Modelling Forest Dynamics Course 2013-2014

(revised March 2026)

ABSTRACT

Overview description of the importance of forest inventory for growth modelling. From measuring trees to the application of different types of growth models.

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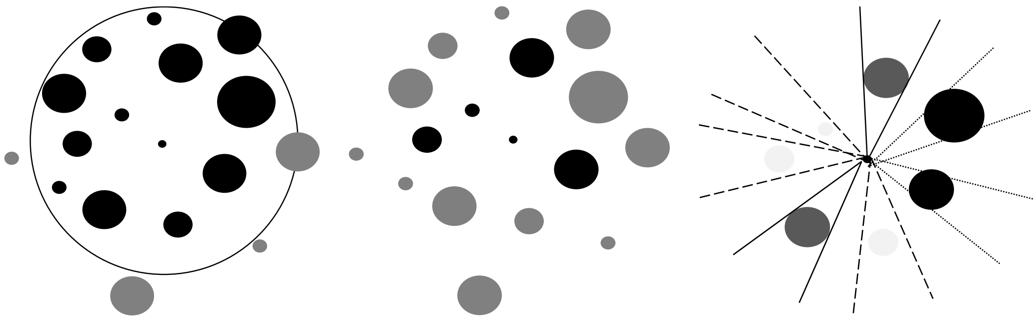
1 Evaluating Stand Variables Based on Inventory Plots

In the course of forest inventory, the objective of characterizing a stand is obtaining the values for a group of stand variables that are usually reported to the hectare. Due to the big dimensions of forest stands, in practice it is impossible to assess all stand variables through the extensive measurement of the whole stand. Therefore, stand level assessments are usually conducted through sampling. In order to carry out the sampling the definition of the population (individuals) implies one of three methods, where the circles in black represent the trees considered for measurement:

a) Measurement of trees in plots of fixed radius;

b) Trees' measurement in plots with a fixed number of trees;

c) Point Sampling, Bitterlich method (*not covered in this course*).



a) Fixed radius plots — b) Fixed number of trees — c) Angle count plots

The first method is the most commonly used because the second usually originates biased estimates of stand variables, although there is the possibility to apply correction factors to minimize the bias. The fixed radius plots' method considers the stand as a population of plots of a given shape and size for which a sample of these plots is characterized. Each of the sampled plots is assessed and dendrometric variables measured or estimated being afterwards reported to the hectare. In general terms stand variables, which are usually obtained as the sum, average or other functions of tree variables can be obtained by three alternative approaches:

a) **Complete enumeration** – when tree level variables required have been measured in all the trees in the plot.

b) **Sample tree** - when tree level variables are only measured for a subset of trees in the plot. (For example, total tree height is usually measured only in sample trees).

c) **Estimates –** when the variable is difficult and/or expensive to measure it can be assessed using a regression fitted between the desired variable and other variables easier to obtain. Development of stand specific regressions require having access to a considerable amount of data from different plots which makes it more difficult to develop being generally obtained doing a bibliographic research.

d) Before introducing the methodologies applied to assess stand variables in plots a set of plot characteristics such as shape and dimension will be considered.

1.1 The Inventory Plot

1.1.1 Plot Shape and Dimension

The most common plot shapes are the rectangle/square, the circle and the strip. The advantage of one shape when compared to another consists of the ratio perimeter/area, which should be the smallest possible to minimize the number of border trees an extremely important source of error. In theory, the best shape is the one that for the same area presents the biggest ratio. For this reason, the circle is the preferred shape (**Table 1**).

Table 1. Relationship between area and perimeter for different plot shapes and different areas.

Plot shape		Perimeters (m) for the following areas (m^2) :				
		400 (m^2)	**500 (m^2)**	**1000 (m^2)**	**1256.64 (m^2)**	**2827.43 (m^2)**
Circle		70.90	79.27	112.10	125.66	188.50
Square	ℓxℓ	80.00	89.44	126.49	141.80	212.69
Rectangular	2ℓxℓ	84.85	94.87	134.16	150.40	225.60
	3ℓxℓ	92.38	103.28	146.06	163.73	245.60
	4ℓxℓ	100.00	111.80	158.11	177.25	265.87

The issue of plot dimension is related to the fact that variation among plots decreases with the increase of its dimension. Thus, sampling error decreases with plot size. If plots are too small it is possible that having one very big tree in one plot and only small trees in other plot to increase the variance between plots. On the contrary, if the plot is big enough to capture the stand's variability, then variance among plots decreases. That is why plot dimension is closely related to stand density and homogeneity. **Table**

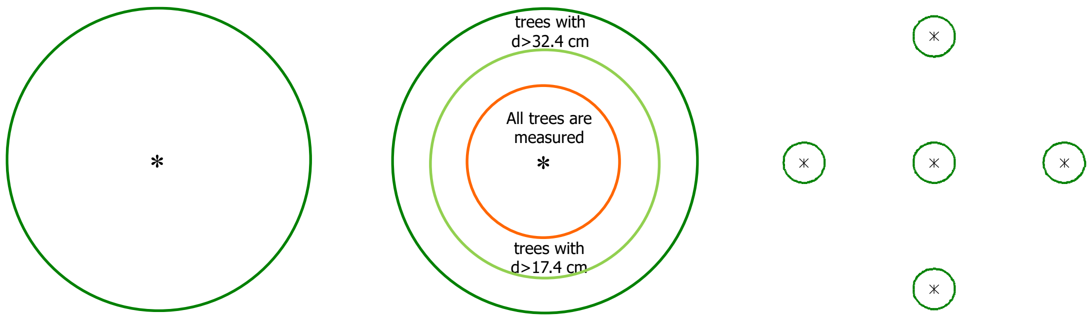
2 shows the most commonly used plot areas in Portugal for the Portuguese main tree species.

Table 2. Plot areas by tree species.

