Linear programming - revisions

27 February 2019

A **binding constraint**

is a constraint satisfied in equality by the optimal activity levels.

Keeping the river clean

A pulp mill makes mechanical and chemical pulp and during the production process it pollutes the river in which it spills its spent waters. The owners would like to minimize pollution, keeping at least 300 people employed at the mill and generating at least 40000€ of revenue per day.

Keeping the river clean - Data

- The maximum capacity of the mill is 300 tons per day to make mechanical pulp and 200 tons per day to make chemical pulp (the mechanical pulp line cannot be used to make chemical pulp, and vice-versa)
- Both mechanical and chemical pulp require the labor of 1 worker for about 1 day, or 1 workday (wd), per ton produced
- Pollution is measured by the biological oxygen demand (BOD). 1 ton of mechanical pulp produces 1 unit of BOD, 1 ton of chemical pulp produces 1.5 units
- The chemical pulp sells at 200€, the mechanical pulp at 100€ per ton.

Keeping the river clean - Decision variables and formulation

x_1 - Amount of mechanical pulp produced (in tons/day, or t/d)

x_2 - Amount of chemical pulp produced (t/d).

$$\min Z = x_1 + 1.5x_2 \quad \text{units of BOD per day} \tag{1}$$

$$subject\,to$$

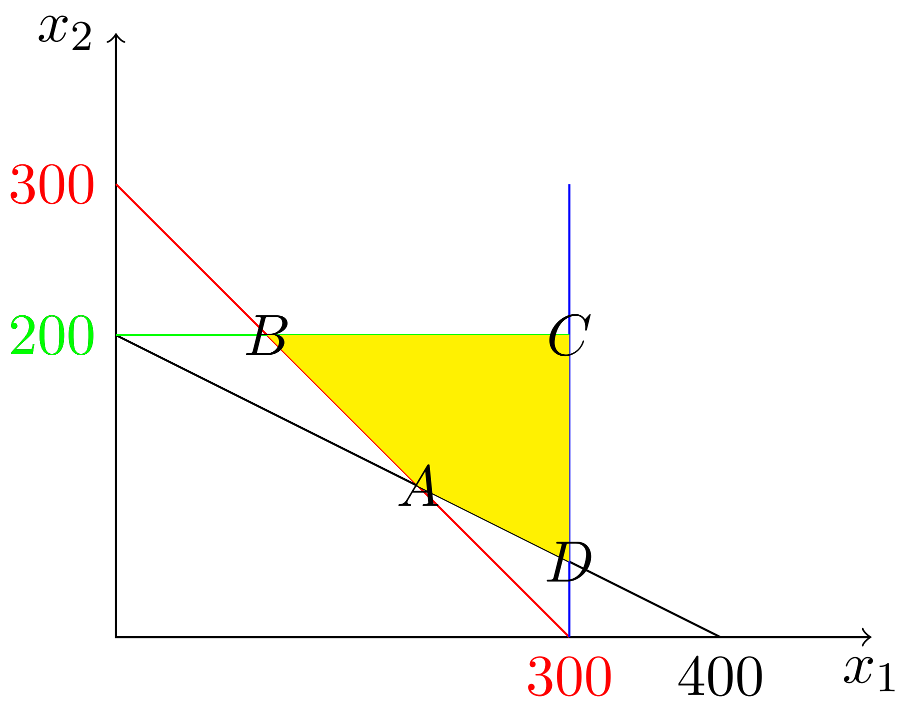
$$x_1 + x_2 \geq 300 \quad \text{workers employed} \tag{2}$$

$$100x_1 + 200x_2 \geq 40000 \quad \text{revenue, euros/day} \tag{3}$$

$$x_1 \leq 300 \quad \text{mechanical pulping capacity, t/day} \tag{4}$$

$$x_2 \leq 200 \quad \text{chemical pulping capacity, t/day} \tag{5}$$

$$x_1, x_2 \geq 0. \tag{6}$$



Optimal solution $A = (200, 100) \longrightarrow Z^* = 350$

$x_1 + x_2 = 300$ and $100x_1 + 200x_2 = 40000$ are **binding**. The other constraints are **non-binding**.

Shadow price - Definition

The **shadow price of a constraint**

measures the impact on the optimal objective value with the (slight) increase of the RHS, remaining the other parameters the same.

Non-binding and binding constraints

The shadow price of a non-binding constraint is equal to zero (there is no impact).

The shadow price of a binding and redundant constraint is equal to zero (there is no impact).

The **range of feasibility for the RHS value of a constraint**

is the range for the RHS value over which the shadow price is applicable.

Columns "Allowable Increase" and "Allowable Decrease" give the amount by which each RHS value (in column "Constraint R.H. Side") can be increased or decreased, respectively, over which the shadow price is applicable.

The **range of optimality for a objective function coefficient**

is the range for the objective function coefficient over which the optimal activity levels do not change, remaining the other parameters the same.

Columns "Allowable Increase" and "Allowable Decrease" give the amount by which each objective function coefficient (in column "Objective Coefficient") can be increased or decreased, respectively, without changing the optimal activity levels.

The **reduced cost of a non-basic variable**, which has zero value in the optimal solution,

provides a measure of how much the objective function would change (a penalty amount) if one unit of this variable were forced into the solution.