INSTITUTO SUPERIOR DE AGRONOMIA

Test of Applied Operations Research - 30 May 2018

Integer Programming - SOLUTION

1. (7val.) As the leader of a wildlife exploration venture, you would like to explore exactly four out of eight possible sites in order to maximize the annual profit. The sites are labeled as s_1, s_2, s_3, s_4, s_5, s_6, s_7 and s_8, and the expected associated annual profits (in 10^4 €) are given in the table below.

Site	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
Profit (10^4 €)	3	4	6	4	2.5	7	2	4.5

a) **Formulate this problem in integer linear programming.**

$$\max Z = 3x_1 + 4x_2 + 6x_3 + 4x_4 + 2.5x_5 + 7x_6 + 2x_7 + 4.5x_8 \quad (1)$$

subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 4 \quad (2)$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \in \{0, 1\}, \quad (3)$$

where

$i = 1, ..., 8$, $x_i = \begin{cases} 1 & \text{if site } s_i \text{ is explored} \\ 0 & \text{otherwise.} \end{cases}$

b) **Find an optimal solution of the problem and calculate the corresponding profit.**
Exploring sites 6, 3, 8 and 2, with the annual profit of 21.5×10^4 €.

c) **Formulate constraints for the following conditions:**

i) **Sites s_3 and s_6 can not be explored simultaneously.**
$x_3 + x_6 \leq 1$.

ii) **If site s_2 is explored, then site s_5 must also be explored.**
$x_2 \leq x_5$.

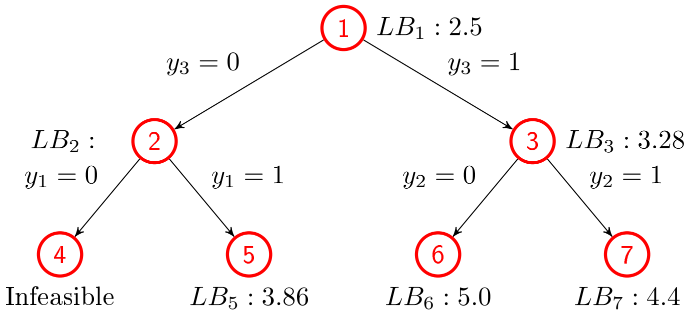
iii) **If site s_3 and s_4 are both explored, then site s_7 must also be explored.**
$x_3 + x_4 - 1 \leq s_7$.

iv) **If site s_3 and s_6 are both explored, then site s_8 can not be explored.**
$x_8 \leq 2 - (x_3 + x_6)$.

2. (3val.) Consider the mixed integer linear programming model

$$\begin{array}{llcl}
\text{Min} & Z = -2x + 3y_1 + 2y_2 + 3y_3 & & \\
\text{s.t.} & x + y_1 + y_2 + y_3 & \geq & 2 \\
& 10x + 5y_1 + 3y_2 + 4y_3 & \leq & 10 \\
& x & \geq & 0 \\
& y_1, \quad y_2, \quad y_3 & \in & \{0, 1\}
\end{array}$$

and an incomplete branch-and-bound for the model, where node 1 represents its linear relaxation (LB_i is the objective function value of the optimal solution obtained at node i, displayed in the table below).



Node #	x	y_1	y_2	y_3
1	0.5	0	1	0.5
2	0.4	0.6	1	0
3	0.43	0	0.57	1
4	-	-	-	-
5	0.29	1	0.71	0
6	0.2	0.8	0	1
7	0.3	0	1	1

a) **Compute LB_2.**
$LB_2 = -2(0.4) + 3(0.6) + 2(1) + 3(0) = 3.0.$

b) **Which nodes can be pruned?**
Node 4 because it is infeasible; node 7 because gives a feasible solution of the mixed integer linear programming model; node 6 because the corresponding solution is already worse than the feasible solution known ($LB_6 > 4.4$). Node 5 cannot be pruned because it is possible to obtain further from this node a feasible solution better than the best feasible solution known ($LB_5 < 4.4$).

c) **Between which values does the objective function value of an optimal solution of the model lie?**
Between 3.86 (the unexplored node) and 4.4 (the best solution found so far).