Applied Operations Research

Solving applications of linear programming with Excel

Question 1.

Land use planning

A river is a major source of water of a certain city, and the council has an annual plan to expand the city's development in the area along the river. A total of 100 acres of land is projected to be needed for residential, business, and recreational use. According to the plan, at least 20 acres of land should be designed to for residential development, 30 acres will be used for business development, and a recreational park will be built on at least 10 acres. The initial investment cost for residential land is 8 million euros for the first 20 acres of land and 300000 euros for extra acre of land thereafter. The initial investment costs for business land and recreational land are 20 million and 12 million euros, respectively, while the costs for additional land are 500000 and 400000 euros per acre, respectively. An acre of residential land can yield a profit of 50000 euros per year, and the expected annual profits for business land and recreational land are 120000 and 150000 euros, respectively. On average, every acre of residential land will use 20 m^3 of water per month, and every acre of business land and recreational land consumes 40 m^3 and 25 m^3 of water per month, respectively. The annual budget is 80 million euros, and the regulation of water from the river is 40000 m^3 per year. The city council wants to find an annual plan with the maximum profit.

1. Formulate this problem as an LP model.
2. Solve the model with the Excel Solver. Which constraints are binding at the optimal solution found (saturated constraints)? Comment.
3. Report the shadow prices for each constraint and comment.
4. Derive the range of feasibility for all RHS values.
5. Determine an expression that gives the optimal objective values for the RHS values mentioned in d) for each constraint.
6. Derive the range of optimality for all objective function coefficients and comment.
7. Solve the problem without using the simplex method.

Question 2.

Fishery planning

A fishery has about 120 tons of fish, including 30 tons of salmon, 50 tons of tuna, and 40 tons of sardines. Every year, fishermen capture a certain amount of fish and sell them to the market. Assume that the average market price for salmon was 6 euros per Kg, for tuna was 5.50 euros per Kg, and for sardines was 5 euros per Kg. To maintain an ecological equilibrium, the fishery manager would like to keep the amount of tuna less than twice the amount of salmon every year. Suppose the overall average reproduction rate of these three fish is 12% per year. The fishery manager would like to keep the expected amount of fish in the fishery after four years to still be at least 120 tons. Additionally, the fishery manager would like to have at least 12 tons of salmon, 25 tons of tuna, and 16 tons of sardines in the fishery at the end of the four years. Find a fishery pattern so that fishermen can maximize their present profit over the 4-year period. Assume that capturing would be made at the end of each year and the annual interest rate is 4%.

1. Formulate this problem as a LP model.
2. Solve the model and criticizes the solution.
3. Add constraints to the model to ensure an even profit over time.

Question 3.

To think

Consider a linear programming model with constraint C_i (model A) and the model where constraint C_i is replaced by constraint $C_j = \frac{1}{10}C_i$ (model B). Let p_i and p_j denote the shadow prices of constraints C_i and C_j, and s_i and s_j the slack values of these constraints.

1. What is the relationship between p_i and p_j?
2. What is the relationship between s_i and s_j?