - 1] = \$2.66$$

We already know that the tree value growth is $30, and the stock holding cost is $29.49. Thus, the net gain from keeping the tree is:

Net holding gain = $30 - $29.49 - $2.66 = -$2.14

Since the net holding gain is negative, we should not keep the tree. Thus, when we properly account for both the stock and the land holding cost, the correct decision is to cut the tree now.

All of the above decisions were analyzed using opportunity costs evaluated in the future (i.e., future values were used). This is because we calculated what we would have had at the end of the 8-year period if the value of the growing stock and the land had been invested, versus what we would have at the end of 8 years if the tree was allowed to grow. The following example shows how the problem could also have been evaluated using present values.

Example

Reconsider the previous example by calculating the present value of the space occupied by the current tree if the tree is cut now, and if the tree is cut in 8 years.

Answer: If the tree is cut now, the present value of the space occupied by the tree is the tree value now plus the LEV.

$$PV_{cut\,now} = SV_0 + LEV = \$80 + \$7.21 = \$87.21$$

If the tree is cut in 8 years, the present value of the space occupied by the tree is the discounted value of the tree value in 8 years plus the discounted LEV.

$$PV_{cut\,in\,8\,yr} = \frac{SV_8}{(1.04)^8} + \frac{LEV}{(1.04)^8} == \frac{\$110}{(1.04)^8} + \frac{\$7.21}{(1.04)^8} = \$85.64$$

With this approach, we also arrive at the conclusion that the tree should be harvested now. Note that the present value of the opportunity cost of not cutting now is $1.56 ($87.21 - $85.64). The future value of this amount is $2.14 – exactly the result obtained earlier with the future value approach.

The criterion for selecting a maximum diameter for the target diameter class distribution should be based on this type of analysis. One can identify the maximum diameter by calculating the value growth and the stock holding and land holding costs over a cutting cycle for trees in each diameter class considered to be a reasonable candidate for the maximum diameter. At some point, the value growth will not be large enough to outweigh the opportunity costs of keeping the tree on the site. Alternatively, an individual tree LEV could be calculated for different harvest ages – just as was done in the example for calculating the land holding cost. The diameter corresponding to the age with the highest individual tree LEV will be the appropriate maximum diameter. Actually, both of these approaches will give the same answer.

6. Selecting the Q Factor and *k*

Specifying a target diameter class distribution with the negative exponential function requires that you specify the values of three parameters: *a*, *k*, and d_{max} (the maximum diameter). How does one decide which values of these three parameters are best for a given stand? One approach is to first identify the maximum diameter, then select a Q factor – which implies a specific value for the *a* parameter. Finally, the *k* value should be selected to ensure that the stand density is consistent with obtaining an appropriate level of regeneration.

Assuming the maximum diameter has already been selected, then the next decision is to select an appropriate Q factor. Recall that selecting the value of the *a* parameter is equivalent to selecting a Q factor and vice versa, so these two decisions are equivalent. Once the Q factor and the maximum diameter have been selected, the selection of *k* is equivalent to specifying a target basal area. To see this, think of holding the Q factor and the maximum diameter constant and shifting *k* up and down. This shifts the entire diameter distribution curve up and down, which, in turn, shifts the stand basal area up and down. So, given that d_{max} and Q have already been chosen, choosing *k* is equivalent to choosing a target basal area. The basal area decision is driven largely by the need to obtain adequate regeneration while maximizing the utilization of the site. The maximum density of the stand that will allow an adequate rate of seedling establishment will vary from species to species. Thus, the selection of *k* (and with it, basal area) is largely a silvicultural question, rather than a management decision. The selection of Q and the maximum diameter are therefore the primary management decisions in specifying the target diameter class distribution.[1]

