Applied Operations Research

Solving an application of Linear Programming with Excel

- 2018/2019 -

1. **Land use planning**

a) **Formulate this problem as a LP model.**

The decision variables are as follows:

x_1 - area to residential development (ha)

x_2 - area to business development (ha)

x_3 - area to recreational development (ha)

The model is as follows:

$$\max\ z = 50000x_1 + 120000x_2 + 150000x_3 \quad (1)$$

$$subject\, to$$

$$x_1 + x_2 + x_3 = 100 \quad (2)$$

$$x_1 \geq 20 \quad (3)$$

$$x_2 \geq 30 \quad (4)$$

$$x_3 \geq 10 \quad (5)$$

$$300000x_1 + 500000x_2 + 400000x_3 \leq 65000000^{\star} \quad (6)$$

$$240x_1 + 480x_2 + 300x_3 \leq 40000 \quad (7)$$

$$x_1, x_2, x_3 \geq 0. \quad (8)$$

$\star \overbrace{8000000 + 300000(x_1 - 20)}^{\text{residential}} + \overbrace{20000000 + 500000(x_2 - 30)}^{\text{business}} + \overbrace{12000000 + 400000(x_3 - 10)}^{\text{recreational}} \leq 80000000$

Expression (1) maximizes the profit. Constraint (2) ensures that the whole land is exploited. Constraints (3), (4) and (5) guarantee that the minimum area for each development (residential, business and recreational) is exploited. Constraint (6) is the annual budget constraint. Constraint (7) is the annual regulation of water constraint. Constraints (8) state the non-negativity requirements on the variables.

b) **Solve the model. Which constraints are binding at the optimal solution found (saturated constraints)? Comment.**

The optimal solution found is $x_1 = 20$ ha, $x_2 = 30$ ha and $x_3 = 50$ ha with the annual profit of 12100000 €. Constraints (2), (3) and (4) are binding. In other words, the total area is used as well as the minimum areas for residential and business developments. With respect to the area for recreational development, 40 h more than the minimum required are used. With respect to the budget and regulation of water, 24000000 € and 5800 m^3, respectively, are not spent.

c) **Report the shadow prices for each constraint and comment.**

The shadow price of a constraint measures the impact on the optimal objective value with the (slight) increase of the RHS, remaining the other parameters the same.

The shadow prices of the non-binding constraints (constraints (5), (6) and (7)) are equal to zero. The shadow prices of the binding constraints are: 150000 €/ha for constraint (2), -100000 €/ha for constraint (3) and -30000 €/ha for constraint (4).

In other words, the increase on the minimum area for recreational development, budget and regulation of water do not have impact on the optimal annual profit. The increase on the total area has a positive impact of 150000 €/ha. The increase on the minimum areas for residential or business development have a negative impact of 100000 €/ha and 30000 €/ha, respectively.

d) **Derive the range of feasibility for all RHS values.**

The shadow prices are valid for RHS increases up to: 19.33 ha (constraint (3)), 40 ha (constraint (4)), 32.22 ha (constraint (5)), 40 ha (constraint (6)) and $+\infty$ (constraints (7) and (8)). Thus,

i) if the total area increased from 100 ha to 119.33 ha, the optimal profit would increase 150000 times 19.33, that is, 2900000 €

ii) if the minimum area for residential development increased from 20 ha to 60 ha, the optimal profit would decrease 100000 times 40, that is, 4000000 €

iii) if the minimum area for business development increased from 30 ha to 62.22 ha, the optimal profit would decrease 30000 times 32.22, that is, 966666.67 €

iv) if the minimum area for recreational development increased from 10 ha to 50 ha, the optimal profit would not change

v) if the budget and the regulation of water increased infinitely, the optimal profit would not change.

