Shadow Price & Sensitivity Analysis

Interpreting Solver outputs

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11 March 2020

Shadow Prices and Sensitivity Analysis

- **Shadow Prices**
- **Sensitivity Analysis**

Exercise

Shadow prices

- LP problems deal with the allocation of resources and when constraints are in the ≤ form, the RHS represents the amount of resources available.
- Consequently, it would be useful to know the contribution of each resource to the performance of our model (Z)
- The shadow price for a given resource measures the rate at which Z could be increased by (slightly) increasing the amount of this resource (bi) if available.
- The optimal tableau gives us the shadow prices (slack variables coeff. in R0).
- Let us follow an example in the upcoming slides:

Shadow prices

Max $Z = 3x_1 + 5x_2$

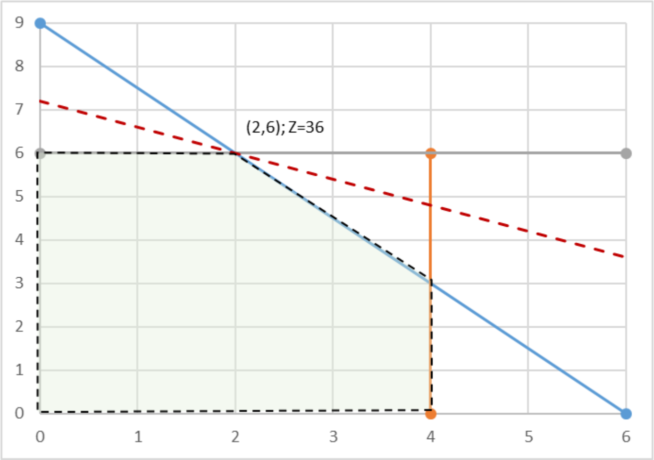
Subject to:

$x_1 \leq 4$

$2x_2 \leq 12$

$3x_1 + 2x_2 \leq 18$

And $x_1 \geq 0; \quad x_2 \geq 0$



X1 number of window batch; X2 number of glass doors batch
Profit of windows batch = 3 profit of doors batch = 5 (K€)

Plant 1- produces the aluminum frames (prod. time available = 4 h/week)
Plant 2- produces the wood frames (prod. time available = 12)
Plant 3- produces the glass and assembles the product (prod. time available = 18)

Resources - the production capacity of each Plant made available (R1, R2, R3), where *bi (RHS)* represents the hours of production time per week

The initial tableau

Row	basic var.	coefficients of: Z	x1	x2	S1	S2	S3	right side
R0	Z	1	-3	-5	0	0	0	0
R1	S1	0	1	0	1	0	0	4
R2	S2	0	0	2	0	1	0	12
R3	S3	0	3	2	0	0	1	18

The optimal tableau

Row	basic var.	coefficients of: Z	x1	x2	S1	S2	S3	right side
R0	Z	1	0	0	0	3/2	1	36
R1	S1	0	0	0	1	1/3	-1/3	2
R2	x2	0	0	1	0	1/2	0	6
R3	x1	0	1	0	0	-1/3	1/3	2

The shadow price for a resource measures the rate at which Z could be increased by (slightly) increasing the amount of resource:

Shadow price for Resource 1 = 0
Shadow price for Resource 2 = 3/2 = 1+1/2
Shadow price for Resource 3 = 1

By increasing the production time in Plant 2 by 1 hour (12 to 13) the optimal solution **(Z) increases** from **36** to 36+3/2 = **37.5** (K€)

Shadow prices

The optimal tableau

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$Z = 3x_1 + 5x_2$

The optimal solution is given by:

(x1, x2, S1, S2, S3) = (2, 6, 0, 0, 0)
resulting in Z = 36

Shadow prices

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For Plant 1 there are two hours of production time left unused (**S1 = 2**), thus this variable and its constraint are called ***not binding.***

In Economics, resources available in **surplus** are called *free goods* and have a ***zero shadow price.***

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For Plant 2 and Plant 3 there is no production time left available (**S2 = 0** and **S3 = 0**), thus these variables and their constraints are called ***binding***, because they bind Z from being increased further.

