Introduction to Operations Research

Marta Mesquita

DCEB - Mathematics Section

February 14, 2023

Forest ressource management

- Forest resource management is the art and science of making decisions with regard to the organization, use, and conservation of forests and related resources.
- Forests may be actively managed for timber, water, wildlife, recreation, or a combination thereof.
- Forest resource managers must make decisions affecting both the very long-term future of the forest and day-to-day activities.

Models

- To make decisions, forest managers use models. Models are abstract representations of the real world that are useful for purposes of thinking, forecasting, and decision making.
- Reality is captured by symbolic variables and by formal algebraic relations between them. Yet, despite or because of their abstraction, mathematical models are very powerful.

Operational Research

- Operational research (OR) is a discipline on the process of making better decision through the development and the application of a wide range of problem-solving methods and techniques.
- OR has been applied to solve agricultural and forest management problems over the decades. Assigning resources on farms, determining a diet to feed animals at minimum cost or planning forest areas to be cut are examples of them.
- The OR methods and techniques involve the construction of mathematical models that aim at describing a problem.

Operational Research

- Operational Research 'ORigin Story'
  https://www.youtube.com/watch?v=ILWbaWrjgU4
- INFORMS O.R. and Analytics Impact
  https://www.youtube.com/watch?v=lq0B6wnAbk0
- EURO - 'Association of European Operational Research Societies'.
  - EURO working group on Agriculture and Forest Management
    https://www.euro-online.org/

Operational Research Models

- Deterministic Model
  - Linear optimization: Linear programming, Integer programming, Transportation and assignment model, Multi-criteria decision-making programming, Network models.
  - Nonlinear optimization: Classical model, Search model, Nonlinear programming.
- Hybrid Model
  - PERT-CPM, Dynamic programming, Inventory model, Simulation model
- Stochastic Model
  - Decision analysis model, Markov model, Queuing Model

Operations Research Models: Linear programming

- Some problems in forest ressource management can be formulated as linear programming models.
- For our immediate purpose, linear programming can be defined as a method to allocate limited resources to competing activities in an optimal manner.
- For example: (i) one manager might want to increase the land area that is growing red pine, but then less land would be available for aspen. (ii) Another manager might want to assign more of his staff to prepare timber sales, but then fewer people would be available to do stand improvement work. He could hire more people, but then he would have too little money.

Linear programming

- No matter what action is choosen, managers always face constraints that limit the range of their options.
- Linear programming may help them by showing which alternatives are possible ("feasible"), and by determining the best one. But this requires that both the management objective as well as the constraints be defined in a precise mathematical manner.
- One can recognize three elements in model development: problem definition, model building and model implementation.

Linear programming

A linear programming problem (LP) is an optimization problem for which:

- We attempt to maximize (or minimize) a linear function of the decision variables. The function that is to be maximized or minimized is called the objective function.
- The values of the decision variables must satisfy a set of constraints. Each constraint must be a linear equation or linear inequality.
- A sign restriction is associated with each variable. For any variable the sign restriction specifies that it must be either nonnegative, nonpositive or unrestricted in sign.

Linear programming: example 1 - problem definition

The protagonist is a poet-forester who lives in the woods of Northern Wisconsin and 10 years ago bought 90 hectares (ha) of woods. The poet finds that sales from the woods come in very handy to replenish a sometimes-empty wallet. So, he has firmly decided to get the most he can out of his woods. To develop his model the poet has put together the following information:

- 40 ha of the land are covered with red pine plantations, the other 50 ha contain hardwoods;
- Since he bought these woods he has spent approximately 800 days managing the red pine and 1500 days on the hardwoods;
- The poet does not want to spend more than half of his time in the woods;
- The total revenue from his forest during 10 years was $36,000 from the red pine land and $60,000 from the northern hardwoods.

- 40 ha of the land are covered with red pine plantations, the other 50 ha contain hardwoods;
- During 10 years, he spent approximately 800 days managing the red pine and 1500 days on the hardwoods;
- The total revenue from his forest during 10 years was $36,000 from the red pine land and $60,000 from the northern hardwoods.

