Applied Operations Research

Simplex method at a glance
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Question 1. Keeping the river clean re-visited

Decision variables:

x_1 - Amount of mechanical pulp produced (in tons/day, or t/d)

x_2 - Amount of chemical pulp produced (t/d).

Formulation:

$$\min Z = x_1 + 1.5x_2 \quad \text{units of BOD per day} \tag{1}$$

$$\textit{subject to}$$

$$x_1 + x_2 \geq 300 \quad \text{workers employed per day} \tag{2}$$

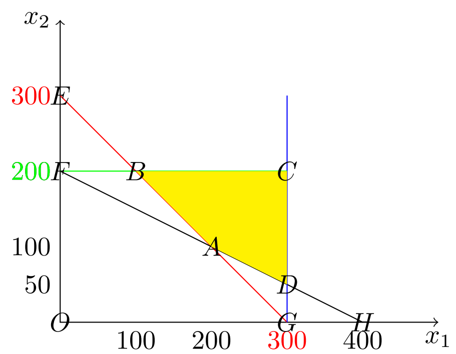
$$100x_1 + 200x_2 \geq 40000 \quad \text{revenue, €/day} \tag{3}$$

$$x_1 \leq 300 \quad \text{mechanical pulping capacity, t/day} \tag{4}$$

$$x_2 \leq 200 \quad \text{chemical pulping capacity, t/day} \tag{5}$$

$$x_1 \geq 0 \tag{6}$$

$$x_2 \geq 0. \tag{7}$$



$x_1 + x_2 = 300$ (2′) $\quad 100x_1 + 200x_2 = 40000$ (3′) $\quad x_1 = 300$ (4′) $\quad x_2 = 200$ (5′) $\quad x_1 = 0$ (6′) $\quad x_2 = 0$ (7′)

Figure 1: Feasible region.

1. How many systems with two equations can you define from the set of equations (2') to (7')?

2. Use the figure above to define the systems mentioned previously, indicate the respective solution sets (there is no need to solve the systems) and highlight, if any, the constraints from (2) to (7) that are violated by the solutions. Suggestion: Complete the table below.

System	Solution	Violated constraints
$\begin{cases} x_1 + x_2 = 300 \\ 100x_1 + 200x_2 = 40000 \end{cases}$	$A(200, 100)$	No
$\vdots$		

3. Consider the "keeping the river clean" problem in the standard form:

$$\min\ Z = x_1 + 1.5x_2 \tag{8}$$
subject to
$$x_1 + x_2 - s_1 = 300 \tag{9}$$
$$100x_1 + 200x_2 - s_2 = 40000 \tag{10}$$
$$x_1 + s_3 = 300 \tag{11}$$
$$x_2 + s_4 = 200 \tag{12}$$
$$x_1, x_2, s_1, s_2, s_3, s_4 \geq 0. \tag{13}$$

Complete the following table:

	(x_1, x_2)	$(x_1, x_2, s_1, s_2, s_3.s_4)$	
A	$(200, 100)$	(200,100,0,0,100,100)	A'
B	$(100, 200)$		B'
C	$(300, 200)$		C'
D	$(300, 50)$		D'
I	$(200, 200)$		I'
J	$(200, 150)$		J'
E	$(0, 300)$		E'

4. What differences seem to exist between points A', B', C' and D', that correspond, respectively, to vertices A, B, C and D, and points I', J'? And between points A', B', C' and D' and E'?

Question 2.

Consider the following linear programming model

$$\begin{array}{llcl} \text{Max} & Z = x_1 - x_2 + x_3 & & \\ \text{s.t.} & 2x_1 - x_2 + 2x_3 & \leq & 6 \\ & -2x_1 + 4x_2 - x_3 & \geq & \alpha \qquad (\alpha \in \mathbb{R}) \\ & x_1 - x_2 + 2x_3 & \geq & 4 \\ & x_1, \quad x_2, \quad x_3 & \geq & 0 \end{array}$$

and the point $P = (0, 2, 4)$.

1. Write the problem in the standard form.

2. Find, if any, a value for α such that P is a vertex of the feasible region and give the corresponding value of the objective function.

3. Consider that P is an optimal solution of the problem for the α value found previously. Comment the following sentence: "The plan $x_1 - x_2 + x_3 = 3$ intercepts the feasible region of the problem."

Question 3.

Consider the following linear programming model

$$\text{Max} \quad Z = x_1 + 2x_2 - x_3$$

$$
\begin{array}{lrrrl}
\text{s.t.} & 2x_1 + 4x_2 & + 3x_3 & \geq 8 \\
& x_1 + x_2 & & \leq 6 \\
& -x_1 + x_2 & & \leq 4 \\
& x_1 & + \ x_3 & \leq 4 \\
& x_1, \quad x_2, & x_3 & \geq 0
\end{array}
$$

1. Write the problem in the standard form.
2. Find an optimal solution of the problem that is obtained from the initial problem by adding the restriction $x_3 = 0$ and indicate the corresponding binding constraints.
3. Does the solution of the previous question correspond to a vertex of the feasible region of the initial problem?