In Economics, because the supply of these resources is ***limited*** they are referred to as *scarce goods* and have a ***positive shadow prices.***

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The information provided by shadow prices is clearly valuable to management when:
reallocations of resources within the organization are considered
When buying/acquiring one more unit of a scarce resource is easy/possible and results in an improvement in Z. In these example **Z = 36** (K€):

acquiring one more unit of resource 2 results in **Z = 37.5**
acquiring one more unit of resource 3 results in **Z = 37**

Max $Z = 3x_1 + 5x_2$

Subject to:

$$x_1 \le 4$$

$$2x_2 \le 12$$

$$3x_1 + 2x_2 \le 18$$

And $x_1 \ge 0; \quad x_2 \ge 0$

Shadow prices

An example of how to solve this LP problem in Excel:

The initial tableau

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The Excel Formulation

Max

Objective function:		
x1	x2	Z
3	**5**	0

	Constraint coeff.		Total		RHS
S1	**1**		0	<=	4
S2		**2**	0	<=	12
S3	**3**	**2**	0	<=	18

x1	x2		
0	0	>=	0

Shadow prices

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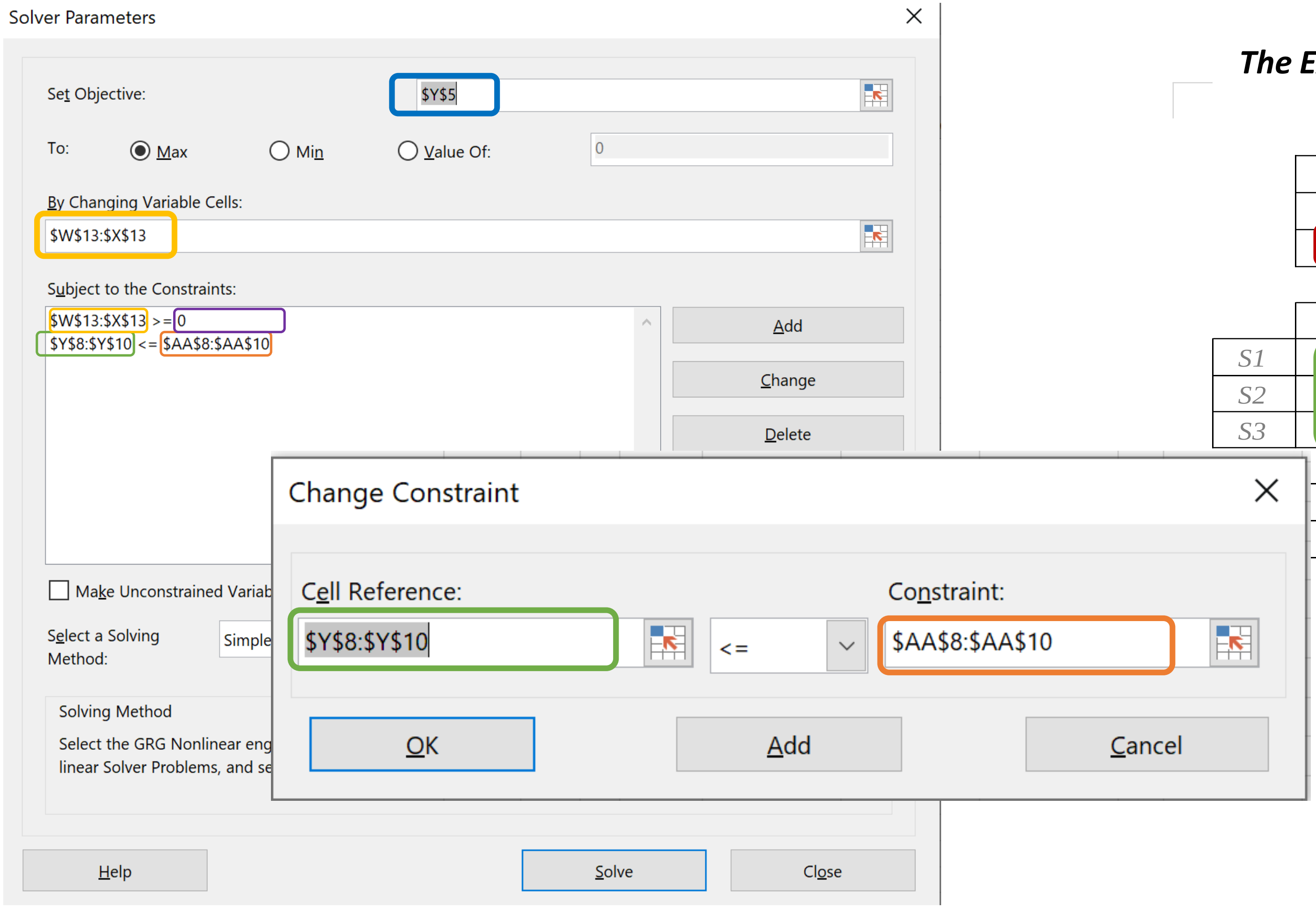