Species	Area (m^2)	Radius (m)
Eucalyptus	400	11.28
Pine and eucalyptus	500	12.64
Pine and young cork oak	1000	17.84
Dense cork oak	1256.64	20
Sparse cork oak	2827.43	30

1.1.2 Plot Type

Different types of plots can be used in an inventory either for practical reasons or for choosing a sampling scheme that optimizes the ratio costs/precision. There are three types of plots:



a) Simple plot

b) Combined or concentric plot

c) Clustered plots

In the case of simple plots all trees have their diameters measured. It is also common that only trees above a certain diameter threshold are measured (5 cm for Eucalyptus and 7.5 cm for other species). Generalizing any Y variable to the hectare is a simple process:

$$\begin{array}{lcl} Y & \text{———} & 10000 \\ Y_p & \text{———} & A_p \end{array} \quad \Rightarrow \quad Y = \frac{10000}{A_p} Y_p$$

The factor $10000/A_p$, by which the Y variable obtained for a plot with area A_p has to be multiplied in order to obtain a corresponding value reported to the hectare, is called area expansion factor.

In the case of combined plots, the sampling unit is composed of various concentric circles where trees are selected to be measured according to the dimension class they belong to. Smaller trees are measured in the smaller circle and other classes are measured in successively larger plots. This was the type of plot used until 1992 in the Portuguese National Forest Inventory (NFI), but is no longer used. However it is very useful for uneven-aged stands for which the plot area required to ensure a good representativeness of big trees has to be large resulting in the measurement of high numbers of small trees.

Supposing 3 concentric plots corresponding to d<12.5 cm, 12.5<=d<22.5 cm 2 d>=22.5 cm, the generalization of Y to the ha would be;

$$\Rightarrow \qquad Y = \frac{10000}{A_{p1}} Y_{p1} + \frac{10000}{A_{p2}} Y_{p2} + \frac{10000}{A_{p3}} Y_{p3}$$

Where A_{pi} is the area for each one of the plots and Y_{pi} the corresponding value for the variable Y. In plot 1 just trees with d<12.5 cm are measured, in plot 2 the measurement focuses trees with d between 12.5 and 22.5 cm and, finally, in plot3 the trees with d>=22.5 cm are the only measured.

The clustered plots consist of a settlement of simple or combined plots or even sampling points around a central point. After the central plot is located and measured another four plots are measured in the direction of the cardinal points at pre-defined distance from the centre of the central plot. It is commonly used for assessing natural regeneration in the NFI and the generalization to the hectare is done as follows:

$$Y = \sum_{i=1}^{n} \frac{10000}{A_{p_i}} Y_{p_i}$$

As all plots present the same area, then:

$$Y = \frac{10000}{A_p} \sum_{i=1}^{n} Y_{p_i}$$

Where A_p is the area of each plot.

Clustered plots can have a different settlement and number of plots, for instance a series of simple plots measured along the perimeter of a square as is the case in the Finish Forest Inventory.

1.1.3 Delimiting the Plot in the Field

After locating the plot centre, the first operation consist of determining the plot limits as accurately as possible which is essential for a correct determination of per hectare values. The incorrect identification of a tree in a 500 m^2 plot corresponds to an error of 20 trees per hectare. The methods will depend on the plot shape and the instrument used in the process. In the case of circular plots delimited using a vertex, the transponder has to be placed in the centre of the plot and all the trees located at a horizontal distance lesser than the plot radius marked facing the plot centre. Special care must be devoted to border trees. Border trees are the ones whose inclusion in the plot raises doubts. The procedure to follow must be the one defined in the field manual and in case this information has been neglected it will be the field crew responsibility to define an inclusion rule and report it along with the measurements. Possible rules are:

a) Considering as part of the plot all trees whose centre at the height of 1.30 m is located within the plot radius
b) Considering as part of the plot all trees whose centre at the base of the tree is located within the plot radius
c) Considering as part of even-numbered plots all trees that touch the border line, whereas for uneven-numbered ones trees should only be considered if totally inside the plot.
d) Considering as part of the plot every-other tree.

Special care must also be taken in the presence of slope. In such cases, the plot area equal to πR^2, when horizontally projected decreases to $\pi R^2 \cos\beta$. **Table 3** shows the errors in plot area obtained for slopes varying between 1º and 50º.

Table 3. Percentage of decrease in horizontal projected areas for a range of slopes up to 50º.

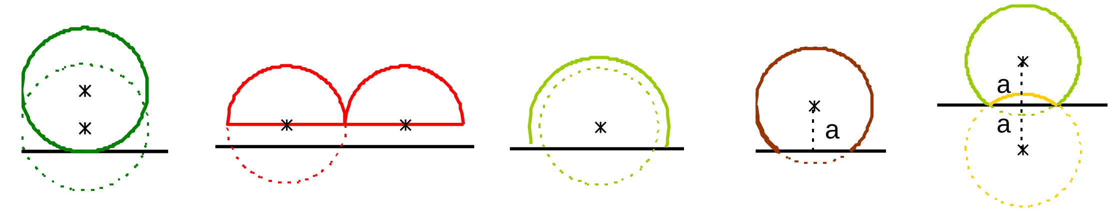
Slope (degrees) β	Slope (%) 100 tanβ	Decrease in projected area (%) 100 (1- cosβ)
1	1.75	0.02
5	8.75	0.38
7.5	13.17	0.86
10	17.63	1.52
15	26.79	3.41
20	36.40	6.03
30	57.74	13.40
50	119.18	35.72

Nowadays, this problem is less important due to the use of instruments that automatically calculate horizontal distances.

1.1.4 Border Plots

The stand border consists of a strip, variable in size, which limits the stand and where growth conditions are different than the ones inside the stand for different light and wind exposures. As a result of this, because the trees located in the stand border also present different characteristics, the border must be properly represented in the sample.

