INSTITUTO SUPERIOR DE AGRONOMIA

Applied Operations Research - 1^{st} test

March 15th, 2019

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1. Consider the following LP problem:

$$\begin{array}{lrl} \text{Max} & Z = x_1 + 1.5x_2 & \\ \text{s.t.} & x_1 + 2x_2 & \leq 8 \\ & 3x_1 - x_2 & \geq 3 \\ & 2x_1 + x_2 & \leq 10 \\ & x_1, \quad x_2 & \geq 0 \end{array}$$

a) Graphically represent the feasible region and the set of feasible solutions with the objective function value equal to 3.

b) Determine an optimal solution and the corresponding objective function value. Explain the process you follow to obtain such a solution.

c) Which constraints are binding at the optimal solution?

d) Rewrite the objective function as $Z = ax_1 + 1.5x_2$ ($a \in \mathbb{R}$). Compute the values of a for which the solution determined in b) remains optimal.

e) Write the problem in the standard form.

f) Define basic feasible solution to this problem. How many basic feasible solutions does the problem have? Give an example of one basic feasible solution and illustrate the previous definition by this example.

g) Does the problem have degenerate basic feasible solutions?

2. Consider a minimization problem P with two variables, x_1 and x_2, and two constraints. Let s_1 and s_2 denote the slack variables associated to these constraints in the standard form. Let u be a basic feasible solution with x_1 and s_2 as basic variables. The system of constraints in the standard form

$\left\{ \begin{array}{lcl} x_1 & = & 1 - s_1 + x_2 \\ s_2 & = & 2 - s_1 - x_2 \end{array} \right.$ and the objective function $Z = 4 - s_1 - x_2$ are both expressed in terms of the non-basic variables.

a) Indicate u and the corresponding vertex.

b) Consider that the simplex algorithm is used to solve P and u is the current basic feasible solution. Is u optimal? If it is not, find which vertex the algorithm goes next and indicate the corresponding objective function value.