1. $f(x) = x e^{-x^2}$

a) $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} x e^{-x^2} \underset{\infty \cdot 0}{=} \lim\limits_{x\to+\infty} \dfrac{x}{e^{x^2}} \overset{R.C}{=} \lim\limits_{x\to+\infty} \dfrac{1}{2x e^{x^2}} = 0$

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} x e^{-x^2} = \lim\limits_{x\to-\infty} \dfrac{x}{e^{x^2}} = \lim\limits_{x\to-\infty} \dfrac{1}{2x e^{x^2}} = 0$

b) $m = \lim\limits_{x\to+\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to+\infty} \dfrac{x e^{-x^2}}{x} = \lim\limits_{x\to+\infty} e^{-x^2} = 0$

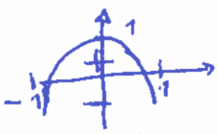
$m = \lim\limits_{x\to-\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to-\infty} \dfrac{x e^{-x^2}}{x} = \lim\limits_{x\to-\infty} e^{-x^2} = 0$

$b = \lim\limits_{x\to+\infty} f(x) - 0\,x = 0$

$b = \lim\limits_{x\to-\infty} f(x) - 0\,x = 0$

assintota à direita e à esquerda $y = 0$

c) $f'(x) = e^{-x^2} + x e^{-x^2}(-2x) = \underbrace{e^{-x^2}}_{>0} \underbrace{(1 - 2x^2)}_{\text{depende}}$



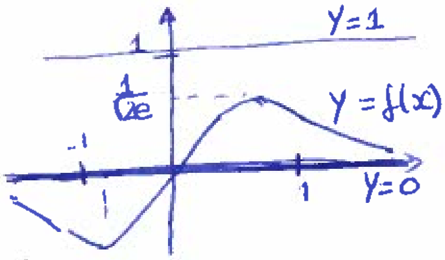
	$-\infty$	$-\frac{1}{\sqrt{2}}$		$\frac{1}{\sqrt{2}}$	$+\infty$
$f'(x)$	$-$	0	$+$	0	$-$
$f(x)$	$\searrow$	MIN	$\nearrow$	MAX	$\searrow$

$1 - 2x^2 = 0 \Leftrightarrow -2x^2 = -1$
$\Leftrightarrow x^2 = \frac{1}{2} \Leftrightarrow x = \pm\frac{1}{\sqrt{2}}$

$f\left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}} e^{-\frac{1}{2}} = -\frac{1}{\sqrt{2e}}$

$f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} e^{-\frac{1}{2}} = \frac{1}{\sqrt{2e}}$

d)



Não!

e) $P f(x) = \frac{-1}{2} P\, {-2x e^{-x^2}} = -\frac{1}{2} e^{-x^2}$

$F(x) = -\frac{1}{2} e^{-x^2} + C$

$F(0) = -\frac{1}{2} e^{0} + C = 1 \Leftrightarrow -\frac{1}{2} + C = 1 \Leftrightarrow C = 1 + \frac{1}{2} = \frac{3}{2}$

$F(x) = -\frac{1}{2} e^{-x^2} + \frac{3}{2}$

f) $F = Pf \Leftrightarrow F' = f$
$G = Pf \Leftrightarrow G' = f$

recta tg ao gráfico de F em x=a tem declive $F'(a) = f(a)$
" " " G " $G'(a) = f(a)$

$\Rightarrow$ são //

② $f(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ \frac{1}{1+x^2}, & x > 1 \end{cases}$

a) $\int_0^2 f(x)\,dx = \int_0^1 3x^2 dx + \int_1^2 \frac{1}{1+x^2} dx = [x^3]_0^1 + [\text{arctg}\, x]_1^2 =$

$P\,3x^2 = \frac{3x^3}{3} = x^3$

$= 1 + \text{arctg}\, 2 - \text{arctg}\, 1 =$
$= 1 + \text{arctg}\, 2 - \frac{\pi}{4}$

