Multiple Objective Decision Analysis

Multiple-Objective Optimization

Susana Barreiro

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Multiple Objective Decision Analysis

- We have assumed so far that **linear programming** encompasses a **single overriding objective** (e.g. maximizing total profit / minimizing total cost).
- Most times this is not realistic since we frequently focus on a variety of objectives, e.g. forest management:
  - to maintain stable profit,
  - increase wood production,
  - diversify ecosystem services,
  - restrain the impact of pests /diseases,
  - minimize erosion,
  - …

Goal programming provides a way of achieving **several objectives *simultaneously*.**

representing some goals by constraints in effect gives them priority over the goal reflected in the objective function, because the objective function is optimized within the feasible region defined by the constraints

Multiple Objective Decision Analysis

- Representing some goals by constraints in effect gives them priority over the goal reflected in the objective function, because the objective function is optimized within the feasible region defined by the constraints

- Deciding which goal should be selected as the objective function and which ones should be reflected by constraints is often arbitrary and difficult

- Goal programming attempts to overcome these limitations still while using linear programming, striving toward selected objectives simultaneously, treating them all in the same manner, although perhaps giving them different weights.

Multiple Objective Decision Analysis

- Decision makers try to balance multiple objectives (e.g., cost, performance, reliability) none of which is obviously the best
- Multiple Objective Decision Analysis (MODA) is an operations research technique for evaluating a decision under multiple, sometimes **competing and conflicting** objectives or criteria.
- MODA provides a process that systematically identifies alternatives and the decision maker's objectives that serves as the measuring device for selecting the preferred alternative given the decision **space**.
- **Multi-objective optimization** leads to a set of optimal solutions because no solution can be considered better than any other regarding all the objectings. These solutions are known as Pareto-optimal solutions

Multiple Objective Decision Analysis

Single objective

Selecting the eucalyptus clone with the highest volume production

Might lead to having:

- Frost sensitiveness
- Low re-sprouting ability
- Medium fiber quality
- Moderate resistance to defoliation
- Production cost

Multi-objective

We may want to optimize with the following objectives

- Volume production
- Frost resistance
- Re-sprouting ability
- Fiber quality
- Resistance to defoliation
- Production cost

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We may want to optimize with the following objectives

- Volume production (obj. 1)
- Frost resistance (obj. 2)
- Re-sprouting ability (obj. 3)
- Fiber quality (obj. 4)
- Resistance to defoliation (obj. 5)
- Production cost (obj. 6)

Multiple Objective Decision Analysis

Each solution will have 6 objective values indicating which

This poses several difficulties but the most significant is how we decide which solution is better than another?

Multi-objective

We may want to optimize with the following objectives

- Volume production (obj. 1)
- Frost resistance (obj. 2)
- Re-sprouting ability (obj. 3)
- Fiber quality (obj. 4)
- Resistance to defoliation (obj. 5)
- Production cost (obj. 6)

Multiple Objective Decision Analysis - Optimization

Multi-objective optimization problems with a number of equality and inequality constraints can be formulated as:

Minimize / Maximize $f_i(x)$ $i = 1,\ldots, N_{obj}$ – number of objective functions

Subject to:

a set of constraints

To simplify the solution process, in optimization problems with a number of objective functions, **additional objective functions are usually handled as constraints**

HOWEVER, the final solutions can be satisfying those constraints they cannot be called optimal with respect to all the objective functions.

Multiple Objective Decision Analysis - Optimization

How do we carry out multi-objective optimization for more than 2 variables?

We can use evolutionary optimization algorithms

- suitable to solve multi-objective optimization problems dealing with a set of possible solutions simultaneously.
- A single run enables finding several elements of the Pareto-optimal set
- Less susceptible to the shape or continuity of the Pareto-front dealing with discontinuous or concave Pareto fronts

There are **non-Pareto** and **Pareto** techniques for multi-objective optimization using evolutionary optimization (e.g. genetic algorithm, simulated annealing, PSO methods

Multiple Objective Decision Analysis - Optimization

Evolutionary Optimization Algorithms

Non-Pareto techniques

- Approaches that do not incorporate directly the concept of **Pareto-optimum**
- Unable to reproduce **certain portions** of the Pareto front
- Efficient and easy to implement, but appropriate to handle **only a few objectives**

Pareto techniques

- Use of non-dominanted ranking and selection to move the population towards the Pareto front
- Require a **ranking procedure** and a technique to **maintain diversity** in the population

Multiple Objective Decision Analysis - Optimization

Assume we have a set of <u>feasible solutions</u> and <u>different objective functions</u>

The Pareto-improvement concept consists in moving from one feasible solution to another so that

- At least **one objective function** returns a **better value**
- And **no other** objective function becomes **worse**

When no further Pareto-improvement can be achieved the Pareto-optimal has been reached

Multiple Objective Decision Analysis

Single objective

obj. - volume production

Multi-objective

obj. 1 - volume production

obj. 2 - Frost resistance

Clone	volume (m^3 ha^{-1})
A	130
B	78
C	109
D	97
E	115

Clone	volume (m^3 ha^{-1})
A	130
B	103
C	109
D	97
E	120

Clone	volume (m^3 ha^{-1})	frost resitance
A	130	2.3
B	103	3.7
C	109	2
D	97	4.4
E	120	2.7

Clone	volume (m^3 ha^{-1})	frost resitance
A	130	2.3
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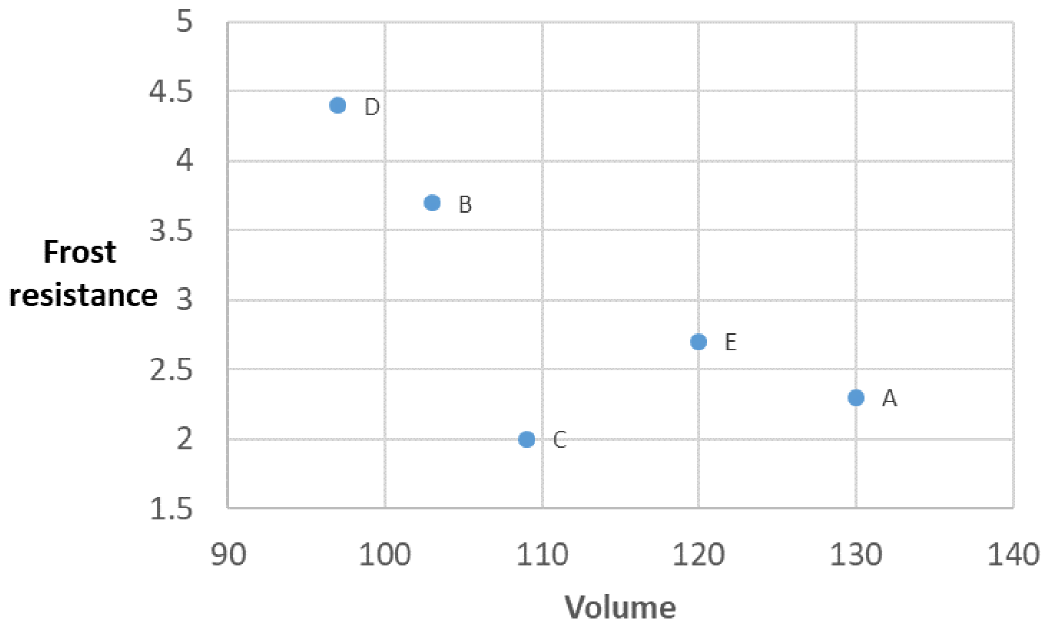
Neither solution is better than the other

Multiple Objective Decision Analysis

Let us plot the objective space:



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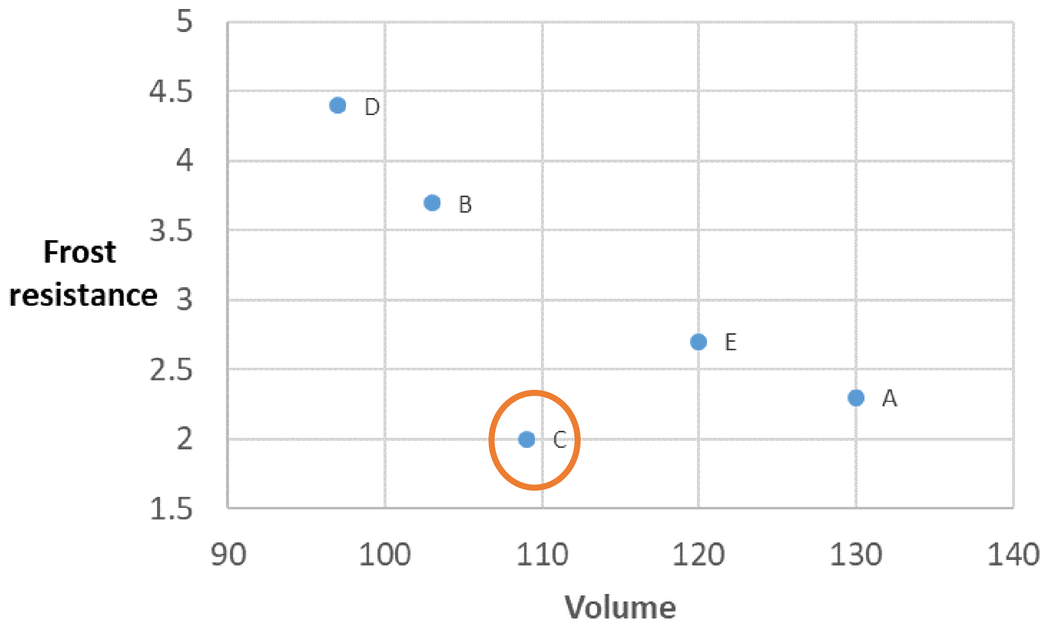