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Objective function:		
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3	**5**	0

	Constraint coeff.		Total		RHS
S1	**1**		0	<=	4
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x1	x2		
0	0	>=	0

Because all the constraint signs are the same, constraint coeff. and their respective RHS can be selected in one step

When each of the constraints has different signs, these must be added one by one.

Shadow prices

Comparing the optimal tableau with the Excel – Solver formulation:

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S1	**1**		2	<=	4
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Solver Results

Solver found a solution. All Constraints and optimality conditions are satisfied.

Keep Solver Solution

Restore Original Values

Reports

Answer

Sensitivity

Limits

Return to Solver Parameters Dialog

Outline Reports

OK

Cancel

Save Scenario...

Solver found a solution. All Constraints and optimality conditions are satisfied.

When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.

The Excel Formulation

Max

Objective function:		
x1	x2	Z
3	**5**	36

	Constraint coeff.		Total		RHS
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S2		**2**	12	<=	12
S3	**3**	**2**	18	<=	18

x1	x2		
2	6	>=	0

There is information we can obtain from the optimal tableau that we don't get directly in the Excel spreadsheet, but further details can be obtained by clicking on the option *Answer* under *Reports.*

Shadow prices

Comparing the optimal tableau with the ***Solver Analysis Report***:

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Max Time Unlimited, Iterations Unlimited, Precision 0.000001
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$Y$11	Total	0	36

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$W$4	S3 x1	0	2	Contin
$X$4	S3 x2	0	6	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$Y$7	S1 Total	2	$Y$7<=$AA$7	Not Binding	2
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For more detailed information e.g (the shadow prices) a different option of the Reports should be selected: ***Sensitivity Analysis***

Sensitivity Analysis

- When defining a LP problem the values used as **model parameters** are **usually estimates**.
- The main purpose of **sensitivity analysis** is to **identify** the sensitive **parameters** (i.e. the ones that cannot be changed without changing the optimal solution) that **require being closely monitored** as the study is implemented.
- If we discover a true value of a sensitive parameter differs from the estimated value, this immediately signals the need to change the solution.

Sensitivity Analysis

The basic idea of **Sensitivity Analysis** is to be able to give answers to questions such as:

1. If the objective function changes, how does the solution change?
2. If resources available change, how does the solution change?
3. If a constraint is added to the problem, how does the solution change?

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Sensitivity Analysis

Using the **SIMPLEX algebra** to perform **Sensitivity Analysis** is possible and enables assessing the impact of changes in the objective function and constraints provided that we have access to the initial and optimal tableaux (bottom one)

	Original variables	Slack variables	
	$x_1 \ \dots \ x_n$	$x_{n+1} \ \dots \ x_{n+m}$	
	$-\mathbf{c}^T$	$\mathbf{0}$	0
$\mathbf{B}$	$\mathbf{A}$	$\mathbf{I}$	$\mathbf{b}$

	Original variables	Slack variables	
	$x_1 \ \dots \ x_n$	$x_{n+1} \ \dots \ x_{n+m}$	
	$\mathbf{c}_B^T\mathbf{B}^{-1}\mathbf{A} - \mathbf{c}^T$	$\mathbf{c}_B^T\mathbf{B}^{-1}$	$z = \mathbf{c}_B^T\mathbf{x}_B$
$\mathbf{B}$	$\mathbf{B}^{-1}\mathbf{A}$	$\mathbf{B}^{-1}$	$\mathbf{x}_B = \mathbf{B}^{-1}\mathbf{b}$