The shadow prices are valid for RHS decreases up to: 40 ha (constraint (3)), 20 ha (constraint (4)), 30 ha (constraint (5)), $-\infty$ (constraint (6)), 24000000 (constraints (7)) and 5800 (constraints (8)). Thus,

i) if the total area decreased from 100 ha to 60 ha, the optimal profit would decrease 150000 times 40, that is, 6000000 €

ii) if the minimum area for residential development decreased from 20 ha to 0 ha, the optimal profit would increase 100000 times 20, that is, 2000000 €

iii) if the minimum area for business development decreased from 30 ha to 0 ha, the optimal profit would increase 30000 times 30, that is, 900000 €

iv) if the minimum area for recreational development decreased infinitely (in fact to zero), the optimal profit would not change

v) if the budget decreased from 80000000 € to 56000000 €, the optimal profit would not change

vi) if the regulation of water decreased from 40000 m^3 to 34200 m^3, the optimal profit would not change.

e) **Determine an expression for each constraint that evaluates the impact on the optimal objective value with the (allowable) variation of the RHS value.**

For constraints

(2): $y = 12100000 + 150000(x - 100),\ x \in [60, 119.33]$

(3): $y = 12100000 - 100000(x - 20),\ x \in [0, 60]$

(4): $y = 12100000 - 30000(x - 30),\ x \in [0, 62.22]$

(5): $y = 12100000,\ x \in [0, 50]\ (] - \infty, 50])$

(6): $y = 12100000,\ x \in [56000000, +\infty[$

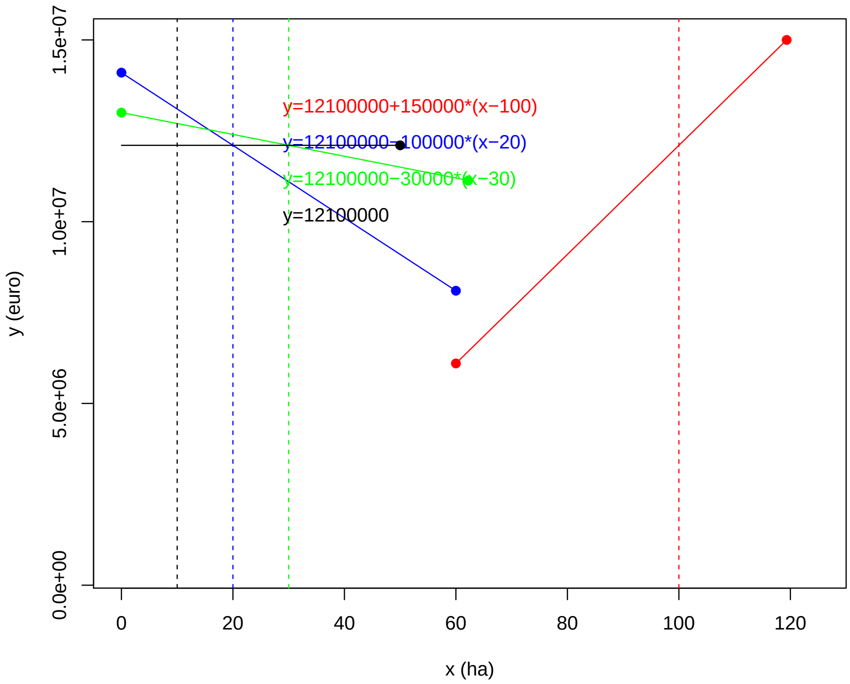

Figure 1: Optimal objective value according to the RHS of constraints (3), (4), (5) and (6). The slope of each line segment is the shadow price of the constraint.

(7): $y = 12100000,\ x \in [34200, +\infty[$,

where y is the optimal annual profit in euros and x is the RHS of each constraint. The slope of each line segment is the shadow price (Figure 1).

f) **Derive the range of optimality for all objective function coefficients.**

Columns "Allowable Increase" and "Allowable Decrease" give the amount by which each objective function coefficient (in column "Objective Coefficient") can be increased or decreased, respectively, without changing the optimal activity levels.

Thus, as long as the values of the objective function coefficients on x_1 (annual residential profit per hectare), x_2 (annual business profit per hectare) and x_3 (annual recreational profit per hectare) are, respectively, in the following intervals, one at a time and remaining the other parameters unchanged, the optimal solution will be the same ($x_1 = 20$ $x_2 = 30$ and $x_3 = 50$):

i) $]-\infty, 150000]$ (the allowable decrease is $+\infty$ and the allowable increase is 100000)

ii) $]-\infty, 150000]$ (the allowable decrease is $+\infty$ and the allowable increase is 30000)

iii) $[120000, +\infty[$ (the allowable decrease is 30000 and the allowable increase is $+\infty$).