It seems reasonable to assume that the amount of regeneration that will occur in an uneven-aged stand will depend primarily on the density and the species composition of the stand. We will make a simplifying assumption that the amount of regeneration in the stand at any given time can be described by a simple function of the stand basal area. Obviously, this function will have a negative slope. Furthermore, for many species, the function should approach zero for basal areas greater than 80 to 90 square feet per acre. As an example, assume that the relationship between seedling establishment (i.e., the number of trees in the 0" diameter class) can also be described using the following negative exponential function:

$$d(0) = k = 300e^{-0.03BA}$$

stand basal area (BA) is a function of *k*, the Q factor, and the maximum diameter. This function can be inverted to give *k* as a function of BA for each given Q factor (holding the maximum diameter constant). Figure 9.2 shows three such relationships for values of Q ranging from 1.1 to 1.3. The regeneration equation has also been overlaid on the graph. The point where the regeneration curve crosses each k(BA) curve indicates the sustainable values of the *k* parameter

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[1] This conclusion, of course, depends on whether one has already accepted the assumption that the diameter class distribution should have a constant Q factor.
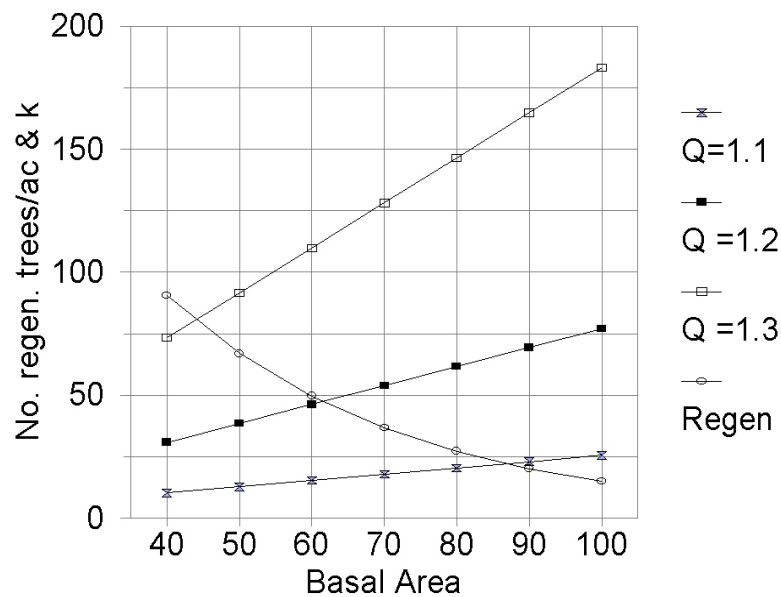
Figure 9.2. Values of *k* corresponding to different stand basal
areas and different Q factors and regeneration levels
by stand basal area.