The optimal tableau (bottom one) will remain optimal as long as:

1. $-\mathbf{c}_N^T + \mathbf{c}_B^T B^{-1} N \geq 0$ *For a maximization problem all entries in R0 should be non-negative for optimality*

2. $B^{-1}\mathbf{b} \geq 0$ *In order to maintain feasibility*

We would use these conditions to check whether the current basis still remains optimal after one (or more)of the parameters is (are) changed.

...but the **calculus can be laborious.**

Sensitivity Analysis

Using the **SIMPLEX algebra** to perform **Sensitivity Analysis** is possible and enables assessing the impact of changes in the objective function and constraints provided that we have access to the initial and optimal tableaux (bottom one)

	Original variables $x_1 \ldots x_n$	Slack variables $x_{n+1} \ldots x_{n+m}$	
	$-\mathbf{c}^T$	$\mathbf{0}$	0
$\mathbf{B}$	$\mathbf{A}$	$\mathbf{I}$	$\mathbf{b}$

	x_{n+m}		
	$\mathbf{c}$	$\mathbf{c}_B^T\mathbf{B}^{-1}$	$z = \mathbf{c}_B^T\mathbf{x}_B$
$\mathbf{B}$	$\mathbf{B}^{-1}\mathbf{A}$	$\mathbf{B}^{-1}$	$\mathbf{x}_B = \mathbf{B}^{-1}\mathbf{b}$

The optimal tableau (bottom one) ... as:

1. $-\mathbf{c}_N^T + \mathbf{c}^T$ ...

...zation problem all entries in ... should be non-negative for optimality

≤ 0

In order to maintain feasibility

We would use these conditions to check whether the current basis still remains optimal after one (or more)of the parameters is (are) changed.

...but the **calculus can be laborious.**

For this reason we'll focus on interpreting the Solver report output!

Sensitivity Analysis

Interpreting the ***Solver Sensitivity Report***:

Max $Z = \boxed{3x_1 + 5x_2}$

Subject to:

$$
\begin{aligned}
x_1 \quad & \leq 4 \\
2x_2 & \leq 12 \\
3x_1 + 2x_2 & \leq 18
\end{aligned}
$$

$x_1, x_2 \geq 0$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$W$4	S3 x1	2	0	3	4.5	3
$X$4	S3 x2	6	0	5	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$Y$7	S1 Total	2	0	4	1E+30	2
$Y$8	S2 Total	12	1.5	12	6	6
$Y$9	S3 Total	18	1	18	6	6

Sensitivity Analysis

Interpreting the ***Solver Sensitivity Report***:

Max $\mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

First, let us analyze the ***Variable Cells*** part of the table:

- The Final Value = ***Optimal Solution***, thus replacing the optimal (x1, X2) in the objective function leads to Z = **3***2 + **5***6 = 36

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$W$4	S3 x1	2	0	3	4.5	3
$X$4	S3 x2	6	0	5	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$Y$7	S1 Total	2	0	4	1E+30	2
$Y$8	S2 Total	12	1.5	12	6	6
$Y$9	S3 Total	18	1	18	6	6

Sensitivity Analysis

Interpreting the ***Solver Sensitivity Report***:

Max $\mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

First, let us analyze the ***Variable Cells*** part of the table:

- The Final Value = ***Optimal Solution***, thus replacing the optimal (x1, X2) in the objective function leads to Z = **3***2 + **5***6 = 36

- The ***allowable increase and decrease*** show how much the ***coeff. of the objective function*** can change before the ***optimal solution*** has to be altered

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$W$4	S3 x1	2	0	3	4.5	3
$X$4	S3 x2	6	0	5	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$Y$7	S1 Total	2	0	4	1E+30	2
$Y$8	S2 Total	12	1.5	12	6	6
$Y$9	S3 Total	18	1	18	6	6

Sensitivity Analysis

Interpreting the ***Solver Sensitivity Report***:

Max $\quad \mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

First, let us analyze the ***Variable Cells*** part of the table:

- The Final Value = ***Optimal Solution***, thus replacing the optimal (x1, X2) in the objective function leads to Z = **3***2 + **5***6 = 36

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease	Upper Limit	Lower Limit
$W$4	S3 x1	2	0	3	+ 4.5	3	7.5	0
$X$4	S3 x2	6	0	5	1E+30	3		

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$Y$7	S1 Total	2	0	4	1E+30	2
$Y$8	S2 Total	12	1.5	12	6	6
$Y$9	S3 Total	18	1	18	6	6

Since the ***Allowable Increase*** for X1 is 4.5 this means that if we increase the objective function coeff. for x1 up to an **Upper Limit** of **7.5** the optimal solution will not change (2, 6)

Sensitivity Analysis

Interpreting the ***Solver Sensitivity Report***:

Max $\quad Z = 3x_1 + 5x_2$

Subject to:

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

First, let us analyze the ***Variable Cells*** part of the table:

- The Final Value = ***Optimal Solution,*** thus replacing the optimal (x1, X2) in the objective function leads to Z = **3***2 + **5***6 = 36

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease	Upper Limit	Lower Limit
$W$4	S3 x1	2	0	3	4.5	3	7.5	0
$X$4	S3 x2	6	0	5	$\boxed{1E+30}$	3	+ ∞	2

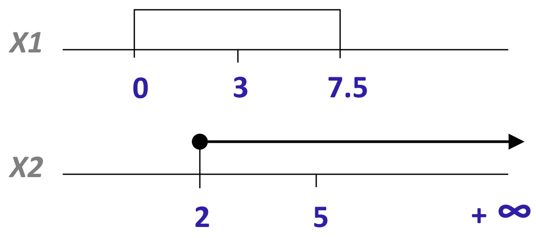
Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$Y$7	S1 Total	2	0	4	1E+30	2
$Y$8	S2 Total	12	1.5	12	6	6
$Y$9	S3 Total	18	1	18	6	6

Since the ***Allowable Increase*** for X1 is 4.5 this means that if we increase the objective function coeff. for x1 up to an **Upper Limit** of **7.5** the optimal solution will not change (2, 6)

Excel usually represents very big numbers by **1E+30** which can be seen as **infinity**

Sensitivity Analysis



Interpreting the ***Solver Sensitivity Report***:

Max $Z = 3x_1 + 5x_2$

Subject to:

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease	Upper Limit	Lower Limit
$W$4	S3 x1	2	0	3	4.5	3	7.5	0
$X$4	S3 x2	6	0	5	1E+30	3	+ ∞	2

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$Y$7	S1 Total	2	0	4	1E+30	2
$Y$8	S2 Total	12	1.5	12	6	6
$Y$9	S3 Total	18	1	18	6	6

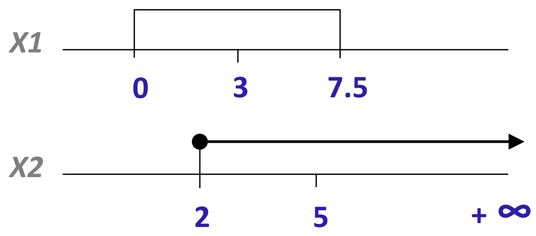
First, let us analyze the ***Variable Cells*** part of the table:

- The Final Value = ***Optimal Solution***, thus replacing the optimal (x1, X2) in the objective function leads to Z = **3***2 + **5***6 = 36

So, what will happen if the **coeff. of X1 increases to 10** ?