Example 1 - Decision variables

- The poet-forester needs to choose the variables to symbolize his decisions.
- The objective function will often give some clue as to what the decision variables should be. We noted earlier that the poet's objective is to maximize his revenues from the property.
- Let us think about a management program for one year. The poet wants to Maximize $Z=$ revenue (in \$) per year.
- The revenue depends on the number of hectares of red pine and hardwoods he is going to manage during a year.
- Decision variables:
  - X_1=the number of ha of red pine to manage
  - X_2=the number of ha of hardwoods to manage

Example 1 - Objective function

- We need an estimate of the yearly revenues generated by each type of forest. Since the poet has earned \$36,000 on 40 ha of red pine and \$60,000 on 50 ha of northern hardwoods during the past 10 years, the average earnings have been \$90 per ha per year for red pine and \$120 per ha per year for northern hardwoods.
  - Objective function: $\max Z = 90X_1 + 120X_2$

Linear programming: example 1 - constraints

- land constraints: the area managed in each timber type cannot exceed the area available
  - $X_1 \leq 40$
  - $X_2 \leq 50$
- working time constraints: the poet does not want to spend more than half his time, let us say 180 days a year, managing his woods. The time he has spent managing red pine during the past 10 years (800 days for 40 ha) averages to 2 days per hectare per year. Similarly, he has spent 3 days per ha per year on northern hardwoods (1500 days on 50 ha during 10 years).
  - $2X_1 + 3X_2 \leq 180$
- sign constraints: none of the decision variables may be negative, since they refer to areas
  - $X_1 \geq 0; X_2 \geq 0$

Linear programming: example 1 - final model

- X_1=the number of ha of red pine to manage
- X_2=the number of ha of hardwoods to manage

$$\max Z = 90X_1 + 120X_2$$
$$X_1 \leq 40$$
$$X_2 \leq 50$$
$$2X_1 + 3X_2 \leq 180$$
$$X_1 \geq 0; X_2 \geq 0$$

See *Joseph Buongiorno, J. Keith Gilless. Decision Methods for Forest Resource Management, Academic Press, 2003*, for a detailed description of this example.

Linear programming: terminology

- A feasible solution is a solution for which all the constraints are satisfied.
- An infeasible solution is a solution for which at least one constraint is violated.
- The feasible region is the collection of all feasible solutions.
- An optimal solution is a feasible solution that has the most favorable value of the objective function.
- For a maximization problem, an optimal solution is a point in the feasible region with the largest objective function value.
- For a minimization problem, an optimal solution is a point in the feasible region with the smallest objective function value.

PROPORTIONALITY:

- The contribution of the objective function from each decision variable is proportional to the value of the decision variable. For example, in the poet forester problem, the contribution of red pine management to revenues is proportional to the area of red pine being managed.
- The contribution of each variable to the left-hand side of each constraint is proportional to the value of the variable. For example, the time the poet must put in managing his land is proportional to the area being managed.

ASSUMPTIONS OF LINEAR PROGRAMMING

ADDITIVITY:

- The contribution to the objective function for any variable is independent of the values of the other decision variables.
- The contribution of a variable to the left-hand side of each constraint is independent of the values of the variable.

This means that the contribution of each variable does not depend on the presence or absence of the others. In example 1, regardless of what the poet-forester does with his northern hardwoods, he will always get $90 per hectare from each hectare of managed red pine, and it will still take him 2 days per hectare per year to manage.

ASSUMPTIONS OF LINEAR PROGRAMMING

DIVISIBILITY:

- Decision variables are continuous.

CERTAINTY:

- A linear programming model is deterministic. It considers that all the parameters are known with certainty

LP: example 2 - Keeping the river clean

A pulp mill makes mechanical and chemical pulp and during the production process it pollutes the river in which it spills its spent waters. The owners would like to minimize pollution, keeping at least 300 people employed at the mill and generating at least 40000€ of revenue per day.

- The maximum capacity of the mill is 300 tons per day to make mechanical pulp and 200 tons per day to make chemical pulp (The two manufacturing processes are independent);
- Both mechanical and chemical pulp require the labor of 1 worker for about 1 day, or 1 workday (wd), per ton produced;
- Pollution is measured by the biological oxygen demand (BOD). 1 ton of mechanical pulp produces 1 unit of BOD, 1 ton of chemical pulp produces 1.5 units;
- The mechanical pulp sells at 100€ per ton, the chemical pulp sells at 200€.

Bibliography

- Wayne L. Winston (Second Edition). *Introduction to Mathematical Programming, Applications and Algorithms*, Duxbury Press.
- Joseph Buongiorno, J. Keith Gilless. *Decision Methods for Forest Resource Management*, Academic Press, 2003.
- https://www.ifors.org/what-is-or/