for each Q factor (given this regeneration relationship). This value of *k* is sustainable because it results in a stand basal area that is also consistent with the regeneration required to sustain the diameter class distribution. For example, with a Q factor of 1.2 and *k* set to 48 the stand basal area will be 62 ft^2/ac. At this basal area, regeneration is approximately equal to 48 trees per acre – just the amount needed for a *k* value of 48 trees per acre. If *k* was set to 39, on the other hand, the stand basal area would be only 50 ft^2/ac, and regeneration would be excessive, at 67 trees/ac. Conversely, if *k* was set to 54, the stand basal area would be 70 ft^2/ac and regeneration would be inadequate, at 37 trees/ac.The point of all this is that there will be only a small range of possible values of *k* for each value of Q for which regeneration will be sustainable. Q and the maximum diameter are therefore the only parameters of the target diameter class distribution that a manager will have much latitude in choosing. So, what are the implications are of choosing different values of Q? Figure 9.3 shows how *k*, stand basal area, and number of trees per acre vary for three values of Q ranging from 1.1 to 1.3. On can see in the figure that, as the Q factor is increased, basal area decreases and the number of trees increases. This means that the average size of the trees gets smaller as Q gets larger. Most people prefer larger trees, and larger trees certainly provide greater economic returns than smaller trees. Thus, it would appear that the best value of Q is the smallest one. Silvicultural reality will limit how low the Q factor can go. Thus, as with *k*, the selection of Q depends more on biological feasibility than on economic analysis. Obviously, Q can never be less than one. However, in most cases, Q factors much less than 1.2 will be difficult to achieve silviculturally.
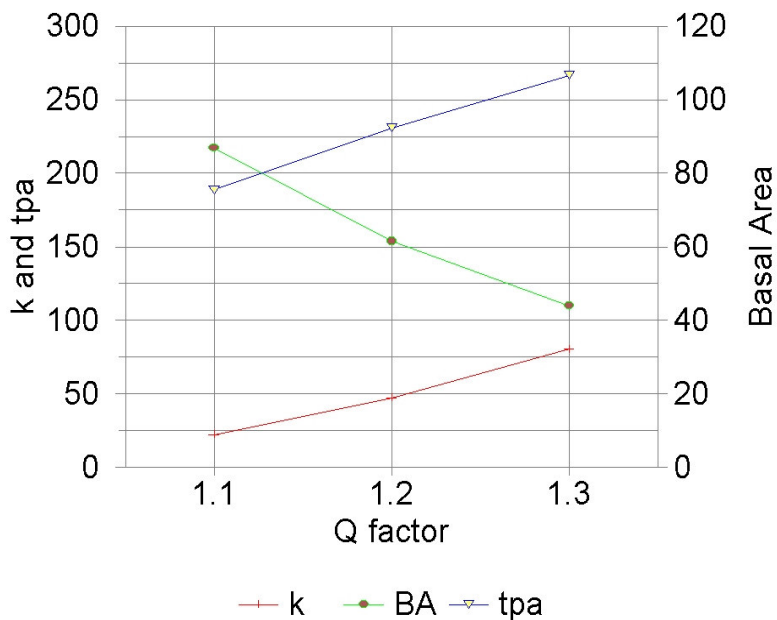
Figure 9.3. Stand characteristics—*k*, basal area (BA), and trees per acre (tpa)—for different values of Q.

7. <u>Individual Tree Selection</u>

The target diameter class distribution is an important guide at the end of the cutting cycle when the time comes to harvest part of the uneven-aged stand. It indicates the number of trees to cut per acre from each diameter class. But, it does not tell us exactly which trees to cut. Most of these decisions must be made in the field with a paint gun in your hand. The undesirable species will often be the first to go. For example, if you are managing an uneven-aged loblolly pine stand, the hardwoods are not only slow growing, but they slow the growth of the pines. Also high on the list of cutters should be trees with diseases or insect infestations, as they may spread their problems to other trees. Next in the priority list will be trees that will never produce a high-quality sawlog. These include trees with a variety of defects, such as low forks, sweep, crook, fire scars, frost cracks, too many large branches, etc. Next come spacing considerations. When two healthy, vigorous trees are clearly competing with each other, one should go.

The general rule you should always have in mind is "keep the trees with the most potential to increase in value." Ideally, the rate of value increase for each tree in the stand should be greater than the alternative rate of return. Otherwise, the capital embodied in the tree will probably be better invested elsewhere. Often, when rehabilitating a stand that has been high-graded, it will be impossible to achieve this goal for several cutting cycles, but ultimately the stand's productivity should be brought up to this standard.

8. Selecting an Optimal Cutting Cycle and Residual Basal Area

We now turn to the decision of how frequently to harvest the excess growing stock from an uneven-aged stand. As mentioned earlier, cutting cycles that are too short will result in excessive harvest costs because the volume removed per unit area will be too small. Cutting cycles that are too long will require very low residual basal areas following the harvest so that the stand will be able to grow longer before the stand density gets too high to allow for adequate regeneration. By the end of a long cutting cycle, the stand will typically be too dense for adequate regeneration to occur. Because longer cutting cycles require lower residual basal areas following the harvest, the cutting cycle decision cannot be separated from the residual basal area decision.