- It will fall outside the allowable interval, thus the optimal solution will change (**Final Values)**

Sensitivity Analysis



Interpreting the ***Solver Sensitivity Report***:

Max $\mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease	Upper Limit	Lower Limit
$W$4	S3 x1	2	0	3	4.5	3	7.5	0
$X$4	S3 x2	6	0	5	1E+30	3	+ ∞	2

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$Y$7	S1 Total	2	0	4	1E+30	2
$Y$8	S2 Total	12	1.5	12	6	6
$Y$9	S3 Total	18	1	18	6	6

First, let us analyze the ***Variable Cells*** part of the table:

- The Final Value = ***Optimal Solution,*** thus replacing the optimal (x1, X2) in the objective function leads to Z = **3***2 + **5***6 = 36

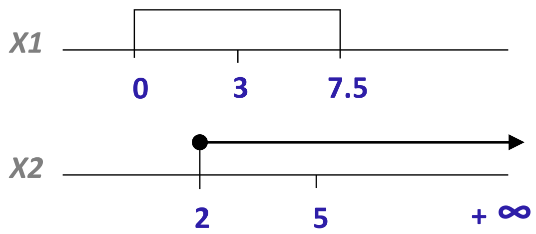
So, what will happen if the **coeff. of X1 increases to 10** ?

- It will fall outside the allowable interval, thus the optimal solution will change (**Final Values)**

And what will happen if the **coeff. of X1 increases to 6** ?

- The optimal solution will remain optimal but Z = **6***2 + **5***6 = 42

Sensitivity Analysis



Interpreting the ***Solver Sensitivity Report***:

Max **$Z = 3x_1 + 5x_2$**

Subject to:

$$x_1 \le 4$$

$$2x_2 \le 12$$

$$3x_1 + 2x_2 \le 18$$

$x_1, x_2 \ge 0$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease	Upper Limit	Lower Limit
$W$4	S3 x1	2	0	4 - 3 = 1	4.5	3	7.5	0
$X$4	S3 x2	6	0	5 - 4 =1	1E+30	3	+ ∞	2

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$Y$7	S1 Total	2	0	4	1E+30	2
$Y$8	S2 Total	12	1.5	12	6	6
$Y$9	S3 Total	18	1	18	6	6

First, let us analyze the ***Variable Cells*** part of the table:

- The Final Value = ***Optimal Solution***, thus replacing the optimal (x1, X2) in the objective function leads to $Z = \mathbf{3}*2 + \mathbf{5}*6 = 36$

And what will happen if **both coeff. X1 and X2 change to 4** (simultaneous changes)?

- This optimality report only applies to individual changes and to answer the question we will have to calculate **100% Rule**:

X1 increases in 1 unit, so: 1 / 4.5 (allowable increase) = 0.22

X2 decreases in 1 unit, so: 1 / 3 (allowable decrease) = 0.33

0.22 + 0.33 = 0.55 %
<100%
Solution remains optimal
$Z = \mathbf{4}*2 + \mathbf{4}*6 = 32$

Sensitivity Analysis

Interpreting the ***Solver Sensitivity Report***:

Max $Z = 3x_1 + 5x_2$

Subject to:

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$W$4	S3 x1	2	0	4 - 3 = 1	4.5	3
$X$4	S3 x2	6	0	5 - 4 =1	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$Y$7	S1 Total	2	0	4	1E+30	2
$Y$8	S2 Total	12	1.5	12	6	6
$Y$9	S3 Total	18	1	18	6	6

First, let us analyze the ***Variable Cells*** part of the table:

- The Final Value = ***Optimal Solution***, thus replacing the optimal (x1, X2) in the objective function leads to $Z = \mathbf{3}*2 + \mathbf{5}*6 = 36$

Reduced Cost column is set to zero for both variables because both products are being produced (2 units of X1 and 6 units of X2).

However, there might be situations for which not producing one of the products is more profitable (**Final Value** = 0). In such situations, the **Reduced Cost** = certain negative amount (for a maximization problem), which represent the reduction in profit that would be obtained if we insisted in producing one unit of that product

Sensitivity Analysis

Interpreting the ***Solver Sensitivity Report***:

Max $\mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

Now, let us analyze the ***Constraints*** part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$W$4	S3 x1	2	0	3	4.5	3
$X$4	S3 x2	6	0	5	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	Upper Limit	Lower Limit
$Y$7	S1 Total	2	0	4	1E+30	2	7.5	0
$Y$8	S2 Total	12	1.5	12	6	6	+ ∞	2
$Y$9	S3 Total	18	1	18	6	6		

Sensitivity Analysis

Interpreting the ***Solver Sensitivity Report***:

Max $\mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$$x_1, x_2 \geq 0$$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$W$4	S3 x1	2	0	3	4.5	3
$X$4	S3 x2	6	0	5	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	Upper Limit	Lower Limit
$Y$7	S1 Total	2	0	4	1E+30	2	+ ∞	2
$Y$8	S2 Total	12	1.5	12	6	6	18	6
$Y$9	S3 Total	18	1	18	6	6	18	12

Now, let us analyze the ***Constraints*** part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged

Increase in Z resulting of an Unit increase in the RHS of a constraint

Sensitivity Analysis

Optimal sol.: (x1, x2) =(0, 6)
Z = 36

Interpreting the ***Solver Sensitivity Report***:

Max $\mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$
$$2x_2 \leq 12$$
$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$W$4	S3 x1	2	0	3	4.5	3
$X$4	S3 x2	6	0	5	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	Upper Limit	Lower Limit
$Y$7	S1 Total	2	0	4	1E+30	2	+ ∞	2
$Y$8	S2 Total	12	1.5	12	6	6	18	6
$Y$9	S3 Total	18	1	18	6	6	18	12

Now, let us analyze the ***Constraints*** part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged

Increase in Z resulting of an Unit increase in the RHS of a constraint

This table allows us to say how much would profit increase (Z) without having to apply Simplex again as long as the change in the **RHS** of a constraint remains between its **Upper** and **Lower Limits**, because this means the **Shadow Price** will hold.

Sensitivity Analysis

Optimal sol.: (x1, x2) =(0, 6)
Z = 36

Interpreting the ***Solver Sensitivity Report***:

Max $\mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$x_{1,}\, x_2 \geq 0$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$W$4	S3 x1	2	0	3	4.5	3
$X$4	S3 x2	6	0	5	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	Upper Limit	Lower Limit
$Y$7	S1 Total	2	0	4	1E+30	2	+ ∞	2
$Y$8	S2 Total	12	1.5	12 + 5 = 17	6	6	18	6
$Y$9	S3 Total	18	1	18	6	6	18	12

Now, let us analyze the ***Constraints*** part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged

Increase in Z resulting of an Unit increase in the RHS of a constraint

This table allows us to say how much would profit increase (Z) <u>without having to apply Simplex again</u> as long as the change in the **RHS** of a constraint remains between its **Upper** and **Lower Limits**, because this means the **Shadow Price** will hold.

Suppose we **increase** the **RHS** of constraint 2 by **5** (from 12 to 17):

5 * **1.5** = **7.5** , thus **Z = 36 + 7.5** = **43.5**

Sensitivity Analysis

Optimal sol.: (x1, x2) =(0, 6)
Z = 36

Interpreting the ***Solver Sensitivity Report***:

Max $\mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$
$$2x_2 \leq 12$$
$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$W$4	S3 x1	2	0	3	4.5	3
$X$4	S3 x2	6	0	5	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	Upper Limit	Lower Limit
$Y$7	S1 Total	2	0	4	1E+30	2	+ ∞	2
$Y$8	S2 Total	12	1.5	12 - 5 = 7	6	6	18	6
$Y$9	S3 Total	18	1	18	6	6	18	12

Now, let us analyze the ***Constraints*** part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged

Increase in Z resulting of an Unit increase in the RHS of a constraint

This table allows us to say how much would profit increase (Z) without having to apply Simplex again as long as the change in the **RHS** of a constraint remains between its **Upper** and **Lower Limits**, because this means the **Shadow Price** will hold.

Suppose we **decrease** the **RHS** of constraint 2 by **5** (from 12 to 17):

- 5 * **1.5** = **7.5** , thus **Z = 36 - 7.5** = **28.5**

Sensitivity Analysis

Optimal sol.: (x1, x2) =(0, 6)
Z = 36

Interpreting the ***Solver Sensitivity Report***:

Max $\mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$
$$2x_2 \leq 12$$
$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$W$4	S3 x1	2	0	3	4.5	3
$X$4	S3 x2	6	0	5	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	Upper Limit	Lower Limit
$Y$7	S1 Total	2	0	4	1E+30	2	+ ∞	2
$Y$8	S2 Total	12	1.5	12	6	6	18	6
$Y$9	S3 Total	18	1	18 + 7 = 11	6	6	18	12

Now, let us analyze the ***Constraints*** part of the table:

- The bottom table addresses the range of feasibility ie the range for the RHS of the constraints that allows the **Shadow Price** to remain unchanged

Increase in Z resulting of an Unit increase in the RHS of a constraint

This table allows us to say how much would profit increase (Z) <u>without having to apply Simplex again</u> as long as the change in the **RHS** of a constraint remains between its **Upper** and **Lower Limits**, because this means the **Shadow Price** will hold.