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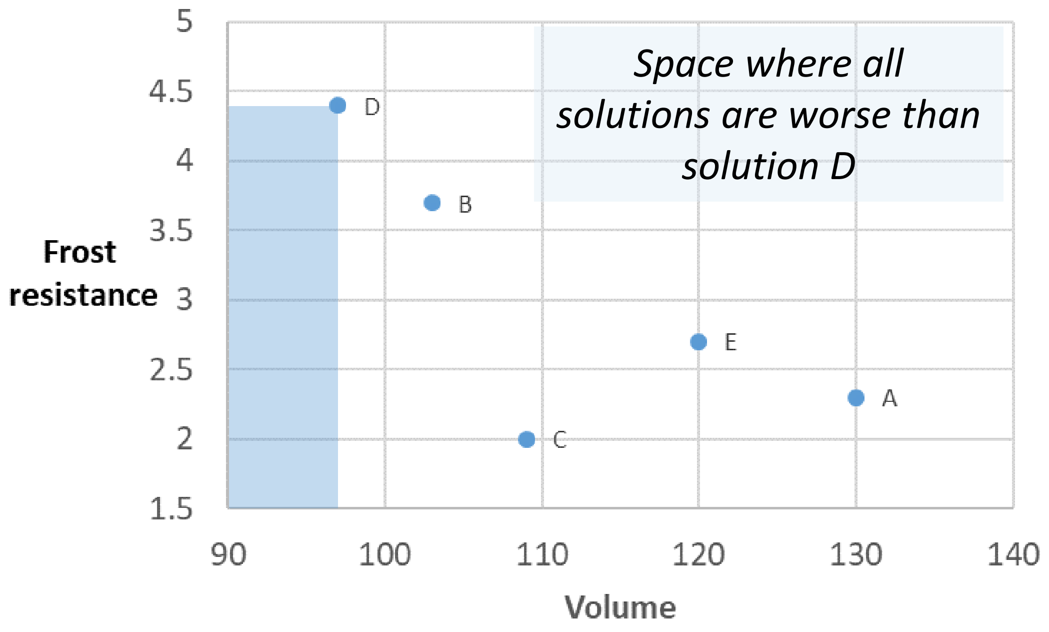

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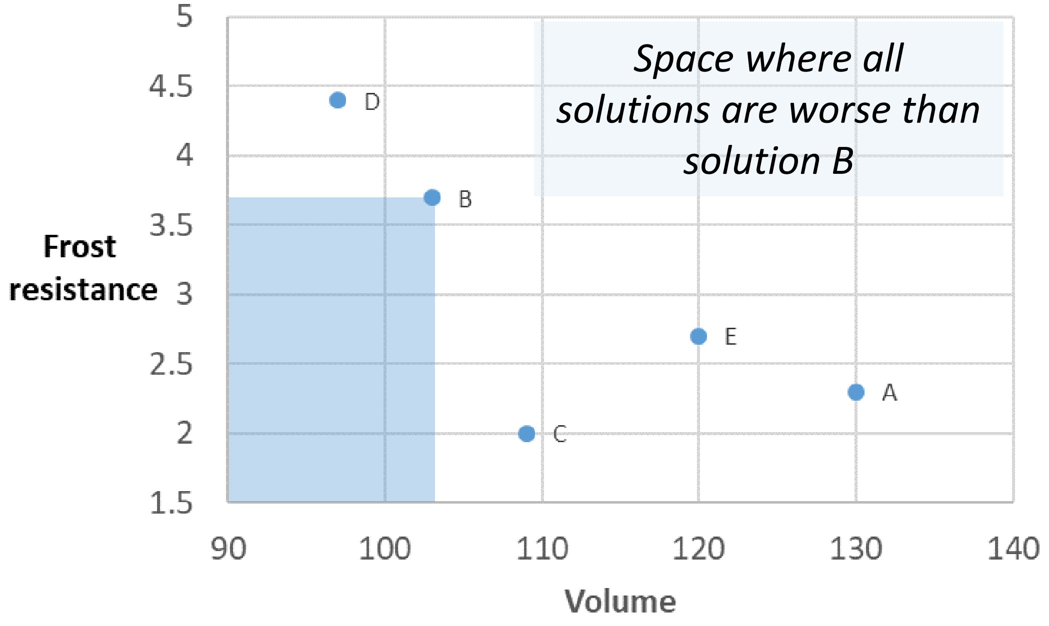

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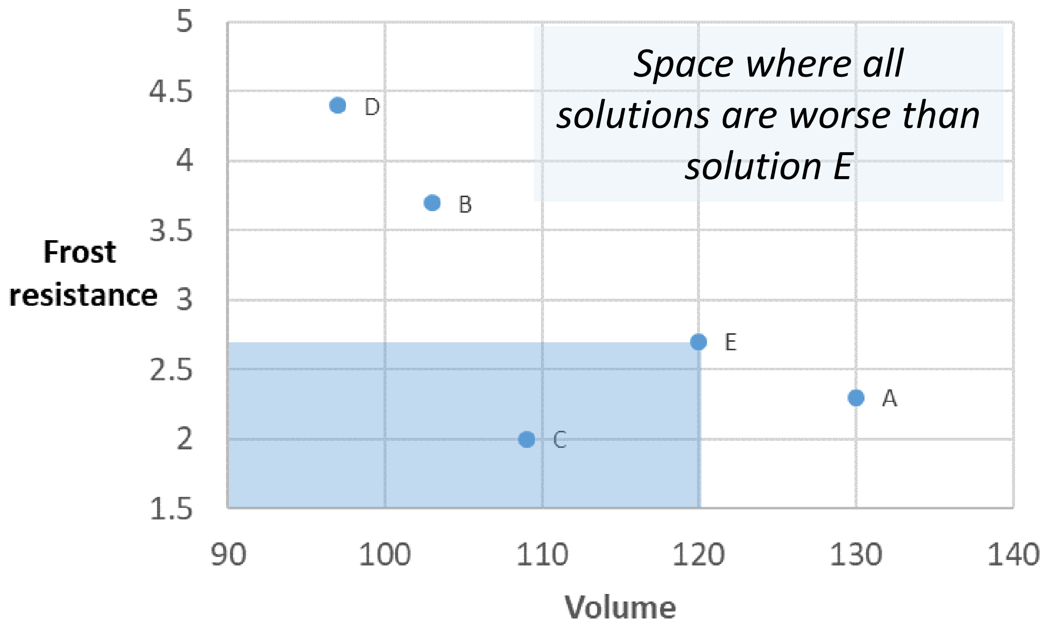

We can get rid of solution C because it does not offer us anything solution E can not, thus we can say ***solution C is dominated by solution E***

Multi-objective

obj. 1 - volume production

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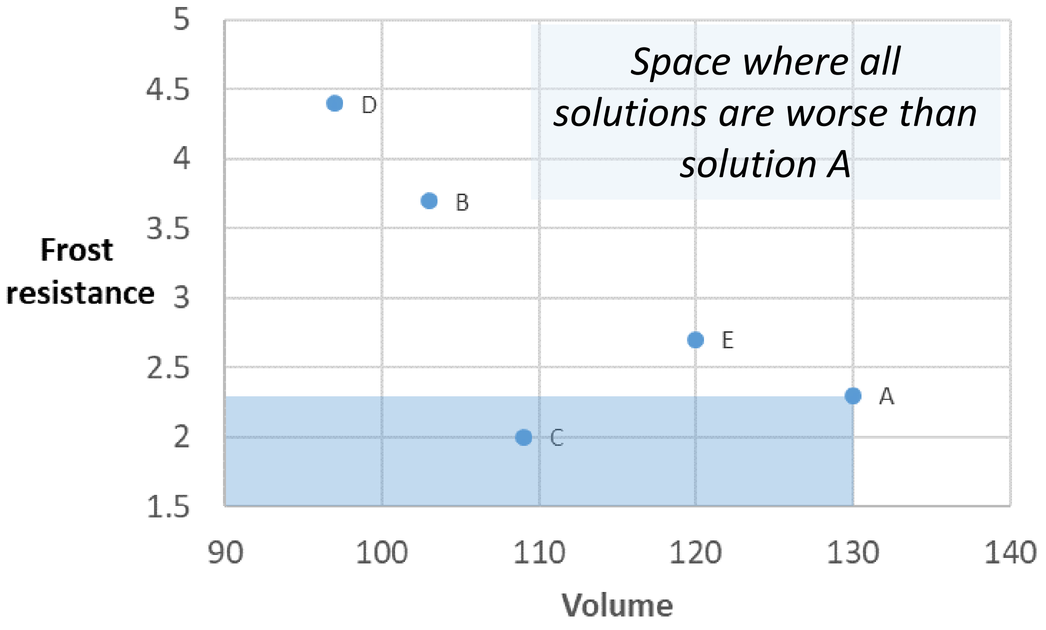