To address the optimal cutting cycle question, let's consider an uneven-aged stand that is overstocked in all diameter classes (an unlikely situation, but a useful simplification). Because each diameter class is overstocked, we can achieve our target diameter class distribution with one cut. After this initial harvest, we will let the stand to grow for one cutting cycle and then cut the excess in each diameter class. At the end of each cutting cycle we will again harvest the stand, returning it again to the target diameter class distribution. Because we will be starting each cutting cycle with the same diameter class distribution, and because the cutting cycle is always the same, the harvest will be the same at the end of each cutting cycle (more or less). We will assume constant real prices and costs, so the same net revenue will be earned at the end of each cycle.

The present value of all our costs and revenues from this management regime gives us the value of the stand, including both the trees and the land. This present value is therefore a Forest Value. The financial analysis of this situation is fairly straightforward. We have two types of revenues:

        1) the revenue from our initial harvest, and
        2) the revenue that we get at the end of each cutting cycle.

We could include a variety of costs, but we will consider only an annual cost (or revenue) and harvest costs here. The formula for the present value of this sequence of revenues and costs makes use of the *infinite periodic series* formula (the same one we use in calculating LEVs).

$$ForVal = NetRev_I + \frac{NetRev_{cc}}{(1+r)^t - 1} + \frac{A}{r}$$

    where: $NetRev_I$ = the initial harvest net revenue,
           $NetRev_{cc}$ = the net harvest revenue at the end of each cutting cycle,
           A = the net annual revenue (negative for costs),
           t = the cutting cycle, and
           r = the real interest rate.

Note the similarities between this equation and the equation for the Forest Value for an even-aged stand that we are going to cut immediately. The first term in the uneven-aged Forest Value

formula is the revenue from the harvest of the current stand. The second two terms are virtually identical to the formula for the LEV (using method 3).

Now, to calculate the optimal residual basal area and cutting cycle, we simply calculate the Forest Value for a range of basal areas and cutting cycles. The combination of the cutting cycle and basal area that gives the highest Forest Value is optimal.

Before we do an example, think about how $NetRev_I$ and $NetRev_{cc}$ should vary depending on the cutting cycle and the residual basal area. First, note that $NetRev_I$ won't depend directly on the cutting cycle, but it will be larger for smaller residual basal areas. (Stop and think about why this is.) $NetRev_{cc}$ should be larger for larger cutting cycles – the stand has had more time to grow, and we are essentially removing the net growth since the last cutting cycle. How $NetRev_{cc}$ should vary for different residual basal areas is ambiguous. A lower basal area gives the stand more room to grow, but there is less growing stock there to put the growth on. For most basal areas that are realistic for uneven-aged management, however, $NetRev_{cc}$ will be larger for larger residual basal areas.

Example

You have a 200 acre uneven-aged forest stand. You need to determine the best cutting cycle and residual basal area for the stand. You are considering three residual basal area levels – 50, 60, and 70 square feet – and three cutting cycles – 5, 10, and 15 years. The volume cut in your initial harvest depends only on the residual basal area. The volume from future harvests, however, will depend on the cutting cycle and the residual basal area following each harvest. Column 2 in Table 9.1 below gives the volume harvested per acre from the initial harvest. The following three columns give the volume harvested per acre from future harvests.

Table 9.1. Harvest volumes for uneven-aged management cutting cycle and residual basal area example.

Residual Basal Area	Volume Harvested (mbf per acre) for Initial Harvest	Volume Harvested (mbf/acre) in Future Harvests		
		5-year Cycle	10-year Cycle	15-year Cycle
50	1.98	0.88	2.42	3.00
60	1.76	1.10	2.53	3.10
70	1.21	1.30	2.64	3.63

The stumpage price for all harvests is $220/mbf, and there are fixed costs for sale preparation, permitting, and moving harvesting equipment to the site totaling $2,000 (for the whole stand) each time a harvest is made.

a. Using a real alternate rate of return of 4%, calculate the Forest Value (per acre) for each cutting cycle and residual basal area combination. Assume that property taxes are $5 per acre per year.