A large number of sample plots are frequently intercepted by the stand border. Several methods have been developed in order to determining the variables' values per area. All the methods consider disregarding plots whose centres are located outside the stand. On the contrary, if the plot centre is part of the stand one of the following methods can be adopted:



A. Translation B. Semi-circle C. Radius expansion D. Evaluations' expansion E. Reflection

Schmid (1969) studied the errors associated to each of these methods and proved that the Reflection method is free from bias, and that both expansion methods are preferable to the Translation and Semi-circle methods. The Evaluations Expansion Method is the one applied by the Portuguese NFI. Field crews are responsible for registering the perpendicular distance from the plot centre to the stand limit represented by *a* in the figures. The plot area can be subdivided in to two areas, the main one representing the area occupied by the stand ($A_{principal}$) being assessed and the remaining area occupied by a different stratum or land use ($A_{remaining}$) based on the values of *a*, plot radius (r) and plot area (A_p).

$$A_{remaining} = r^2 \left(\arccos\left(\frac{a}{r}\right)\right) - a \sqrt{r^2 - a^2}$$

$$A_{principal} = A_p - A_{remaining}$$

1.2 The Number of Trees per Hectare

One of the first variables to assess in a plot of a given area is the number of trees in the plot. Knowing the number of trees in a plot of area A_p (N_p), the number of trees per hectare is given by:

$$N = N_p \frac{10000}{A_p} = N_p \, EF$$

For simplicity reasons it is common to use the symbol *n* when referring to the number of trees in the plot instead of N_p. The value $10000/A_p$ refers to the area Expansion Factor (EF), since it is the value by which the stand variable calculated in a A_p area plot has to be multiplied in order to obtain the corresponding value per hectare.

N refers to the number of living trees in a stand. Natural regeneration is not accounted for, because most of the seedlings are likely to die. On the other hand, there is also the need for setting a threshold for young trees which can no longer be considered regeneration. For these, a diameter threshold is defined above which trees are measured (often 2.5 cm, 5 cm or 7.5 cm).

Other number of trees can be accounted for as described for N, namely: number of dead trees (N_{dead}); number of thinned trees (N_{thin}); number of planted trees (N_{pl}); and ingrowth, the number of trees that were not accounted for in the previous inventory for being below the threshold (N_{ing}).

1.3 Diameter Distribution, Basal Area and Mean Diameter

1.3.1 Diameter Distribution

The diameter distribution of a stand corresponds to determining the frequency of trees according to different diameter classes. Diameter classes usually have a range of 5 cm, however, the range can be adjusted depending on the species and the trees' dimensions. The first diameter class to be considered has a central value of 5, the "*5 cm Class*", and is represented as [2.5 - 7.5 [, meaning that trees with a diameter of 7.5 cm must be accounted for in the class with a central value of 10, the "10 cm Class" (thus represented as [7.5 - 12.4]).

In the field, the diameter distribution is filled in as tree diameters are measured and trees placed in the corresponding diameter class:

Plot nr: 2		Diameter Distribution							Diameters' Measurement								
d Class	Main Tree species: Pine					Other:			Main Tree species: Pine							Other:	
2.5-7.4									8.7	26.0							
7.5-12.4	II								45.7	31.3							
12.5-17.4	I								21.0	29.3							
17.5-22.4	I								40.0	28.0							
22.5-27.4	III								30.0	15.0							
27.5-32.4	~~IIII~~	II							37.3	22.5							
32.5-37.4	I								28.3	29.3							
37.5-42.4	I								25.3	9.2							
42.5-47.4	I								31.9								

The first tree to be measured has a diameter of 8.7 (in yellow) being represented by a vertical bar in the first box of class [7.5 ; 12.4], the "10 cm Class" (in yellow). Each measured tree is represented by a vertical bar in the corresponding diameter class. Only the 5th tree in each group of 5 trees is represented by a horizontal bar closing the set of 5 trees. The 6th tree in this same class is represented in the next box corresponding to the same class. The procedure is described for the "30 cm Class" (in grey): the first tree in this class has a diameter of 30.0 cm (in red) and the 6th, to which corresponds a vertical bar in the next box, of 28.0 cm (in red). The 1st, 6th, 11th trees (in red) in a diameter class represent the sample trees based on the modified Draudt method.

Afterwards, the diameter distribution is generalized to the hectare and can be represented by a histogram. The diameter distribution is an indicator of stand structure providing precious information about the silvicultural options to take in a given stand. The next table (**Table 4**) shows examples of rules to follow while managing cork oak stands based on the simulations carried out using the SUBER model (Tomé et al., 2004).

Draudt Method and Modified Draudt Method

According to the Draudt method, trees are distributed by k diameter classes being the total number of sample trees (m) proportionally distributed to the frequencies by class. The total number of sample trees per class will vary with the number of classes and with the corresponding frequencies. In practice, implementing the Draudt method is quite complex since it requires to:

1. Measure the diameter of all trees in the plot;
2. Grouping these trees by diameter classes;
3. Calculating the quadratic mean diameter (dg) of the trees in each class;
4. Locating in the field the m_i sample trees which are closest of each of the dg's;
5. Measuring the height(s) of these trees.

Given the difficulties, several countries adopted a simplification of this method based on the assumption that if the range of diameter classes is small, any tree can be considered a sample tree. In this way, it is possible to select the sample trees while measuring the diameters:

1. Measure the diameter of all trees in the plot;
2. Grouping these trees by diameter classes and selecting the 1st of every five trees in each diameter class (1st, 6th, 11th,…);
3. Measuring the height(s) of these trees.