We can get rid of solution C because it does not offer us anything solution A can not, thus we can say ***solution C is dominated by solution A***

Multi-objective

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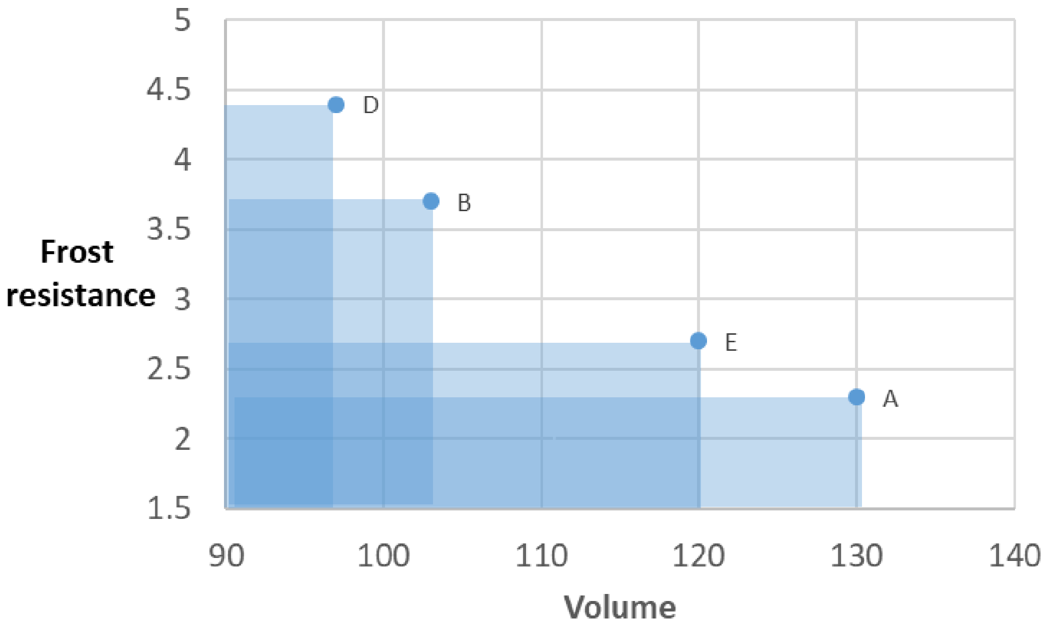
Multiple Objective Decision Analysis

Let us plot the objective space:

*After shading all the areas we can see that no solutions are inside the shaded area, thus **all remaining solutions are non dominated***



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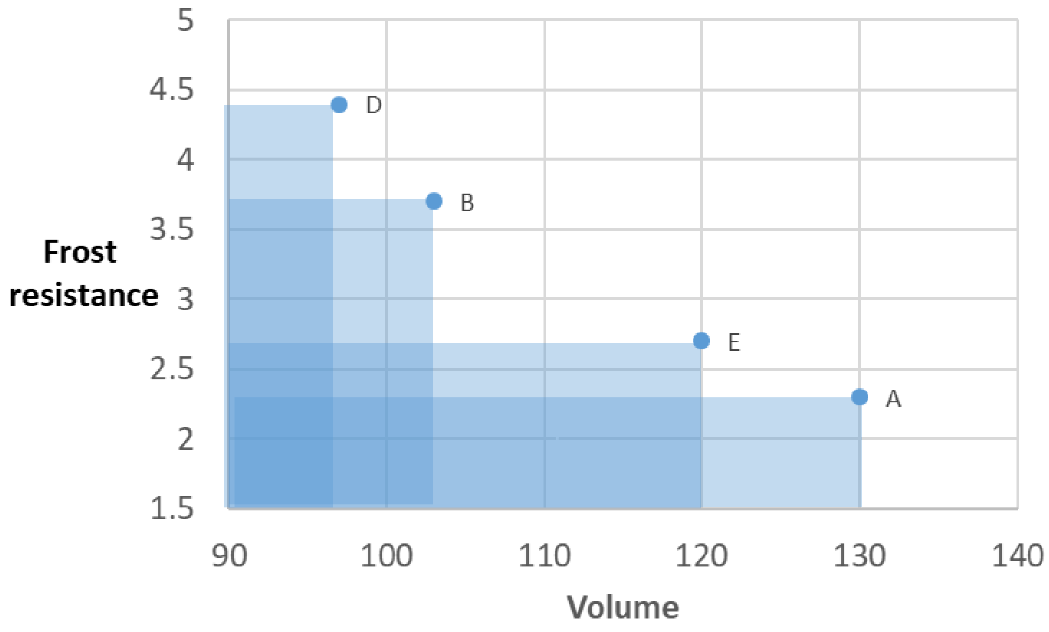

So, which of these solutions is the best?

Multi-objective

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obj. 2 - Frost resistance

Clone	volume ($m^3\ ha^{-1}$)	frost resitance
A	130	2.3
B	103	3.7
D	97	4.4
E	120	2.7

Because no solution dominates another, no solution is completely better than the other UNLESS we have some additional information about the decision makers preferences…
We may know he does not want frost resistance < 2.5

Multiple Objective Decision Analysis

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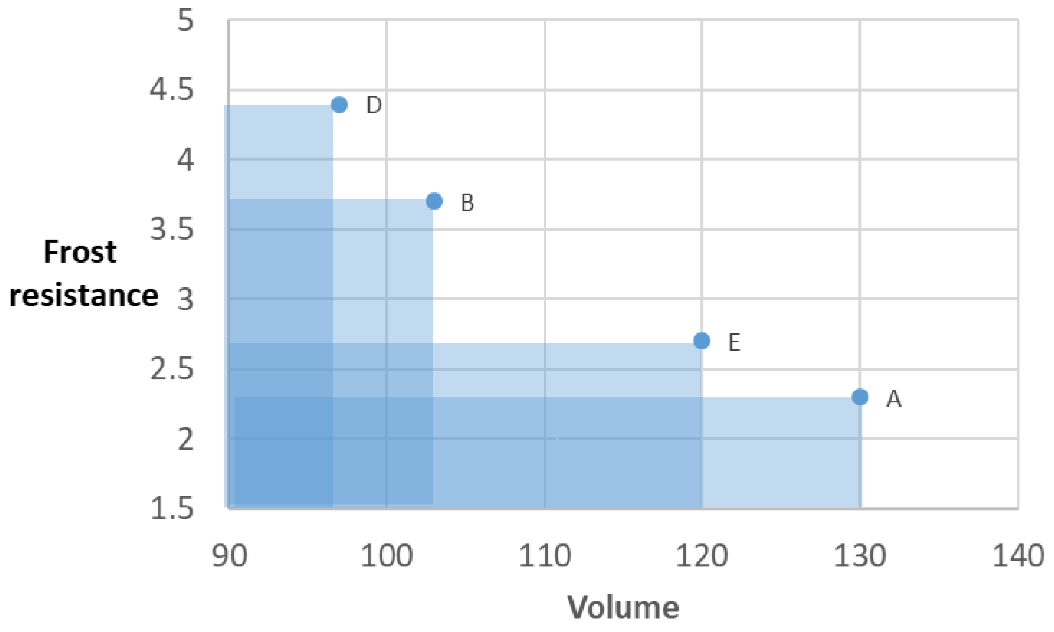
After shading all the areas we can see that no solutions are inside the shaded area, thus ***all remaining solutions are non dominated***



So, which of these solutions is the best?