Answer: Here is a sample calculation for a 5-year cutting cycle and a residual basal area of 50:

First, note that the fixed cost of $2,000 will be spread over 200 acres. Therefore, the fixed cost amounts to $10/ac. Now, we can calculate $NetRev_I$ and $NetRev_{cc}$:

$NetRev_I$ = $220/mbf×1.98 mbf/ac - $10/ac = $425.60/ac

$NetRev_{cc}$ = $220/mbf×0.88 mbf/ac - $10/ac = $183.60/ac

Now, plug these values into the uneven-aged management Forest Value equation:

$$ForVal_{BA=50,cc=5} = \$425.60 + \frac{\$183.60}{(1.04)^5 - 1} - \frac{5}{0.04} = \$1,148.04$$

The Forest Values for the remaining combinations of cutting cycles and residual basal areas are given in Table 9.2.

Table 9.2. Forest Values for uneven-aged management cutting cycle and residual basal area example (200 acre tract).

Residual Basal Area	Forest Value (per acre)		
	5-year Cycle	10-year Cycle	15-year Cycle
50	$1,148.00	$1,388.40	$1,112.10
60	$1,323.00	$1,390.40	$1,091.20
70	$1,405.10	$1,319.80	$1,115.80

The best residual basal area and cutting cycle for this stand is clearly 70 ft^2/ac with a cutting cycle of 5 years because it gives the highest Forest Value: $1,405.10.

b. What if we had 80 acres instead of 200?

Answer: With 80 acres, the fixed cost of $2,000 now amounts to $25/ac. The values of $NetRev_I$ and $NetRev_{cc}$ are now:

$NetRev_I$ = $220/mbf×1.98 mbf/ac - $25/ac = $410.60/ac

$NetRev_{cc}$ = $220/mbf×0.88 mbf/ac - $25/ac = $168.60/ac

Now, these values can be plugged into the uneven-aged Forest Value equation:

$$ForVal_{BA=50,cc=5} = \$410.60 + \frac{\$168.60}{(1.04)^5 - 1} - \frac{5}{0.04} = \$1,063.80$$

The Forest Values for the remaining combinations of cutting cycles and residual basal areas are given in Table 9.3.

Table 9.3. Forest Values for uneven-aged management cutting cycle and residual basal area example (200 acre tract).

Residual Basal Area	Forest Value (per acre)		
	5-year Cycle	10-year Cycle	15-year Cycle
50	$1,063.80	$1,342.10	$1,078.40
60	$1,238.80	$1,344.10	$1,057.50
70	$1,320.90	$1,273.50	$1,082.10

The best residual basal area and cutting cycle for this stand is clearly 60 ft^2/ac with a cutting cycle of 10 years.

9. Study Questions on Uneven-Aged Management

1. What is the defining characteristic of an uneven-aged stand?

2. Why should maintaining an uneven-aged stand generally not be considered an objective of management?

3. What are the advantages and disadvantages of uneven-aged management relative to even-aged management?

4. What are the major management decisions in uneven-aged management? How are they similar to the major decisions in even-aged management? How are they different?

5. What is a diameter class distribution?

6. What are the desirable characteristics of a target diameter class distribution for uneven-aged management?

7. What is the Q factor in uneven-aged management? What aspect of the diameter class distribution of an even-aged stand does it determine?

8. What are the processes that move trees into and out of diameter classes? Why must these processes be balanced in uneven-aged management?

9. Regeneration must be a constant process in uneven-aged management. How is this accomplished? How do regeneration concerns affect each of the major management parameters in uneven-aged management?

10. Why does the negative exponential function give a useful functional form for describing the target diameter class distribution for uneven-aged management?