1.3.2 Stand Basal Area

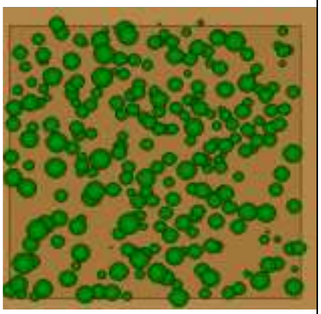
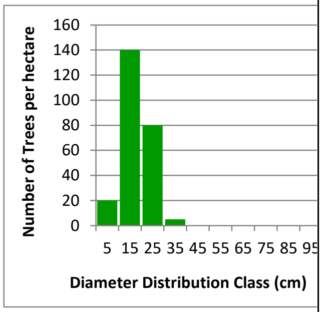
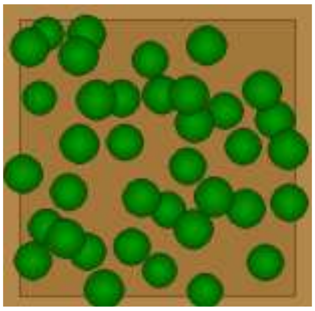
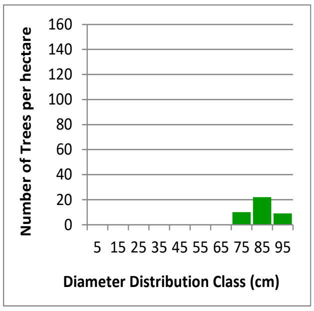
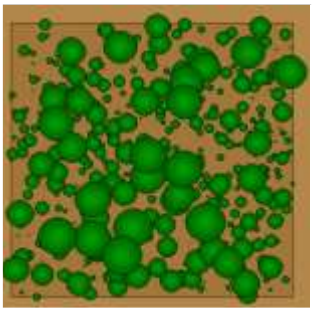
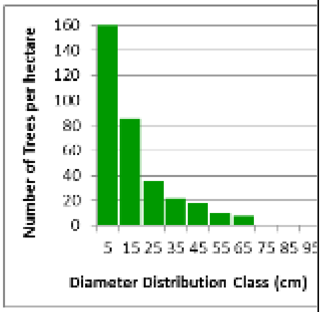
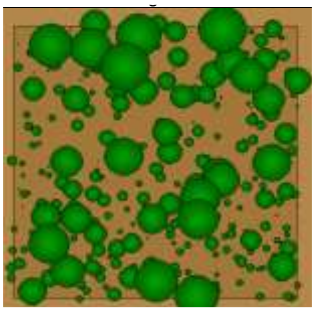
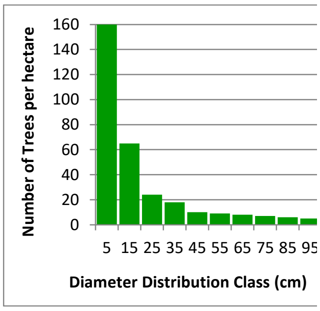
The basal area of a stand is defined as the sum of all the tree basal areas in it. This variable is not only essential in the calculus and/or estimation of several stand variables, but it is also an important stand density indicator. The basal area (G_p) of a plot with a given A_p area, can be determined as:

$$G_p = \sum_{i=1}^{np} \frac{\pi}{40000}\, d_i^2 \qquad (m^2 ha^{-1})$$

Where d_i is the diameter of tree i; np is the number of trees in the plot. Similarly to what was described for determining the stand density, stand basal area is obtained by

$$G = G_p \frac{10000}{A_p} = G_p \, EF \qquad \left(m^2 \, ha^{-1}\right)$$

Table 4. Diameter distribution and stand variables for 4 cork oak stands managed differently.

Even-aged stand - 27 years		**Even-aged stand - 153 years**	
			
Crown Cover =	49%	Crown Cover =	49%
Stand Density =	250 trees ha^{-1}	Stand Density =	37 trees ha^{-1}
Basal Area =	7 m^2 ha^{-1}	Basal Area =	20 m^2 ha^{-1}
Quadratic mean d =	19 cm	Quadratic mean d =	83 cm
Average crown diameter =	5 m	Average crown diameter =	14 m
Virgin Cork weight =	833 kg ha^{-1}	Virgin Cork weight =	40 kg ha^{-1}
Mature Cork weight =	1153 kg ha^{-1}	Mature Cork weight =	5394 kg ha^{-1}
Uneven-aged stand - Structure 1		**Uneven-aged stand - Structure 2**	
			
Crown Cover =	58%	Crown Cover =	58%
Stand Density =	181 trees ha^{-1}	Stand Density =	143 trees ha^{-1}
Basal Area =	12.5 m^2 ha^{-1}	Basal Area =	15 m^2 ha^{-1}
Quadratic mean d=	30 cm	Quadratic mean d=	36 cm
Average crown diameter =	6 m	Average crown diameter =	7 m
Virgin Cork weight=	342 kg ha^{-1}	Virgin Cork weight=	274 kg ha^{-1}
Mature Cork weight =	3076 kg ha^{-1}	Mature Cork weight =	3760 kg ha^{-1}

1.3.2.1 Assessing Stand Basal Area

As previously mentioned, stand variables can be determined using alternative approaches. In the case of basal area, two alternative approaches can be applied: complete enumeration and diameter distribution. The first approach requires having measured the diameters of all trees in the plot so that the corresponding basal areas can be determined and summed to calculate the plot basal area as in the following example:

Property: *Fernão Ferro*
Plot: *74*
Date: *1997*

Tree Number	d (cm)	g (m^2)
23	11.1	0.0097
24	11.5	0.0104
28	13.9	0.0152
12	14.5	0.0165
13	14.6	0.0167
11	17.1	0.0230
10	18.2	0.0260
9	19.6	0.0302
25	20.3	0.0324
7	21.1	0.0350
6	21.5	0.0363
29	21.6	0.0366
27	21.7	0.0370
8	22.1	0.0384
26	22.4	0.0394
19	22.7	0.0405
20	22.7	0.0405
18	23.2	0.0423
32	23.2	0.0423
31	23.4	0.0430
14	23.5	0.0434
5	24.1	0.0456
15	24.1	0.0456
22	24.2	0.0460
4	25.2	0.0499
1	25.4	0.0507
2	25.4	0.0507
21	25.7	0.0519
17	26.0	0.0531
3	26.4	0.0547
30	27.5	0.0594
16	30.6	0.0735
	G_p =	1.2356 m^2
	G =	**24.71** $m^2\ ha^{-1}$

With g calculated using the expression: PI()/40000*d^2

(with d representing DBH)

With G_p as the sum of g and G as G_p times the expansion factor that in this case is 20 (10000/500)

The other approach was widely used in the XX century before the computers' age. Back in those days, calculations took time and therefore it was common to determine stand variables using aggregated data. However, nowadays despite having access to fast computers it is still important to understand this approach because old records will present the calculus in this format being impossible to recalculate stand variables using the current methodologies. When we can only have access to diameter distributions the basal area (G_p) of a plot with an A_p area is determined as follows:

$$G_p = \sum_{j=1}^{k} f_j\, g_j = \sum f_j\, \frac{\pi}{4}\, \frac{d_j^2}{4} = \frac{\pi}{4} \sum_{i=1}^{k} f_j\, d_j^2,$$