Multi-objective

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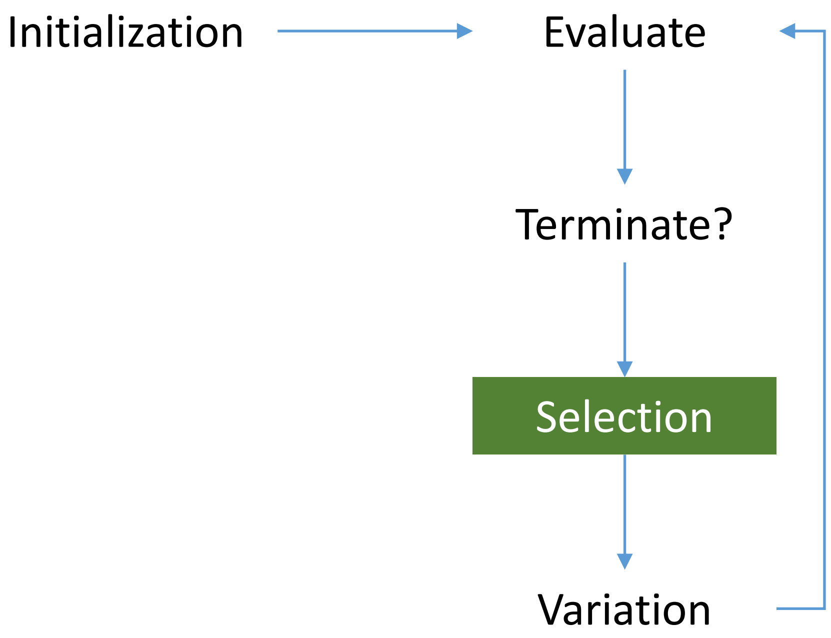
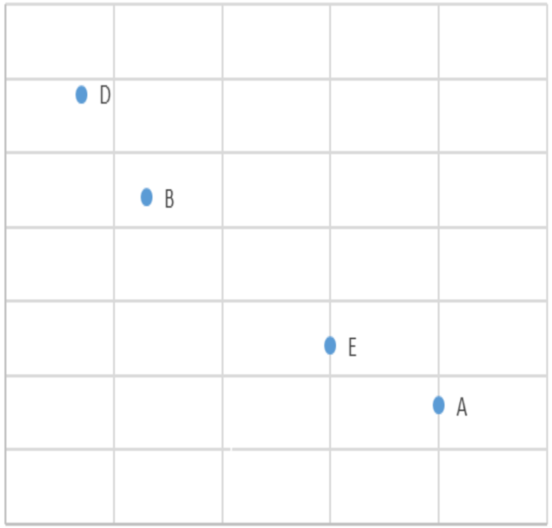
There are several ways to reduce the number of solutions, but this still DOES NOT tell you which is best

Multiple Objective Decision Analysis

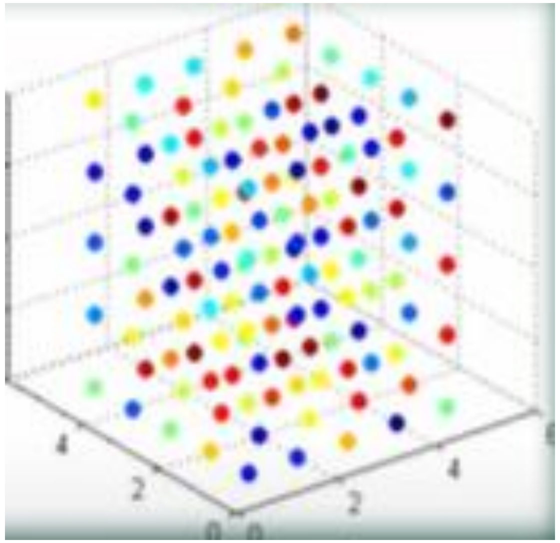
Challenges: Selecting the best solution, Visualization



2D



3D



How we visualize the objective space is not complicated for 2 variables, but for 3 it gets difficult to see which solutions are dominated and for 4…

Multiple Objective Decision Analysis - Optimization

Evolutionary Optimization Algorithms

Non-Pareto techniques

- Aggregating approaches
- Vector evaluated genetic algorithm (VEGA)
- Lexicographic ordering
- Target vector approaches

Pareto techniques

- Multi-objective genetic algorithm (MOGA)
- Non-dominated sorting genetic algorithm-II (NSGA-II)
- Multi-objective particle swarm optimization (MOPSO)
- Strength Pareto evolutionary algorithm (SPEA-II)

Multiple Objective Decision Analysis - Optimization

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Multiple Objective Decision Analysis - Optimization

Non-dominated sorting genetic algorithm-II (NSGA-II)

This method was proposed by Deb et al. in 2000

Key features:

- Emphasizes non-dominated sorting
- Uses a diversity preserving mechanism
- Performs crowding comparison
- Uses elitist principle: some parents go directly to the next generation based on the above mentioned conditions

Multiple Objective Decision Analysis - Optimization

Non-dominated sorting genetic algorithm-II (NSGA-II)

The algorithm uses an **evolutionary process** including selection, genetic crossover, and genetic mutation. *(In the following example we will skip this part)*

The population of solutions is sorted into a hierarchy of sub-populations based on the ordering of Pareto dominance.

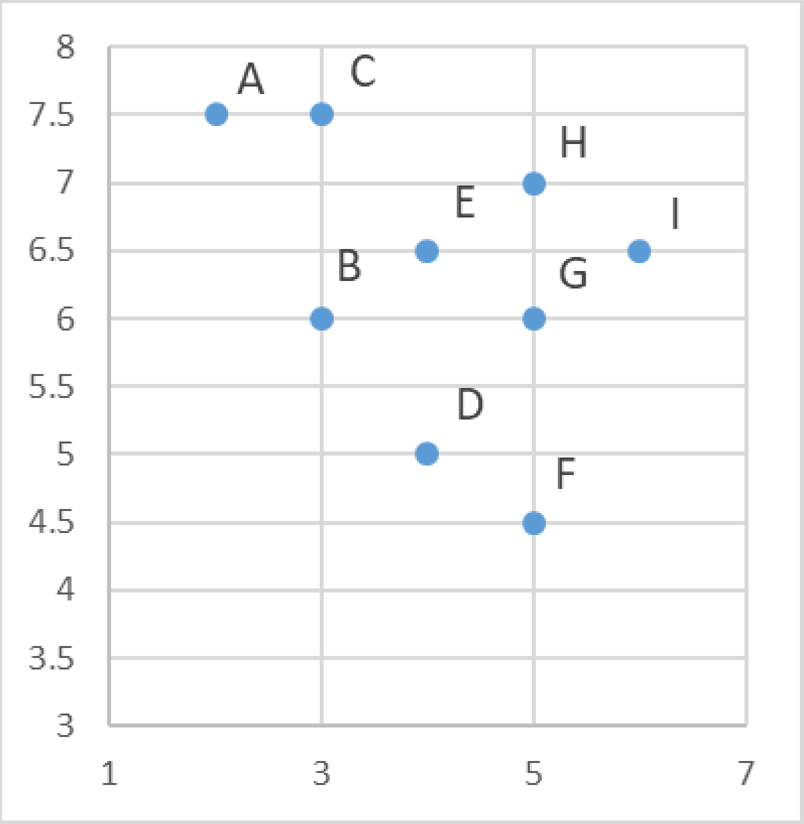
Similarity between members of each sub-group is evaluated on the Pareto-front

Similarity measures are used to promote a diverse front of non-dominated solutions.

Multiple Objective Decision Analysis - Optimization

Non-dominated sorting genetic algorithm-II (NSGA-II)

- non-dominated sorting



A&B

A is better in terms of time, but
B is better in terms of cost

THUS, A&B are a **non-dominated** set

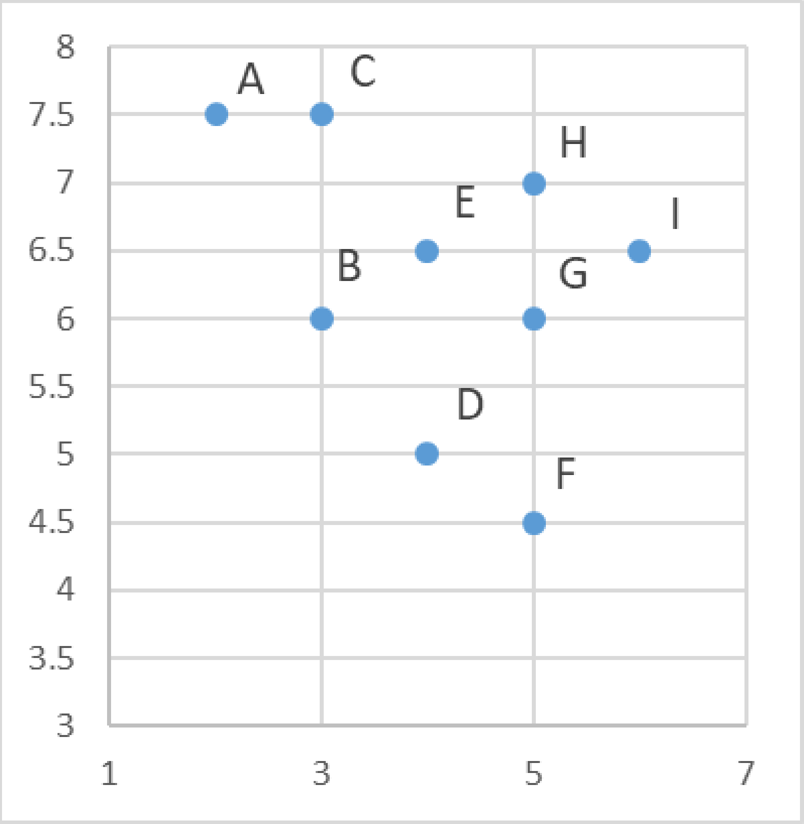
Minimize Time & **Minimize** Cost

soil preparation	Time spent (hours)	Cost (100€)
A	2	7.5
B	3	6
C	3	7.5
D	4	5
E	4	6.5
F	5	4.5
G	5	6
H	5	7
I	6	6.5