11. What are the three parameters that you need to specify a negative exponential target diameter class distribution?

12. Why should we consider selecting the value for the parameter k a silvicultural decision, rather than a management decision?

13. What stand characteristics do stands with high Q factors have compared with stands with low Q factors?

14. Why is it more desirable from an economic perspective to use as low a value of Q as possible?

15. From an economic perspective, is an uneven-aged stand with a high Q quotient likely to be more or less profitable than a stand with a low Q quotient? Why?

16. If lower Q factors are desirable, what prevents managers from using a Q factor lower than 1—or even as low as 1.2 in many cases?

17. What factors must be balanced in selecting the cutting cycle?

18. Why would you not want to use a cutting cycle that is too long or too short?

19. Why will we generally need to use a lower residual basal area with longer cutting cycles?

20. If you want to use a long cutting cycle, does that mean you will want to leave a larger or smaller residual basal area than with a shorter cutting cycle? Why?

21. Why is the optimal cutting cycle longer and the optimal residual basal area lower in example 2 when we have a smaller area?

22. What opportunity cost is not considered by the financial maturity criterion?

23. Explain what the tree holding cost land the land holding cost are.

24. What similarities can you see between the decision whether to cut an individual tree and the decision to cut a whole stand? What differences are there?

10. Exercises

1. What is the diameter class distribution function for a stand with a constant Q factor and 70 trees per acre in the one-inch diameter class and 24 trees per acre in the 10-inch diameter class?

2. What is the diameter class distribution function for a stand with a constant Q factor and 33 trees per acre in the five-inch diameter class and 13 trees per acre in the 10-inch diameter class?

3. a. What is the diameter class distribution function for a stand with a constant Q factor of 1.15 and 55 trees per acre in the one-inch diameter class?

    b. Create a table showing the number of trees and basal area in each 1" diameter class for the diameter class distribution from part a (assume that the largest diameter class is 20"). At the bottom of the table, indicate the total number of trees and the total basal area per acre.

    c. The number of trees that can be sustained in the 1" diameter class depends on the basal area of the stand—if the basal area is too high, there will not be enough regeneration, and if the basal area is too low there will be too much regeneration. Assume that the relationship between the number of trees that can be sustained in the 1" diameter class and the basal area of the stand can be described with the following equation:

    $$f(1) = 2,800\ e^{-0.04\ BA}$$

    Can the diameter class distribution described in parts **a** and **b** be sustained? Why, or why not?

4. You have been asked to manage an uneven-aged loblolly pine stand. You have used data that have been collected on the site to develop the following regeneration equation:

    $$\textit{Expected no. trees in the } 1'' \textit{ diameter class } = 800\,e^{-0.039\,BA}$$

    You want the stand to have a Q factor of 1.2 and a maximum 1-inch diameter class of 18 inches. Identify a target diameter class distribution for this stand that will be consistent with the above regeneration equation. (*Hint:* find the value for a that gives you a Q factor of 1.2. Next, make a guess at what k should be. Use that k to calculate the basal area of the stand.

Then plug that basal area into the regeneration function. If the expected number of trees in the 1-inch class is too large, choose a smaller k. If the expected number of trees in the 1-inch class is too small, choose a larger k.) Find a and k such that the expected number of trees in the 1-inch class is within 1 tree of the desired number.

5. You have a 40 acre forested woodlot that you would like to manage using uneven-aged management. After an initial (one-time) improvement cut which will <u>cost</u> $150/acre, you expect that you will be able to manage the forest on a 10-year cutting cycle, with the first profitable harvest coming in 10 years. You project that each of these profitable harvests should net an average real value of $450/acre. The taxes on the land are $4/acre per year, and the annual management cost will be $2/acre per year. Your alternate rate of return is 5%.

a. What is the present value (per acre) of this woodlot?

b. If you owned this woodlot only for the timber revenue and someone offered to buy it for $500 per acre, should you sell it? Why?

c. Assume that owning this land provides you with other types of value—e.g. the satisfaction of owning woods and hunting opportunities. Now, let's say that someone offered you $600 an acre for the land, and you turned them down. Let's assume you understood what the income-producing potential of the land is and you made the decision not to sell anyway. How much value per year must the other, non-financial values be worth to you (for the whole woodlot—40 acres).

d. Assuming you didn't own the land, what is the maximum you could afford to pay per acre to rent the land (considering only the timber values)?