Suppose we decrease the **RHS** of constraint 3 by **7**, from 18 to **11**, please note that the Allowable Decrease is 6 making the **RHS** new value fall outside the **Lower Limit**. Therefore the **Shadow Price** is <u>no longer valid</u> and for that reason we <u>can not tell what would happen to profit.</u>

Sensitivity Analysis

Interpreting the ***Solver Sensitivity Report***:

Max $\mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$x_1, x_2 \geq 0$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$W$4	S3 x1	2	0	3	4.5	3
$X$4	S3 x2	6	0	5	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$Y$7	S1 Total	2	0	4	1E+30	2
$Y$8	S2 Total	12	1.5	12	6	6
$Y$9	S3 Total	18	1	18	6	6

Now, let us analyze the ***Constraints*** part of the table:

- In this example, **all constraints are ≤** thus all of them are associated to **SLACK variables** and the value of the slack variable can be obtained as: **|RHS – Final Value|:**

For S1: | 4 - 2 |= 2
For S2: |12 - 12|= 0
For S3: |18 - 18|= 0

(≥ constraints are associated to **SURPLUS variables**)

Sensitivity Analysis

Interpreting the ***Solver Sensitivity Report***:

Max $\mathbf{Z = 3x_1 + 5x_2}$

Subject to:

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$x_{1,}\ x_2 \geq 0$

Now, let us analyze the ***Constraints*** part of the table:

- In this example, **all constraints are ≤** thus all of them are associated to **SLACK variables** and the value of the slack variable can be obtained as: **|RHS – Final Value|:**

For S1: $|\ 4 - \ 2\ | = 2$

For S2: $|12 - 12| = 0$

For S3: $|18 - 18| = 0$

(≥ constraints are associated to **SURPLUS variables**)

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$W$4	S3 x1	2	0	3	4.5	3
$X$4	S3 x2	6	0	5	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease	
$Y$7	S1 Total	2	0	4	1E+30	2	non-binding
$Y$8	S2 Total	12	1.5	12	6	6	binding
$Y$9	S3 Total	18	1	18	6	6	binding

The **Binding Constraints** are the ones for which the **RHS** = **Final Value**, ie

- the ones with SLACK (or SURPLUS) = 0
- **Shadow Prices** <> 0

Sensitivity Analysis - Excercise

- A company produces 3 different products: A, B and C. Each product has to go under 3 processes consuming different amounts of time along the way. The time available for each process is described in the table below.

Assuming the selling profits for products A, B and C are 2, 3 and 4€ per unit formulate the problem in EXCEL and use Solver to find the optimal solution.

Compare your results with the solution you obtained previously in class using SIMPLEX. Then based on the Solver Sensitivity Report, answer the following:

Process	**Total number of hours available**	**Number of hours needed to produce each product**		
		A	B	C
I	12000	5	2	4
II	24000	4	5	6
III	18000	3	5	4

a) What would happen to profit if we increased the production of product A to 1 unit? Justify your answer stating how much would profit increase or decrease.
b) What will happen to profit and the optimal solution if we increase the selling price of product A to 5? Justify
c) What will happen to profit and the optimal solution if we increase the selling price of product B to 4? Justify
c) What will happen to profit and the optimal solution if we increase the selling price of product B to 4? Justify
d) What will happen to profit and the optimal solution if we decrease the selling price of product C by 1? Justify
e) What will happen to profit if the total number of hours available for process I is increased by 500? Justify
f) What will happen to profit if the total number of hours available for process II is decreased to 21000? Justify
g) Analyzing the report which resources would you classify as scarce? Justify