Multiple Objective Decision Analysis - Optimization

Non-dominated sorting genetic algorithm-II (NSGA-II)

- non-dominated sorting



A&B

A is <u>better</u> in terms of time, but
B is <u>better</u> in terms of cost

THUS, A&B are a **non-dominated** set

A&C

A is <u>better</u> in terms of time, but
C is <u>equally good</u> in terms of cost

THUS, A **dominates** C

... (see Excel)

Minimize Time & **Minimize** Cost

soil preparation	Time spent (hours)	Cost (100€)
A	2	7.5
B	3	6
C	3	7.5
D	4	5
E	4	6.5
F	5	4.5
G	5	6
H	5	7
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Multiple Objective Decision Analysis - Optimization

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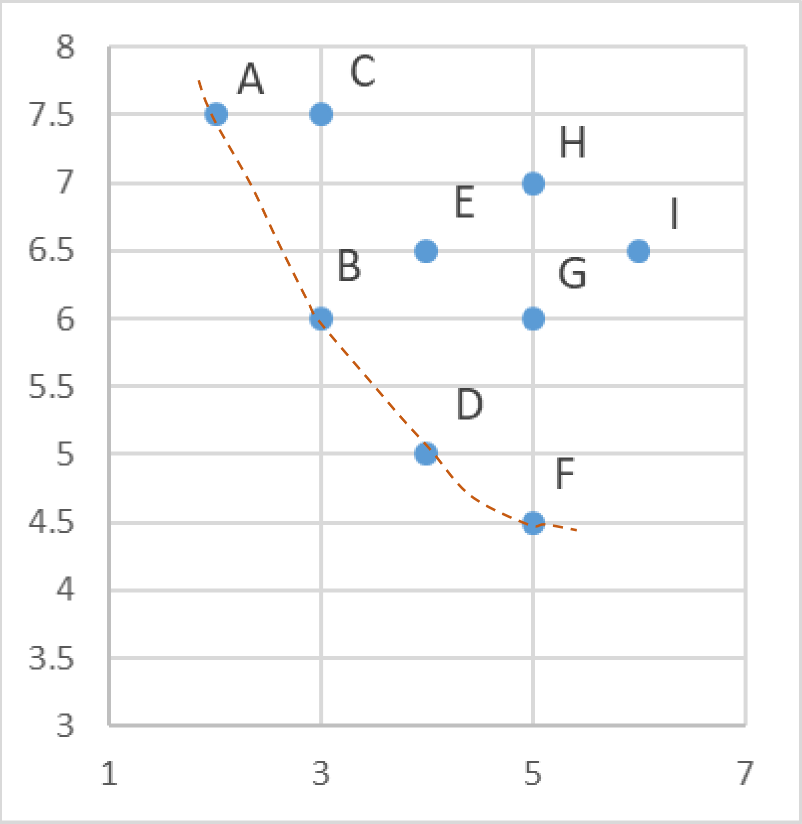
- non-dominated sorting



A **dominates** C
B **dominates** C, E, G, H, I
D **dominates** E, G, H, I
F **dominates** G, H, I

THUS

A, B, D, F form a **non-dominated** set

A, B, D, F form the **Pareto-optimal front**

Minimize Time & **Minimize** Cost

soil preparation	Time spent (hours)	Cost (100€)
A	2	7.5
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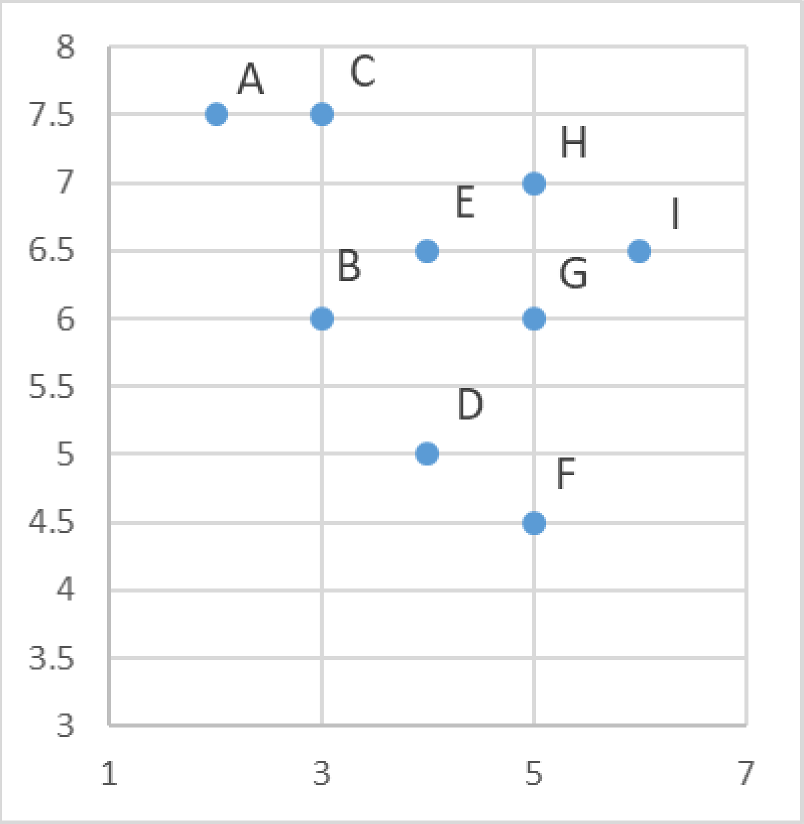
Multiple Objective Decision Analysis - Optimization

Non-dominated sorting genetic algorithm-II (NSGA-II)

- non-dominated sorting



The line is the Pareto-optimal front and the solutions in it are called the **Pareto-optimal**

Solutions along the line are non-dominated

All **Pareto-optimal** solutions are non-dominated

soil preparation	Time spent (hours)	Cost (100€)
A	2	7.5
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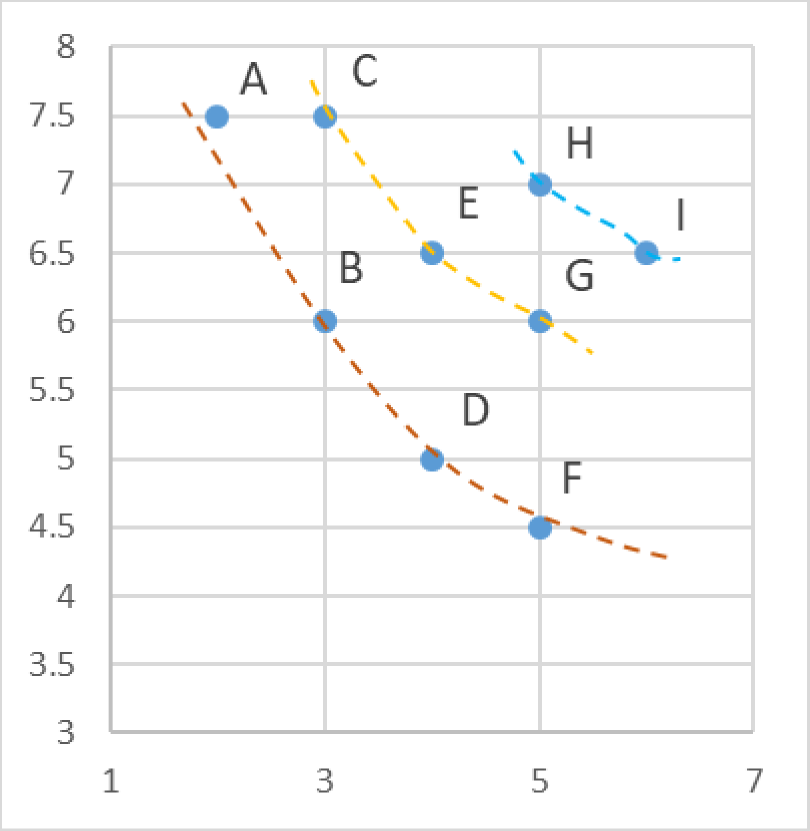
Multiple Objective Decision Analysis - Optimization