*6. You have a 40-acre forest stand that you want to manage using an uneven-aged system. You need to determine the best cutting cycle and residual basal area for the stand. You are considering three residual basal area levels: 80, 90, and 100 square feet, and you are considering three cutting cycles: 5, 10, and 15 years. The revenue from your initial harvest depends only on the residual basal area. The revenue for future harvests, however, will depend on the cutting cycle and the residual basal area following each harvest. Column 2 in Table 9.4 gives the net revenue (excluding fixed costs) per acre from the initial harvest. The following three columns in the table give the net revenue (excluding fixed costs) per acre from future harvests. There are fixed costs for sale preparation, permitting, and moving harvesting equipment to the site totaling $2,000 (for the whole stand) each time a harvest is made.

a. Using a real alternate rate of return of 4%, calculate the Forest Value (per acre) for each cutting cycle and residual basal area combination. Assume that property taxes are $5 per acre per year.

b. What is the best residual basal area and cutting cycle for this stand?

Table 9.4. Harvest volumes for uneven-aged management Problem 6.

Residual Basal Area	Net Revenue (per acre) for Initial Harvest	Net Revenues (per acre) for Future Harvests		
		5-year Cycle	10-year Cycle	15-year Cycle
80	240	160	365	595
90	275	185	400	610
100	220	200	410	590

7. You have a 200 acre forest stand that you want to manage using an uneven-aged system. You need to determine the best cutting cycle and residual basal area for the stand. You are considering three residual basal area levels: 70, 80, and 90 square feet, and you are considering three cutting cycles: 5, 10, and 15 years. The volume harvested in your initial harvest depends only on the residual basal area. The volume harvested in future harvests, however, will depend on the cutting cycle and the residual basal area following each harvest. Column 2 in the Table 9.5 gives the volume harvested per acre from the initial harvest. The following three columns give the volume harvested per acre from future harvests. The stumpage price for all harvests is $220/mbf, and there are fixed costs for sale preparation, permitting, and moving harvesting equipment to the site totaling $2,000 (for the whole stand) each time a harvest is made.

Table 9.5. Harvest volumes for uneven-aged management Problem 7.

Residual Basal Area	Volume Harvested (mbf per acre) for Initial Harvest	Volume Harvested (mbf/acre) in Future Harvests		
		5-year Cycle	10-year Cycle	15-year Cycle
70	1.98	0.88	2.42	3.00
80	1.76	1.10	2.53	3.10
90	1.21	1.30	2.64	3.63

a. Using a real alternate rate of return of 4%, calculate the Forest Value (per acre) for each cutting cycle and residual basal area combination. Assume that property taxes are $5 per acre per year.

b. What is the best residual basal area and cutting cycle for this stand?

8. You are going through an uneven-aged stand deciding which trees to cut. You come on a large southern red oak and you estimate that it would yield 1,200 board feet if cut now. If you wait ten years, you estimate that it will yield 1,700 bd. ft. Current oak prices are

$200/mbf, and you expect this price will remain constant in real terms. Your real alternate rate of return is 3.5%. Yields for the tree that would likely replace this tree when it is cut down are shown in Table 9.4.

a. Fill in Table 9.6 to determine the opportunity cost of using the land currently occupied by the oak tree.

Table 9.6. Yield estimates and LEV calculations for future trees that will occupy this site.

Rotation	Yield	LEV	Rent
40	600		
50	1,300		
60	2,000		
Maximum LEV	---		

b. Fill in Table 9.7, which considers the problem of whether or not to cut the oak tree down now in terms of future values (10 years from now).