Where k stands for the number of diameter classes, f_j represents the frequency in class j and d_j and g_j are, respectively, the central diameter of class j and its corresponding basal area. Look at the example below where using the data from “Fernão Ferro property” basal area was calculated using aggregated data by diameter classes:

Property: *Fernão Ferro*
Plot: *74*
Date: *1997*

d Classes	central diameter	fi (n_p)	n	gi	Gp
7.5-12.4	10	2	40	0.007854	0.314159
12.5-17.4	15	4	80	0.017671	1.413717
17.5-22.4	20	9	180	0.031416	5.654867
22.5-27.4	25	15	300	0.049087	14.72622
27.5-32.4	30	2	40	0.070686	2.827433
			N = 640	G=	24.93639 $m^2\,ha^{-1}$

With gi calculated using the expression:
PI()/40000*(central diameter)^2
and
G_p calculated as gi* (10000/500)

G results from summing up the G_p values in each diameter class

1.3.3 Mean Basal Area and Quadratic Mean Diameter

The mean basal area results from averaging the basal areas of all the trees in a plot or a stand. It is an important stand variable because (being the volume and biomass linearly related to it) the trees with a basal area close to the average basal area can be considered as the most representative trees of the plot or the stand.

The mean basal area (gm) can be determined either using the plot basal area or the stand basal area:

$$gm = \frac{G_p}{N_p} = \frac{G}{N}$$

To the mean basal area corresponds a diameter: the quadratic mean diameter (dg). This diameter is used to locate the average trees in the plot. Please note that this diameter is different from averaging all tree diameters, presenting a slightly higher value. The difference between the average diameter and quadratic mean diameter increases with the variability of diameters observed in the plot. The quadratic mean diameter is calculated as:

$$gm = \pi \ \frac{dg^2}{4} \qquad \Rightarrow \qquad dg = 100 \ \sqrt{\left(\frac{4 \ gm}{\pi}\right)} = 100 \ \sqrt{\frac{4 \ G}{\pi \ N}}$$

It can also be calculated as the quadratic average of diameters:

$$dg = 100 \ \sqrt{\frac{\sum_{i=1}^{n} d_i^2}{n}}$$

Using the same data as in the previous examples, mean basal area and quadratic mean diameter can be calculated:

$$gm = \frac{G}{N} = \frac{24.71}{640} = 0.0386 \ \text{m}^2$$

$$dg = 100 \ \sqrt{\frac{4 \ G}{\pi \ N}} = 100 \sqrt{\frac{4 \ x \ 24.71}{\pi \ 640}} = 22.17 \ \text{cm}$$

1.4 S*to*cking and Stand Density

Despite not being synonyms, these terms are usually used with the same meaning. The stand density is a quantitative measure of woody material per area, whereas the stocking refers to an evaluation of stand density with regard to a given management. In this sense, stands can be classified as under-stocked, fully stocked or over-stocked.

Stocking and stand density assessments are extremely important because the need for thinning as well as the thinning intensity can often be based on these stand variables.

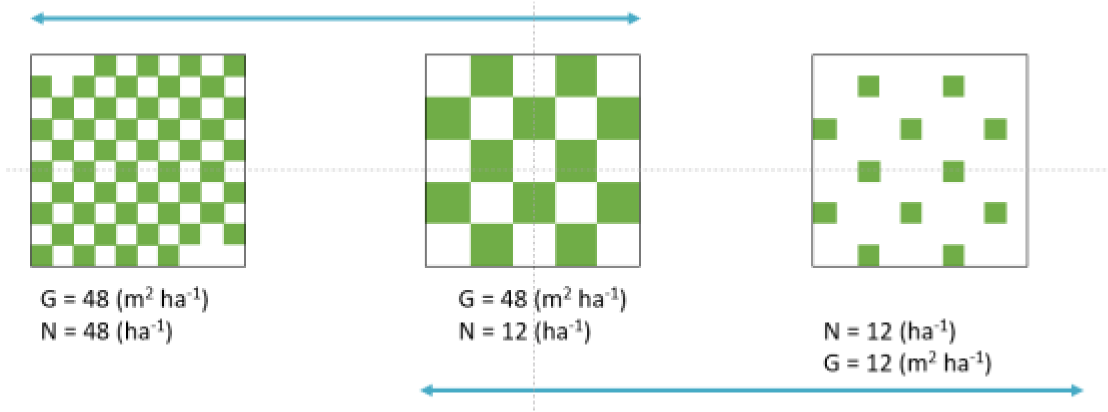
1.4.1 Assessing Stocking

Stocking assessment relies on determining or defining the suitable stand density for a given tree species, in a certain place, being managed with a specific purpose. Consequently it is a quite subjective and difficult to define concept. According to Bickford (1979):

> *The stocking that results in maximum yield is the ideal that every forest manager would like to have if he only knew what it was and could recognize it if he saw it.*

The terminology under-stocked and over-stocked is still quite common among forest managers.

Basal area and stand density are stand density measures. However, neither basal area nor the number of trees per hectare alone give a clear idea about the stocking of a stand requiring the use of other density measures. Two stands can have the same stand density and one be competing for resources while the other is still far from reaching that point. The same is valid for two stands with the same basal area, they can face different competition levels depending on the number of trees per ha.



1.4.2 Assessing Stand Density

1.4.2.1 Basal Area

Basal area is one of the most commonly used stand variables for assessing stand density. Apart from being easily obtained in the course of forest inventory, it is also objective and easy to understand by forest managers. Basal area is a good measure of stand density expressing the need for thinning. The thinning intensity can be defined through the residual basal area, which means the basal area left in the stand after thinning. However, the practical implementation can sometimes be a hard task because the forest manager has to select the trees to be thinned in order to reach the desired residual basal area.

1.4.2.2 Number of Trees per Hectare

The number of trees per hectare is not usually used in natural stands, because stand density does not always relate with this variable. On the other hand, in planted forests it is the most used. It is easy to determine and allows an objective definition of thinning intensity. Once the forest manager has decided to thin, either based on the basal area or any other measure, he knows the number of trees per hectare to be removed in the thinning, which is much easier than removing trees until a certain basal area is attained.