Non-dominated sorting genetic algorithm-II (NSGA-II)

- non-dominated sorting



If we repeat the comparison procedure leaving A, B, D and F out we conclude C, E and G form a **non-dominated** set

Pareto-optimal front 2 **Rank 2**

Pareto-optimal front 1 **Rank 1**

soil preparation	Time spent (hours)	Cost (100€)
A	2	7.5
B	3	6
C	3	7.5
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Multiple Objective Decision Analysis - Optimization

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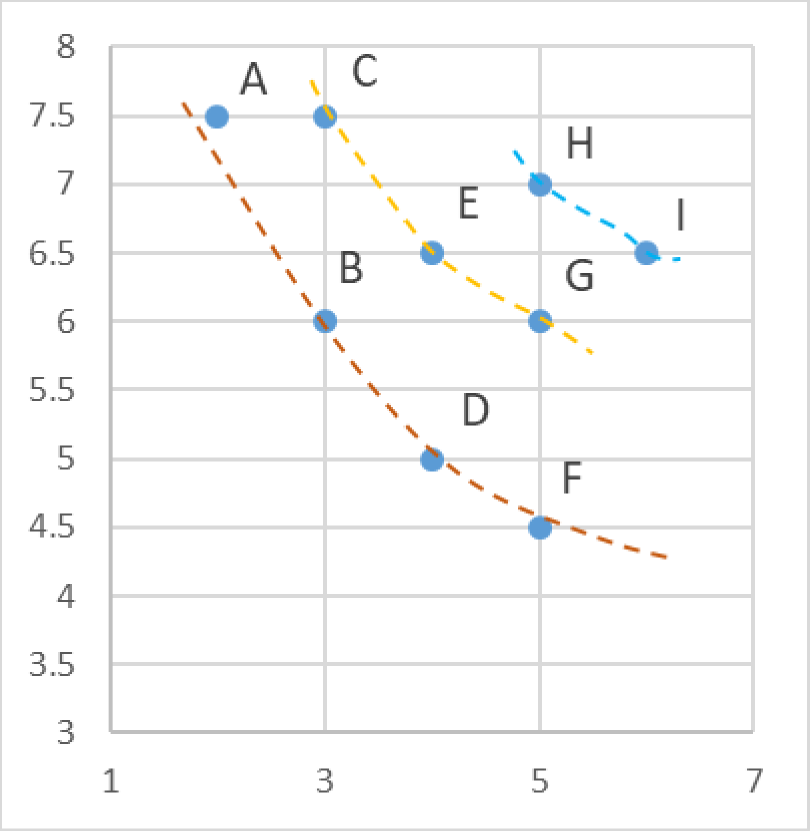

Pareto-optimal front 3 **Rank 3**

Pareto-optimal front 2 **Rank 2**

Pareto-optimal front 1 **Rank 1**

When selecting **Rank 1 has priority over 2** and 2 over 3, etc

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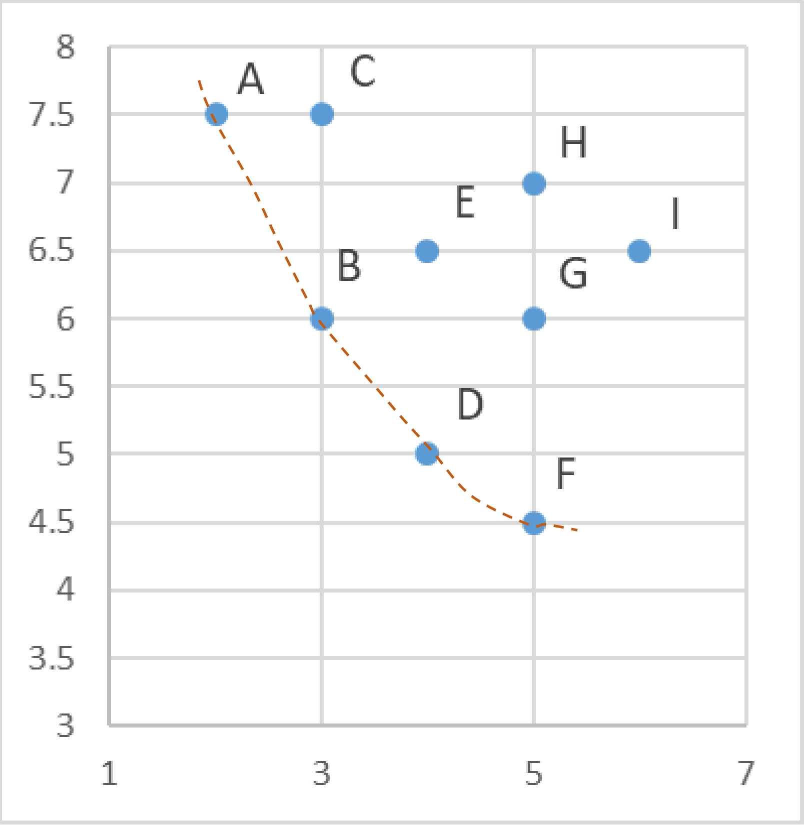
Multiple Objective Decision Analysis - Optimization

Non-dominated sorting genetic algorithm-II (NSGA-II)

- Population diversity and crowding



Pareto-optimal front 1 **Rank 1**

How to choose solutions in the same Pareto-optimal front ?

If we had to **choose 3 out of 4**, which would we choose?

We have to ensure population diversity so choosing solutions along the font will serve the purpose

A and F being at the extremes will be chosen, but we have to decide among B and D

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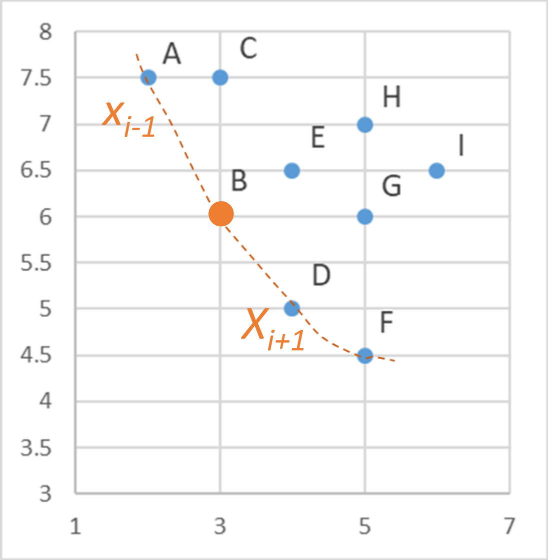
Multiple Objective Decision Analysis - Optimization

Non-dominated sorting genetic algorithm-II (NSGA-II)

- Population diversity and crowd distance (CD)



Pareto-optimal front 1

1) Sort all the solutions in the Pareto front in ascending order for each objective $(f_m(x_{i+1}) > f_m(x_{i-1}))$ and compute CD:

$$CD_{im} = \frac{(f_m(x_{i+1}) - f_m(x_{i-1}))}{(f_m(x_{max}) - f_m(x_{min}))}$$

2) Repeat the process for each objective, the distance will result of the sum of the CD for all options/solutions

$$CD_i = \sum_{m=1}^{M} CD_{im}$$

3) Given 2 solutions, the one preferred is the one with the highest CD value

Where m is the number of objectives, I is the number of options/solutions in the Pareto-optimal front and f_m the values of each objective under each option

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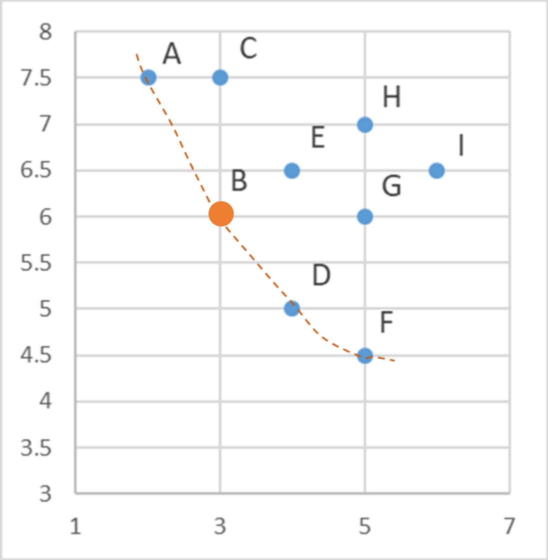