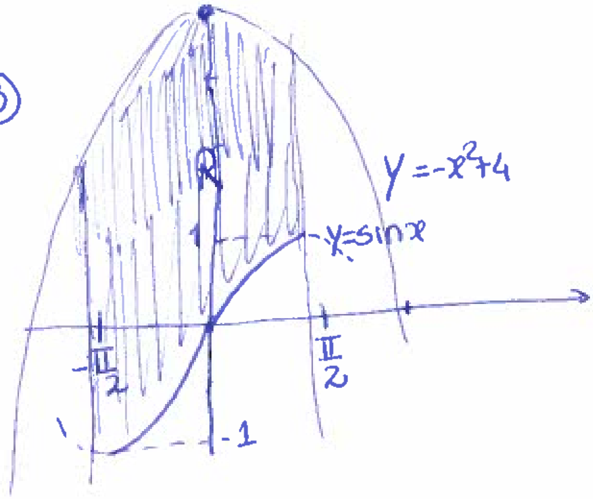
b) $\int_0^x f(t)\,dt = \begin{cases} \int_0^x 3t^2 dt & 0 \le x \le 1 \\ \int_0^1 3x^2 dx + \int_1^x \frac{1}{1+t^2} dt, & x > 1 \end{cases}$

$[t^3]_0^x = x^3$

$[\text{arctg}\, t]_1^x =$
$\text{arctg}\, x - \text{arctg}\, 1 =$
$\text{arctg}\, x - \frac{\pi}{4}$

$= \begin{cases} x^3 & 0 \le x \le 1 \\ 1 + \text{arctg}\, x - \frac{\pi}{4}, & x > 1 \end{cases}$

③



$-x^2+4=0 \Leftrightarrow x^2=4 \Leftrightarrow x=\pm 2$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -x^2+4-\sin x \, dx = \left[-\frac{x^3}{3}+4x+\cos x \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = -\frac{\left(\frac{\pi}{2}\right)^3}{3}+4\frac{\pi}{2}+\cos\frac{\pi}{2} -$$

$$-\left(-\frac{\left(-\frac{\pi}{2}\right)^3}{3}+4\left(-\frac{\pi}{2}\right)+\cos\left(-\frac{\pi}{2}\right) \right) = -\frac{\pi^3}{24}+2\pi-\frac{\pi^3}{24}+2\pi = 4\pi-\frac{2\pi^3}{24}=4\pi-\frac{\pi^3}{12}$$

④ a) $AB^T = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 & -1 \\ 0 & 3 & 0 \\ 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$

b) $AB^T=6I \Leftrightarrow \frac{1}{6}AB^T=I \Leftrightarrow A\left(\frac{1}{6}B^T\right)=I$

$\Rightarrow A^{-1}=\frac{1}{6}B^T = \begin{bmatrix} 1/2 & -1/6 & -1/6 \\ 0 & 1/2 & 0 \\ 0 & -1/6 & 1/3 \end{bmatrix}$

c) i) u é sol. de $Ax=b \Leftrightarrow Au=b \Leftrightarrow \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix} = b$

ii) $\exists A^{-1} \Leftrightarrow x=A^{-1}b$ e o sistema é det. (tem uma solução apenas)

$\Rightarrow \nexists$ outra solução de $Ax=b$ para além de u

d) $\begin{cases} (x_1,x_2,x_3)\,|\,(2,0,0)=0 \\ (x_1,x_2,x_3)\,|\,(1,2,1)=0 \end{cases} \Leftrightarrow \begin{cases} 2x_1=0 \\ x_1+2x_2+x_3=0 \end{cases} \Leftrightarrow \begin{cases} x_1=0 \\ 2x_2+x_3=0 \end{cases}$

$\Leftrightarrow \begin{cases} x_1=0 \\ x_3=-2x_2 \\ x_2=\forall \end{cases}$ $\qquad (0, x_2, -2x_2)$

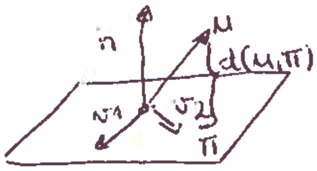
$\|(0,x_2,-2x_2)\| = \sqrt{x_2^2+4x_2^2} = \sqrt{5x_2^2}$

$\|(0,x_2,-2x_2)\| = \sqrt{5} \Leftrightarrow \sqrt{5x_2^2}=\sqrt{5} \Leftrightarrow 5x_2^2=5$

$x_2^2=1 \Leftrightarrow x_2=\pm 1$

$(0,1,-2)$ e $(0,-1,2)$

e)



$$d(u,\pi) = \| \mathrm{proj}_n u \| =$$

$$\mathrm{proj}_n u = \frac{n|u.n}{n.n} = \frac{(0,1,-2)|(1,1,1)}{(0,1,-2)|(0,1,-2)}(0,1,-2) = \frac{1-2}{1+4}(0,1,-2) =$$

$$= -\frac{1}{5}(0,1,-2) = \left(0,-\frac{1}{5},\frac{2}{5}\right) //$$