1.4.2.3 Percent Crown Cover

This is a very useful density measure particularly in stands characterized by low densities. Percent crown cover (%cc) results from expressing the sum of crown areas as a percentage of plot area:

$$\%cc = \frac{\sum \frac{\pi \ cw^2}{4}}{A_p} \ 100$$

For example, Natividade (1950) suggests that cork oak stands should always be managed keeping percent crown cover below 58% to secure that sufficient light reaches the trees' crowns. The calculus of the percent crown cover is illustrated bellow for a cork oak permanent plot with A_p=2827 m^2 where the four crown radius were measured. When the crown radius are not measured a crown diameter equation has to be used.

Property: *Herdade do Chaparro*

Permanent Plot: *8*

Date: *1989*

Tree Number	Tree Species	Crown Radius				Mean Crown Radius	Tree Crown Area
		North	South	East	West		
1	Sb	3	4.4	2.3	4.7	3.6	40.72
2	Sb	5.2	7.4	4.5	6.2	5.8	106.60
3	Sb	4.6	6.2	2.7	3.9	4.4	59.45
4	Sb	1.9	3	3.1	3.3	2.8	25.07
5	Sb	2	4.2	4.1	5.4	3.9	48.40
6	Sb	5	4.9	5.6	4.9	5.1	81.71
7	Sb	2.5	6.5	2.2	4.2	3.9	46.57
8	Sb	6.6	9.1	8.3	7.8	8.0	198.56
9	Sb	2.4	4.2	5.1	3.8	3.9	47.17
10	Sb	2.9	5.5	2.6	3.8	3.7	43.01
11	Sb	5.4	3.6	2.8	4.3	4.0	50.90
12	Sb	5.6	5.7	8.1	4.7	6.0	114.04
13	Sb	3.7	6.1	4.7	2	4.1	53.46
14	Sb	5.1	7.1	6.7	5.5	6.1	116.90
15	Sb	6.7	7.1	7.2	6.6	6.9	149.57
16	Sb	5.1	5.5	4.3	5.4	5.1	80.91
17	Sb	5.1	4.5	5.7	4.5	5.0	76.98
18	Sb	6.1	5.9	5.4	5.1	5.6	99.40
19	Sb	2.9	4.8	4	3.6	3.8	45.96
20	Sb	7.5	9.6	5.3	7.9	7.6	180.27
21	Sb	2.6	1.4	1.9	2.4	2.1	13.53
22	Sb	2	0.3	1.8	0.8	1.2	4.71
23	Sb	0.8	2.1	2	1.6	1.6	8.30
24	Pm	3.4	2.9	3.5	3	3.2	32.17
25	Pm	4.8	3.4	4.3	3.9	4.1	52.81
26	Pm	1.9	1.7	2.1	1.7	1.9	10.75
27	Pm	2.6	2.5	2.7	2	2.5	18.86
28	Pm	1.4	1.7	1.6	1.9	1.7	8.55
29	Pm	5.3	6.1	5.8	4.6	5.5	93.31
30	Pm	2.1	2.3	2	2.4	2.2	15.21
31	Pm	2.2	2.7	2.2	2.4	2.4	17.72

A_p = 2827 m^2

Total Crown Area 1941.55 m^2

%CC **68.68** %

The Mean Crown Radius (crd) is obtained by averaging the radius measured in the cardinal directions.

The tree crown area is then determined using the expression: PI()*crd^2

The Total Crowns Area results from summing up all the Tree Crown Areas

The %CC is then calculated as the racio between the Total Crowns Area and the plot area (Ap): Total Crowns Area/A_p*100

1.4.2.4 Stand Density Index

Both the basal area and the number of trees per hectare are incomplete measures of stand density. Two stands with the same basal area can have distinct stand densities as long as these stands have a different number of trees per hectare or a different age.

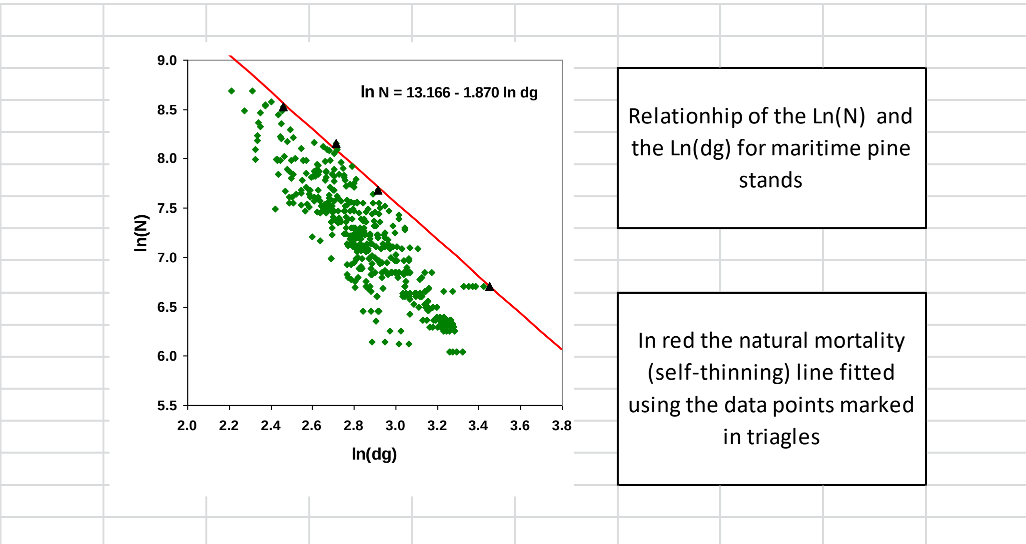
In order to tackle the limitations of the previous measures, other measures were developed combining more than one stand variable. The stand density index (SDI) is based in the two components of basal area: number of trees per hectare and quadratic mean diameter. This index assesses stand density by comparing the stands characteristics with those of a stand with the maximum stand density (under self-thinning). Reineke (1933) observed that the relationship between the logarithm of the number of trees per hectare and the logarithm of the quadratic mean diameter in maximum stocked stands is usually linear. Moreover, he also observed that for stands with the maximum stock this line presents a slope close to -1.605:

$$\log_{10} N = -1.605 \ \log_{10} dg + k ,$$

Where N is the number of trees per hectare, dg is the quadratic mean diameter and k is a species specific constant. This line is usually defined as reference curve and it is assumed that in maximum stocked stands the relative growth rate of the number of trees (negative) and the relative growth rate of the quadratic mean diameter (positive) are proportional:

$$-\frac{1}{N}\frac{dN}{dt} = b\frac{1}{dg}\frac{ddg}{dt}$$

This relationship is shown in the graphic below for data from a thinning trial established in Portuguese maritime pine stands. Natural logarithms were used since the slope is not affected by the logarithm base. All trials include some non-thinned control plots. The graphic shows a wide dispersion of data for each dg value indicating that not all the plots have reached the self-thinning line. For the higher values of dg a horizontal evolution is observed indicating that this plot is still far from self-thinning. To estimate the natural mortality line the points marked with triangles are used. The slope obtained is slightly smaller than Reineke's, although it is in line with the value of -1.815 found by Luís et al. (1991) for maritime pine using National Forest Inventory data, or the value of -1.997 obtained by Oliveira (1985) for the mountainous an sub-mountainous regions of Portugal. These results evidenced the need of installing more of these trials in order to obtain more precise information on the natural mortality line.



SDI is based on the difference between the number of trees per ha corresponding to the maximum stock (given by the expression above) and the number of trees per ha of the target stand.

Moreover, SDI assumes that for an under-stocked stand the relationship between log N and log dg is parallel but with a lower intercept than the one observed for the fully-stocked stands. The intercept can be obtained by the expression:

$$k = \ln N + 1.870 \ln dg \quad \text{(\textit{using the value obtained for the pine stand})}$$

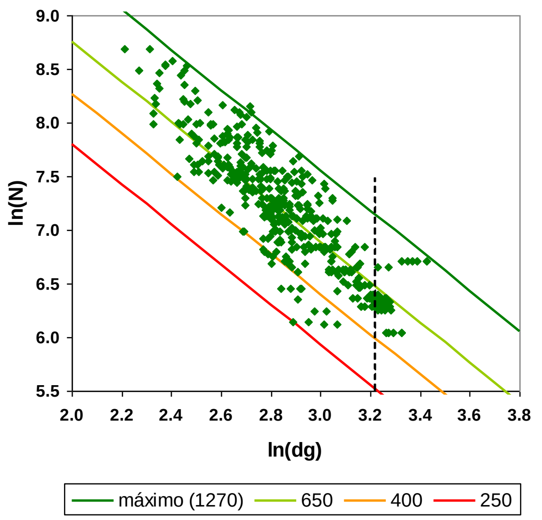
For standardization the calculus of SDI is done based on the number of trees per hectare the stand would have for a dg=25 cm. The SDI for any given stand with dg=25 is:

$$\ln SDI = -1.870 \ln 25 + k$$

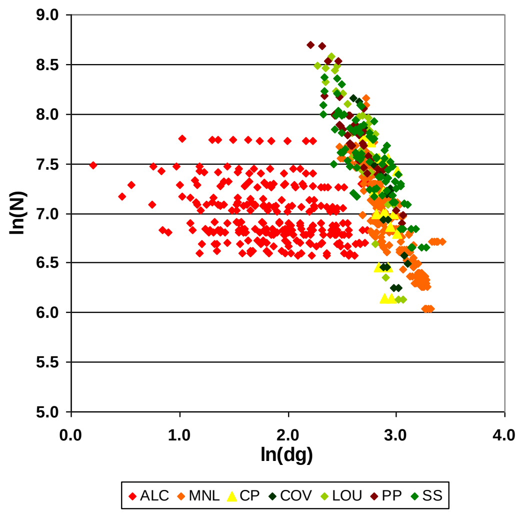
Replacing k by the value obtained for the target stand, the expression for SDI can be obtained:

$$\ln SDI = -1.870 \ln 25 + \ln N + 1.870 \ln dg$$

$$SDI = N \left(\frac{dg}{25} \right)^{1.870}$$



Natural mortality line and the lines corresponding to stands with different SDI values.



In red we have maritime pine under-stocked stands approaching the natural mortality line; the remaining different colours refer to different trials.

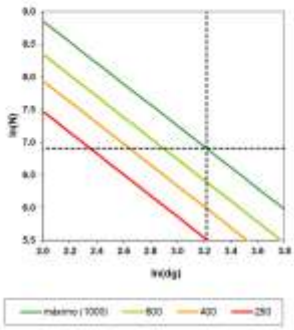
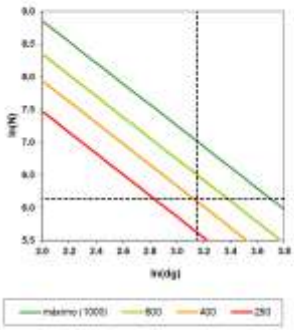
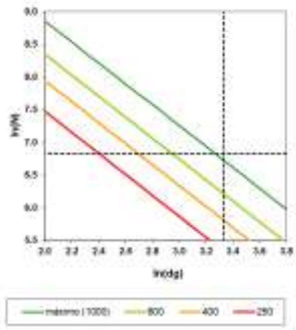
For a given dg value, the maximum SDI is obtained for the N corresponding to a fully-stocked stand; if the number of trees in the stand is used in the expression. SDI provides a stand density measure, independent from tree dimension.

The left hand side graphic shows the mortality line fitted for maritime pine using the same data set as before as well as the lines corresponding to stands with different SDI values.

When thinning are not carried out, in the mid-term stands will tend to approach the natural mortality line as shown in the right hand side graphic.