Pareto-optimal front 1

$$CD_{im} = \frac{(f_m(x_{i+1}) - f_m(x_{i-1}))}{(f_m(x_{max}) - f_m(x_{min}))}$$

$$(f_m(x_{i+1}) > f_m(x_{i-1}))$$

decide among B and D

Let us look at the crowding distance for B

soil preparation	Time spent (hours)	Cost (100€)
A	$\boxed{2}$	$\boxed{7.5}$
B	3	6
D	$\boxed{4}$	$\boxed{5}$
F	5	4.5

CD_B (Cost) = (7.5 -5) / (7.5 -4.5) = 2.5/3 = 0.83

CD_B (Time) = (4 -2) / (6 -2) = 2/4 = 0.5

CD_B (Cost + Time) = 0.83 + 0.5 = 1.33

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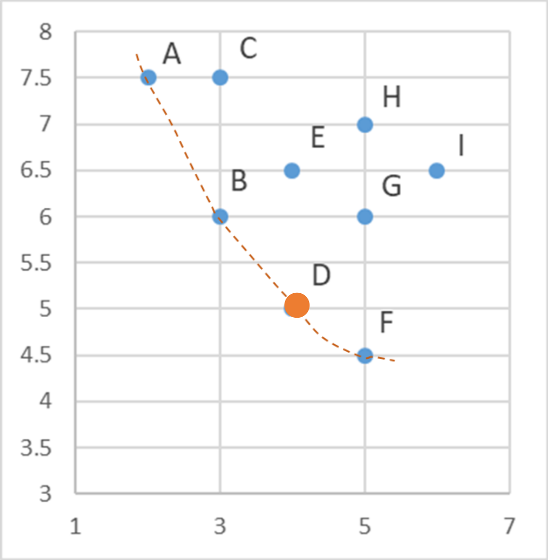


Pareto-optimal front 1

$$CD_{im} = \frac{(f_m(x_{i+1}) - f_m(x_{i-1}))}{(f_m(x_{max}) - f_m(x_{min}))}$$

$$(f_m(x_{i+1}) > f_m(x_{i-1}))$$

decide among B and D

Let us look at the crowding distance

soil preparation	Time spent (hours)	Cost (100€)
A	2	7.5
B	$\boxed{3}$	$\boxed{6}$
D	4	5
F	$\boxed{5}$	$\boxed{4.5}$

CD_D (Cost) = (6 -4.5) / (7.5 -4.5) = 1.5/3 = 0.5

CD_D (Time) = (5 -3) / (6 -2) = 2/4 = 0.5

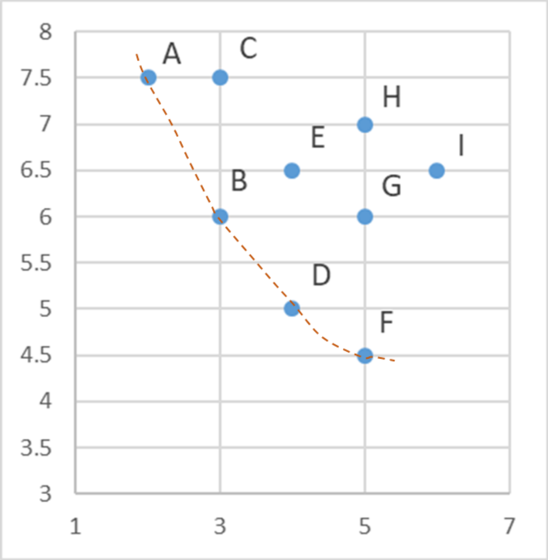
CD_D (Cost + Time) = 1.00

soil preparation	Time spent (hours)	Cost (100€)
A	$\boxed{2}$	$\boxed{7.5}$
B	3	6
C	3	7.5
D	4	5
E	4	6.5
F	5	$\boxed{4.5}$
G	5	6
H	5	7
I	$\boxed{6}$	6.5

Multiple Objective Decision Analysis - Optimization

Non-dominated sorting genetic algorithm-II (NSGA-II)

- Population diversity and crowd distance (CD)



Pareto-optimal front 1

decide among B and D

Let us look at the crowding distance

CD_B (Cost) = (7.5 -5) / (7.5 -4.5) = 2.5/3 = 0.83
CD_B (Time) = (4 -2) / (6 -2) = 2/4 = 0.5
CD_B (Cost + Time) = 0.83 + 0.5 = 1.33

CD_D (Cost) = (6 -4.5) / (7.5 -4.5) = 1.5/3 = 0.5
CD_D (Time) = (5 -3) / (6 -2) = 2/4 = 0.5
CD_D (Cost + Time) = 1.00

$CD_A = CD_F$ = infinity

CD_B = 1.33, $CD_A = CD_F$ = infinity

soil preparation	Time spent (hours)	Cost (100€)
A	2	7.5
B	3	6
C	3	7.5
D	4	5
E	4	6.5
F	5	4.5
G	5	6
H	5	7
I	6	6.5

Multiple Objective Decision Analysis - Optimization

Non-dominated sorting genetic algorithm-II (NSGA-II)

- NSGA is an **extension** of the Genetic Algorithm for multiple objective function optimization.
- Genetic algorithms provide an alternative approach to optimization in the case of more complex problems:

  - advantages - works well for problems with chaotic or ill-defined behavior and also those problems with local maxima or minima that would, perhaps, trap a conventional search algorithm; fitting several loops simultaneously

  - disadvantages - its random nature may not always find the absolute optimum solution, but offers a greater chance of quickly finding a relatively good one.

Multiple Objective Decision Analysis - Optimization

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Requires software implementation !

Multiple Objective Decision Analysis - Optimization

Addressing Multicriteria Forest Management With Pareto Frontier Methods: An Application in Portugal

Jose G. Borges, Jordi Garcia-Gonzalo, Vladimir Bushenkov, Marc E. McDill, Susete Marques, and Manuela M. Oliveira

The practice of multicriteria forest management planning is often complicated by the need to explicit a priori goals and preferences aims at describing an approach that may take advantage of a posteriori preference modeling to facilitate the specification of objectives in a typical forest management planning framework. The goal is to provide information about nondominated points i (FSCS) so that decisionmakers may take advantage of trade-off information. The emphasis is on demonstrating the potential o decisions when three or more criteria are considered. The approach combines the use of mathematical programming and interactiv how the estimation refinement method may be used to approximate the Pareto frontier of a typical model I linear programmi feasible goals method/interactive decision maps method may be used to retrieve a solution selected by stakeholders from interac frontier. Results are discussed for a large-scale test application encompassing over 1 million ha of cork and holm oak forest e

Keywords: forest management planning, cork oak, multiple criteria decisionmaking, Pareto frontier methods

The approach combines the use of mathematical programming and interactive decision maps techniques
(…)
The **feasible goals method/interactive decision maps** (**FGM/IDM**) technique is used to provide interactive and animated visualization of the Pareto frontier generated by the ERM

Conclusion: **FGM/IDM technique** may be used to develop and display the Pareto frontier in the case of problems with more than three forest management planning objectives

Multiple Objective Decision Analysis - Optimization

How do we carry out multi-objective optimization for more than 2 variables?

We can also use **Multi Objective Linear Programming (MOLP)**

Tradeoffs concept

Part of the challenge is in expressing the value trade-offs among the objectives.

These trade-offs indicate which objectives are relatively more important to the decision maker.

Which 2 objectives does a decision maker prefer: "decrease production costs" or "increase product quality"?

- Does he prefer them equally?
- Is one more important?
- How much more important is it?

Incorporating **trade-offs** into the decision-making process is valuable because it ensures decisions **integrate** the **decision maker's values**

Multiple Objective Decision Analysis - Optimization

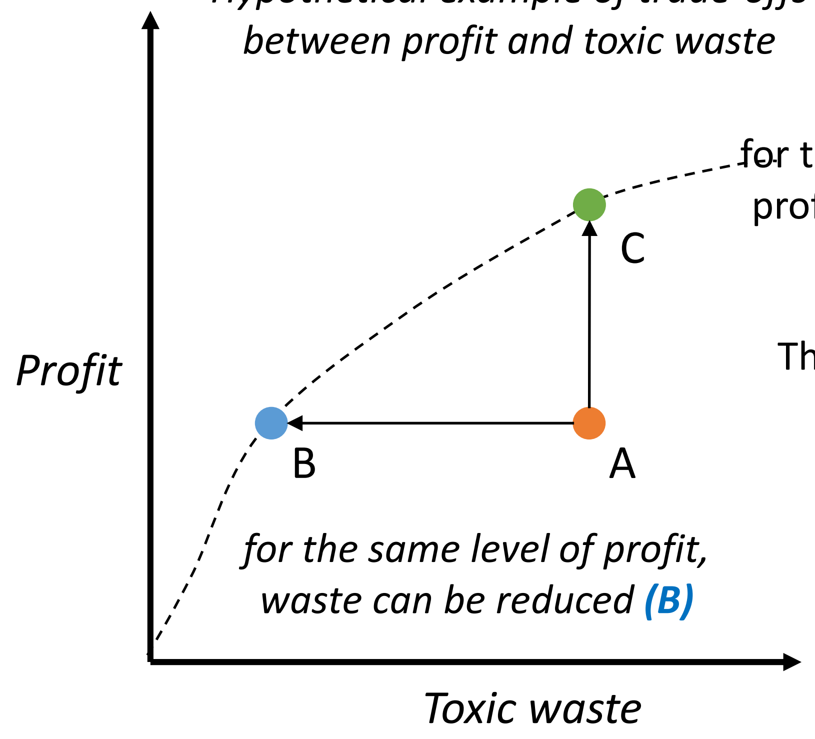
Multi Objective Linear Programming

- Take the following example:

Untreated pulp and paper mill effluents are very toxic to most aquatic life, treatment can reduce toxicity but at a cost. Increasing profit usually leads to the production of more toxic waste. Ideally the mill managers will wish to maximize profit reducing toxic waste production.