Table 9.7. Tree Cutting Analysis with Future Values

Tree Value Now	Tree Value in 10 yr	Tree Value Growth	Stock Holding Cost	Land Holding Cost	Total Holding Cost	Net Holding Gain	Decision

10. You own a sugar maple tree that now contains 1,200 bd. ft. Over the next 10 years, you expect it to increase by 500 bd. ft. Assume that:

i) You wish to maximize your present net worth over an infinite planning horizon using a real alternate rate of return of 4 percent;

ii) You expect sugar maple timber to be worth $200 in real terms now and in the future;

iii) After you cut this tree, another sugar maple will become established that will have a volume of 600 bd. ft. after 50 years, 1,200 bd. ft. after 60 years, and 1,700 bd. ft. after 70 years.

a. Should you wait 10 years (or more) to cut the tree, or should you cut it now? First, you need to make some LEV calculations to determine the land holding cost. Fill in Table 9.8 based on the projected yield for the maple tree that will replace the current tree. What is the optimal rotation and LEV for future maple trees on the site?

Table 9.8. Individual-tree LEV and rent calculations for Problem 10.

Rotation	Yield	LEV	Rent
50			
60			
70			

b. Now, analyze the choice of holding the tree in terms of future values by filling in Table 9.9:

Table 9.9. Future-value analysis of the cutting decision in Problem 10.

Tree value now	Tree value in 10 years	Tree value growth	Stock holding cost	Land holding cost	Total cost	Net gain from holding

c. Now make the calculations in terms of present values by filling in the following table:

Table 9.10. Present-value analysis of the cutting decision in Problem 10.

	Cut n=0	Leave n=10
Stumpage Value		
Discounted Stumpage Value		
Maximum LEV		
Discounted LEV		
Present Net Worth		

11. You are going through an uneven-aged stand deciding which trees to cut. You come on a large southern red oak and you estimate that it would yield 1,200 board feet if cut now. If you wait ten years, you estimate that it will yield 1,700 bd. ft. Current oak prices are $200/mbf, and you expect this price will remain constant in real terms. Your real alternate rate of return is 3.5%. There are two possible trees that could replace this tree when it is cut

down. Yields for those trees are shown in Table 9.11. You are quite sure that if you wait ten years that the current tree will be replaced by Tree A. However, if you cut the tree now, it may be replaced by either Tree A or Tree B. In Tables 9.12 and 9.13 analyze the tree cutting decision for two cases:

 I) the current tree will be replaced by Tree A when it is cut whether it is cut now or ten years from now;

II) the current tree will be replaced by Tree B if it is cut now and it will be replaced by Tree A when it is cut, if it is cut ten years from now.

a. Fill in Table 9.11 to determine the opportunity cost of using the land currently occupied by the oak tree.

Table 9.11. Yield Estimates and LEV Calculations for Tree A and Tree B.

Rotation	Tree A			Tree B		
	Yield	LEV	Rent	Yield	LEV	Rent
40	600			300		
50	1,300			900		
60	2,000			1,300		
Maximum LEV	---			---		

b. Fill in Table 9.12, which considers the problem of whether or not to cut the oak tree down now in terms of future values (10 years from now).

Table 9.12. Tree Cutting Analysis with Future Values.

	Tree Value Now	Tree Value in 10 yr	Tree Value Growth	Stock Holding Cost	Land Holding Cost	Total Holding Cost	Net Holding Gain	Decision
Case I								
Case II								

c. Fill in Table 9.13, which considers the problem of whether or not to cut the oak tree down now in terms of present values.

Table 9.13. Tree Cutting Analysis with Present Values

	Case I		Case II	
	n=0	n=10	n=0	n=10
Stumpage Value (current tree)				
Discounted Stumpage Value				
Maximum LEV (next generations)				
Discounted Maximum LEV				
Present Net Worth (Discounted SV plus discounted LEV)				