INSTITUTO SUPERIOR DE AGRONOMIA

Test of Applied Operations Research - 11 May 2016/17

Number: Name:

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1. (5val.) A forest is composed of three even-aged compartments of ponderosa pine. The area in each compartment is shown in Table 1, along with projected per-hectare lumber volumes during the next three 5-year periods. The silviculture is even-aged management, with clearcutting followed by planting. During the management plan, land that is cut will not be cut again, and no more than half of the entire forest area may be cut. The owner of the forest wants to maximize the total volume harvested.

		Lumber volume, b_{ij} (m^3/ha)			Total lumber volume, c_{ij} (m^3)		
	Area, a_i	Period j			Period j		
Compartment i	(ha)	1	2	3	1	2	3
1	2450	88	99	117	215600	242550	286650
2	3760	117	157	198	439920	590320	744480
3	8965	82	93	111	735130	833745	995115

Table 1: Projected lumber volume for a ponderosa pine forest.

a) Consider that the volume harvested in periods 2 and 3 can increase up to 10% and decrease up to 10% of the volume harvested in periods 1 and 2, respectively. Additionally, assume that each compartment can be partially cut. Formulate the problem as a linear program.

b) Assume that the entire area of each compartment is subject to the same decision in each period, or cutting or doing nothing. Consider that the owner does not want to cut both compartments 2 and 3 in the same period. Formulate the problem as an integer program.

2. (5val.) Consider the following LP problem:

$$\begin{aligned}
\text{Max} \quad & Z = 10x_1 + 5x_2 + 9x_3 - 3x_4 + 7x_5 && (1)\\
\text{s.t.} \quad & 2.5x_1 + x_2 + x_3 + x_4 + x_5 \leq 100 && (2)\\
& x_3 \leq 70 && (3)\\
& x_4 \geq 25 && (4)\\
& x_3 - x_5 \geq 0 && (5)\\
& x_1, \quad x_2, \quad x_3, \quad x_4, \quad x_5 \geq 0. && (6)
\end{aligned}$$

The Excel Solver solved this problem with an objective function value of $Z = 590$ and the optimal values of the decision variables $x_1 = 0$, $x_2 = 0$, $x_3 = 70$, $x_4 = 25$, $x_5 = 5$.

a) Complete the gray boxes of the answer report in Table 2.

b) Consider the following information about sensitivity analysis:

i) As long as x_1's objective function coefficient is not greater than 17.5, x_1 will remain 0 in the optimal solution;

ii) If x_4's objective function coefficient is decreased by infinity, x_4 will remain 25 in the optimal solution;

iii) If one unit of x_2 were forced into the current solution, the optimal objective function value would decrease to 588 ;

iv) The optimal objective function value decreases to 570 if the RHS of constraint (4) increases to 27;

v) The RHS of constraint (2) can be increased to 165, or decreased to 95, without changing the current shadow price.

Complete the gray boxes of the sensitivity report in Tables 3 and 4.

-	Name	Cell value	Status	Slack
-	(2)			
-	-	-	-	-
-	-	-	-	-
-	(5)			

Table 2: Constraints (answer report by the Excel Solver).

-	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
-	x_1		-			-
-	x_2			-	-	-
-	x_3	-	-	-	-	-
-	x_4		-		-	
-	x_5	-	-	-	-	-

Table 3: Variable cells (sensitivity report by the Excel Solver).

–	Name	Final Value	Shadow Price	Constant R.H. Side	Allowable Increase	Allowable Decrease
-	(2)		-			
-	(3)	-	-	-	-	-
-	(4)				-	-
-	(5)	-	-	-	-	-

Table 4: Constraints (sensitivity report by the Excel Solver).

3. (10val.) Consider the following LP problem (P):

$$
\begin{aligned}
\text{Max} \quad & Z = x_1 + 2x_2 + 3x_3 && (1)\\
\text{s.t.} \quad & x_1 + x_2 + x_3 \leq 5 && (2)\\
& 2x_1 - x_2 + x_3 = 2 && (3)\\
& x_1 + x_2 \geq 1 && (4)\\
& x_1, \quad x_2, \quad x_3 \geq 0. && (5)
\end{aligned}
$$

a) Use the Big M method to obtain a starting basic feasible solution for (P). Identify the first tableau for the simplex method and the corresponding basic feasible solution.

b) Write the dual problem of (P).

c) The primal optimal solution for (P) is $x_1 = 0$, $x_2 = 1.5$ and $x_3 = 3.5$. Use complementary slackness conditions to obtain the dual optimal solution.