The decision maker has to decide the most desirable level of trade-offs between profit and waste

Hypothetical example of trade-offs between profit and toxic waste



for the same level of waste, profit can be increased **(C)**

The solution indicated by **point A** is clearly undesirable because

Each point on the curve corresponds to a possible level of profit and the minimum amount of waste that must be produced to achieve it

Multiple Objective Decision Analysis - Optimization

Multi Objective Linear Programming

- Take the following example:

Untreated pulp and paper mill effluents are very toxic to most aquatic life, treatment can reduce toxicity but at a cost. Increasing profit usually leads to the production of more toxic waste. Ideally the mill managers will wish to maximize profit reducing toxic waste production.

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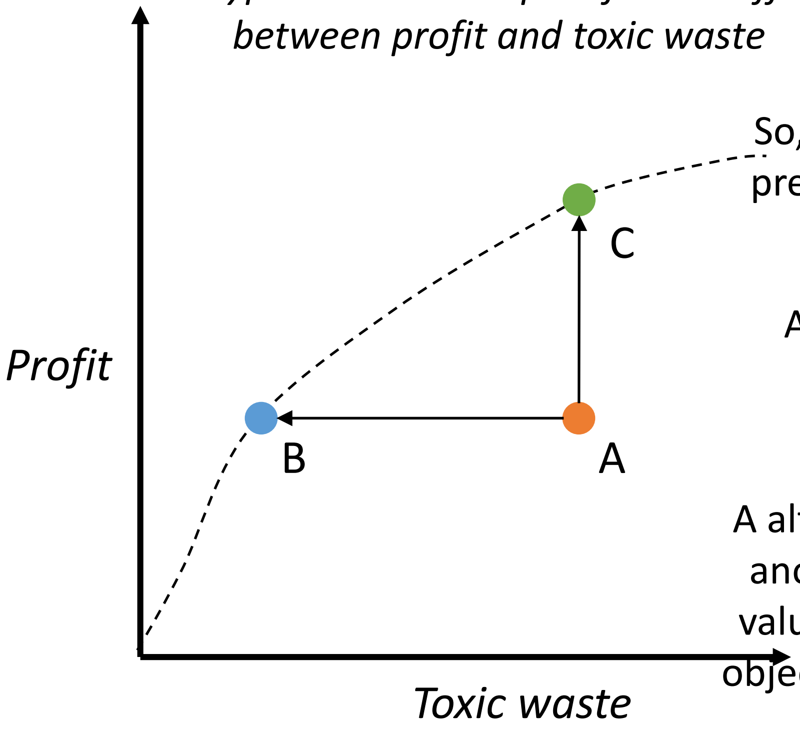


So, points **B** and **C** are both preferable to (or dominate) **point A**

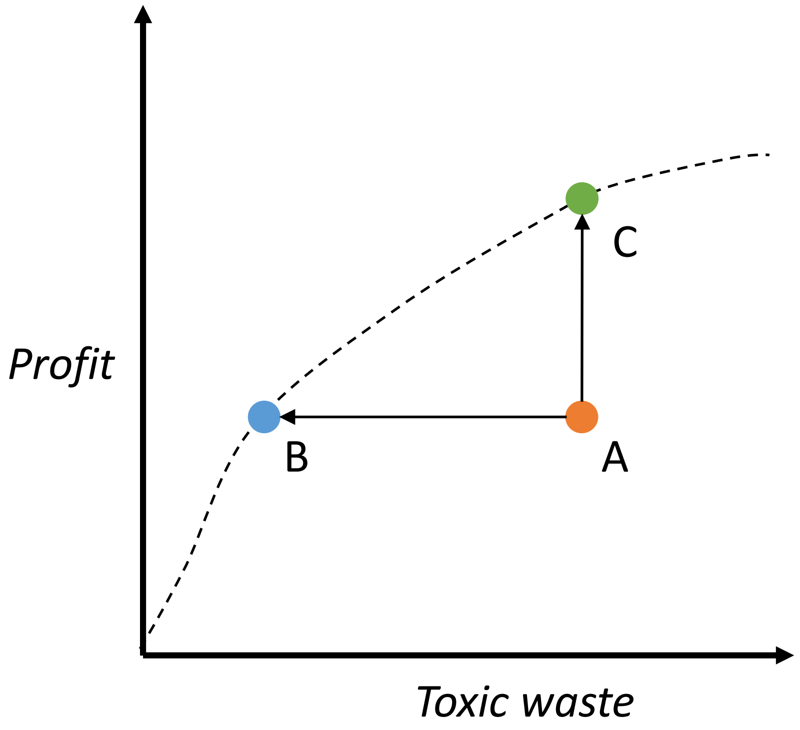
All points along the curve connecting B and C are preferable to A

A alternative is dominated if another produces a better value for at least one of the objectives without worsening the other objectives

Each point on the curve corresponds to a possible level of profit and the minimum amount of waste that must be produced to achieve it

Multiple Objective Decision Analysis - Optimization

Multi Objective Linear Programming



Decision makers should only want to consider decision alternatives that are non-dominanted.

MOLP guarantees the decisions presented to the decision maker are non-dominanted.

MOLP can be viewed as special types of GP problems where appart of solving the problem, we must also determine target values for each goal or objective.

Thus solving these problems also requires we use the minimization and maximization objectives described in previous classes.

Multiple Objective Decision Analysis - Optimization

Multi Objective Linear Programming

- Some ideas to form goal function:

Objective 1: Maximize profit (OF1)
Objective 2: Minimize toxic waste production (OF2)

1st Goal Form:

Goal function = Max (OF1) and add OF2 as constraint

Produced hazards <= Max allowable

Multiple Objective Decision Analysis - Optimization

Multi Objective Linear Programming

- Some ideas to form goal function:

Objective 1: Maximize profit (OF1)
Objective 2: Minimize toxic waste production (OF2)

2nd Goal Form:

Goal function = Max (OF1/OF2) or Goal function = Min (OF2/OF1)

Multiple Objective Decision Analysis - Optimization

Multi Objective Linear Programming

- Some ideas to form goal function:

Objective 1: Maximize profit (OF1)

Objective 2: Minimize toxic waste production (OF2)

Absolut values are used to avoid the existence of big positive deviation in one OF to cancel big negative deviation in the other

3rd Goal Form (weighted normalized method):

Goal function = Min (W1 * abs (Actual OF1- Optimum OF1) / Optimum OF1) +
W2 * abs (Actual OF2- Optimum OF2) / Optimum OF2)

Multiple Objective Decision Analysis - Optimization

Multi Objective Linear Programming

- Example:

Objective 1: Maximize profit (OF1)

Objective 2: Minimize toxic waste production (OF2)

3rd Goal Form (weighted normalized method):

Goal function = Min (W1 * abs (Actual OF1- Optimum OF1) / Optimum OF1) +
W2 * abs (Actual OF2- Optimum OF2) / Optimum OF2)

Multiple Objective Decision Analysis - Optimization

- Exercise 1

  Blackstone Mining Company operates 2 coal mines Wythe and Giles. The manager is anticipating a demand increase for coal in the coming year and he wants to schedule extra shifts of workers to the mines. Each extra shift has an extra cost of 40000/month at Wythe and 32000/month at Giles.

  The extraction methods lead to the production of toxic water. Running an extra shift at Wythe leads to the production of 800 gallons and 1250 gallons at Giles.

  Despite safety guidelines are followed 0.2 life threatening accidents are expected at Wythe and 0.45

	Coal production a month by a shift of workers (ton)		Increase in demand
	Wythe	**Giles**	
production of **high**-level coal	12	4	48
production of **medium**-level coal	4	4	28
production of **low**-level coal	10	20	100

Determine the number of extra shifts at each of the mines that minimizes costs, toxic waste production and life threatening accidents