Applied Operations Research

Solving an application of Linear Programming with Excel

(one resolution)

- 2018/2019 -

2. **Fishery planning**

a) **Formulate this problem as a LP model.**

The decision variables are as follows:

x_{it} - amount of fish (in tonne) from species i captured at the end of year t, $i = 1$ (salmon), $i = 2$ (tuna), $i = 3$ (sardine), $t = 1, 2, 3, 4$

S_{it} - amount of fish (in tonne) from species i in the fishery at the end of year t, $i = 1.2.3$, $j = 1, 2, 3, 4$ (auxiliary variables).

The present profit p_{it} (in euros) obtained per tonne of fish captured, according to species i and year t, is as follows:

	Present profit p_{it} (€/t)			
x_{it}	$t = 1$	$t = 2$	$t = 3$	$t = 4$
$i = 1$	5769.23	5547.34	5333.98	5128.83
$i = 2$	5288.46	5085.06	4889.48	4701.42
$i = 3$	4807.69	4622.78	4444.98	4274.02

The model is as follows:

$$\max z = \sum_{i=1}^{3} \sum_{t=1}^{4} p_{it} x_{it} \quad (1)$$

subject to

$$x_{11} \leq 1.12 \times 30 \quad (2)$$

$$x_{21} \leq 1.12 \times 50 \quad (3)$$

$$x_{31} \leq 1.12 \times 40 \quad (4)$$

$$S_{11} = 1.12 \times 30 - x_{11} \tag{5}$$

$$S_{21} = 1.12 \times 50 - x_{21} \tag{6}$$

$$S_{31} = 1.12 \times 40 - x_{31} \tag{7}$$

$$x_{12} \leq 1.12S_{11} \tag{8}$$

$$x_{22} \leq 1.12S_{21} \tag{9}$$

$$x_{32} \leq 1.12S_{31} \tag{10}$$

$$S_{12} = 1.12S_{11} - x_{12} \tag{11}$$

$$S_{22} = 1.12S_{21} - x_{22} \tag{12}$$

$$S_{32} = 1.12S_{31} - x_{32} \tag{13}$$

$$x_{13} \leq 1.12S_{12} \tag{14}$$

$$x_{23} \leq 1.12S_{22} \tag{15}$$

$$x_{33} \leq 1.12S_{32} \tag{16}$$

$$S_{13} = 1.12S_{12} - x_{13} \tag{17}$$

$$S_{23} = 1.12S_{22} - x_{23} \tag{18}$$

$$S_{33} = 1.12S_{32} - x_{33} \tag{19}$$

$$x_{14} \leq 1.12S_{13} \tag{20}$$

$$x_{24} \leq 1.12S_{23} \tag{21}$$

$$x_{34} \leq 1.12S_{33} \tag{22}$$

$$S_{14} = 1.12S_{13} - x_{14} \tag{23}$$

$$S_{24} = 1.12S_{23} - x_{24} \tag{24}$$

$$S_{34} = 1.12S_{33} - x_{34} \tag{25}$$

$$S_{21} \leq 2S_{11} \tag{26}$$

$$S_{22} \leq 2S_{12} \tag{27}$$

$$S_{23} \leq 2S_{13} \tag{28}$$

$$S_{24} \leq 2S_{14} \tag{29}$$

$$\tag{30}$$

$$S_{14} + S_{24} + S_{34} \geq 120 \tag{31}$$

$$S_{14} \geq 12 \tag{32}$$

$$S_{24} \geq 25 \tag{33}$$

$$S_{34} \geq 16 \tag{34}$$

$$x_{it}, S_{it} \geq 0,\ i = 1, 2, 3,\ t = 1, 2, 3, 4. \tag{35}$$

Expression (1) maximizes the present profit. Constraints (2) to (4) ensure, for each species, that the amount of fish captured in the first year does not exceed the maximum amount that may exist in the fishery. Constraints (5) to (7) determine, for each species, the amount of fish that is in the fishery at the end of year 1. Constraints (8) to (25) guarantee the same for the remaining years. Constraints (26) to (29) ensure, for each year, that the amount of tuna in the fishery is less than or equal to twice the amount of salmon. Constraint (31) states that the total amount of fish in the fishery at the end of the 4-year period is greater than or equal to 120 t. Constraints (32) to (34) establish, for each species, a minimum amount of fish in the fishery at the end of the period. Constraints (35) state the non-negativity requirements on the variables.

b) **Solve the model and criticize the solution.**

The optimal solution obtained is displayed in the following table. The present profit of this solution is 335609.54 €.

	x_{it} (t)			
x_{it}	$t = 1$	$t = 2$	$t = 3$	$t = 4$
$i = 1$	0	0	0	28.19
$i = 2$	0	0	0	40.64
$i = 3$	0	0	0	0

There is no profit in the first three years. Sardines are not selected to be captured during the 4-year period, which can threaten the ecological equilibrium.

c) **Add constraints to the model to ensure an even profit over time.**

$$\sum_{i=1}^{3} p_{it}x_{it} \leq 1.15 \sum_{i=1}^{3} p_{i,t-1}x_{i,t-1},\ t = 2,3,4 \tag{36}$$

$$\sum_{i=1}^{3} p_{it}x_{it} \geq 0.85 \sum_{i=1}^{3} p_{i,t-1}x_{i,t-1},\ t = 2,3,4 \tag{37}$$

Constraints (36) and (37) ensure that the profit obtained in each period neither is larger than 1.15 times the profit obtained in the previous period nor smaller than 0.85 times this profit.

The optimal solution obtained with these new constraints is displayed in the following table. The present profit of this solution is 273736.51 €.

Not only there is a profit every year as also sardine is captured.

	x_{it} (t)			
x_{it}	$t = 1$	$t = 2$	$t = 3$	$t = 4$
$i = 1$	7.85	0	0	1.50
$i = 2$	4.51	0	0	3.01
$i = 3$	0	17.20	15.21	8.33