The SDI value for a stand with dg=22.1 cm and N=320, is given by:

$$SDI = N\left(\frac{dg}{25}\right)^{1.87} = 320\left(\frac{22.1}{25}\right)^{1.87} = 254$$

<table>
<tr><td>Suppose you have a stand with N=1000 and dg=25</td><td>Suppose you have a stand with N=463 and dg=23.36</td><td>Suppose you have a stand with N=920 and dg=28</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>
<table>
<tr><td>SDI</td><td>1000</td><td></td></tr>
<tr><td></td><td></td><td>ln</td></tr>
<tr><td>N</td><td>1000</td><td>6.907755</td></tr>
<tr><td>dg</td><td>25</td><td>3.218876</td></tr>
</table>
</td><td>
<table>
<tr><td>SDI</td><td>415.3685</td><td></td></tr>
<tr><td></td><td></td><td>ln</td></tr>
<tr><td>N</td><td>463</td><td>6.137727</td></tr>
<tr><td>dg</td><td>23.36</td><td>3.151025</td></tr>
</table>
</td><td>
<table>
<tr><td>SDI</td><td>1102.901</td><td></td></tr>
<tr><td></td><td></td><td>ln</td></tr>
<tr><td>N</td><td>920</td><td>6.824374</td></tr>
<tr><td>dg</td><td>28</td><td>3.332205</td></tr>
</table>
</td></tr>
<tr><td>If SDI reached the maximum, it has reached the self-thinning line, beyond that trees will start competing for resources</td><td>If SDI is below the maximum, the stand is far from the self-thinning line and trees are not competing for resources</td><td>If SDI is above the maximum the stand is beyond the self-thinning line and trees are competing for resources</td></tr>
<tr><td colspan="3">In any of these cases you may choose to thin depending on your management objective, but the SDI value tells you nothing about the thinning weight to consider</td></tr>
</table>

1.4.2.5 Crown Competition Factor

The crown competition factor (CCF), Krajicek et al. (1961), reflects the relationship between the area available in the stand and the maximum area trees would have access to if they were open-grown (without competition).

It requires knowing the relationship of crown width (cw) and diameter for open-grown trees, which tends to be linear and assumes that the crowns of open-grown trees are circular:

$$cw_l = b_0 + b\ d_l \qquad\qquad ca_l = \pi\ \frac{cw_l^2}{4}$$

CCF results of the sum of ca_l of all the trees in the plot expressed as a percentage of the plot area (A_p):

$$CCF = \frac{100}{A_p}\sum_{i=1}^{n} ca_l\ ,$$

Where A_p is the area of the plot.

If calculated based on the diameter distribution:

$$CCF = \frac{100}{Ap}\sum_{j=1}^{k} fj\ ca_{lj}\ ,$$

Where k represents the number of diameter classes, fj the frequency in class j and ca_{lj} the diameter class mid-point.

In Portugal not many studies on the crown dimension of open-grown maritime pine trees have been carried out. Alegria (1994) fitted a model for open-grown pine trees in Oleiros, Castelo Branco and Proença-a-Nova:

$$cw_l = 0.335229 + 0.171785\ d$$

Property: *Proença-a-nova*

Date: *1989*

Tree Number	Tree Species	d	Isolated Trees: Crown width cw_l	Isolated Trees: Crown area ca_l
1	*Pb*	*11.1*	2.24	3.95
2	*Pb*	*11.5*	2.31	4.19
3	*Pb*	*13.9*	2.72	5.82
4	*Pb*	*14.5*	2.83	6.27
5	*Pb*	*14.6*	2.84	6.35
6	*Pb*	*17.1*	3.27	8.41
7	*Pb*	*18.2*	3.46	9.41
8	*Pb*	*19.6*	3.70	10.76
9	*Pb*	*20.3*	3.82	11.48
10	*Pb*	*21.1*	3.96	12.32
11	*Pb*	*21.5*	4.03	12.75
12	*Pb*	*21.6*	4.05	12.86
13	*Pb*	*21.7*	4.06	12.97
14	*Pb*	*22.1*	4.13	13.41
15	*Pb*	*22.4*	4.18	13.74
16	*Pb*	*22.7*	4.23	14.08
17	*Pb*	*22.7*	4.23	14.08
18	*Pb*	*23.2*	4.32	14.66
19	*Pb*	*23.2*	4.32	14.66
20	*Pb*	*23.4*	4.35	14.90
A_p =	500 m^2		FCC=	**43.42**

The crown width (cw_l) is estimated with the equation: cwl = 0.335229 + 0.171785 d

(with d representing DBH)

The tree crown area (ca_l) is then determined using the expression: PI()/4*cw^2

The CCF is then calculated as the ratio between the Total Crowns Area and the plot area (Ap): Isolated Crowns Area/A_p*100

1.4.2.6 Relative Spacing

The relative spacing (RS) relates the average distance between trees and the height of dominant trees. It is based on the assumption that the relationship between the distances between trees and dominant height expresses stand density:

$$RS = \frac{\text{average distance between trees}}{\text{hdom}}$$

Assuming a squared spacing, each tree has an area of:

$$\text{Tree Area} = \frac{10000}{N},$$

Therefore, the average distance between trees will be given by:

$$\text{Average distance between trees} = \sqrt{\frac{10000}{N}}$$

Then the relative spacing index can be written in the following form, commonly designated as Wilson's factor (Wilson, 1946):

$$FW = \frac{\sqrt{10000/N}}{hdom} = \frac{100}{hdom\ \sqrt{N}}$$

The Wilson's factor has been widely used in the management of maritime pine stands in Portugal, for its easy applicability. Leiria National Forest, the Portuguese forest area with the oldest management plan, has been using this index for determining the need for thinning as well as the thinning intensity. Each stand is inventoried every 5 years and Wilson's factor calculated. If this value is lower than the limit considered then a thinning should be carried out (for several years the value of Fw=0.25 was used, lately lower values have been preferred). The thinning intensity is then defined by the difference between the number of trees corresponding to the limit Wilson's factor and the Wilson's factor value found for the target stand. Suppose we are in the presence of a stand with 756 trees per ha and a dominant height of 16 m. The Wilson's factor will be given by:

$$Fw = \frac{100}{16\ \sqrt{756}} = 0.227 < 0.25 \qquad \Rightarrow \qquad \text{thinning required}$$

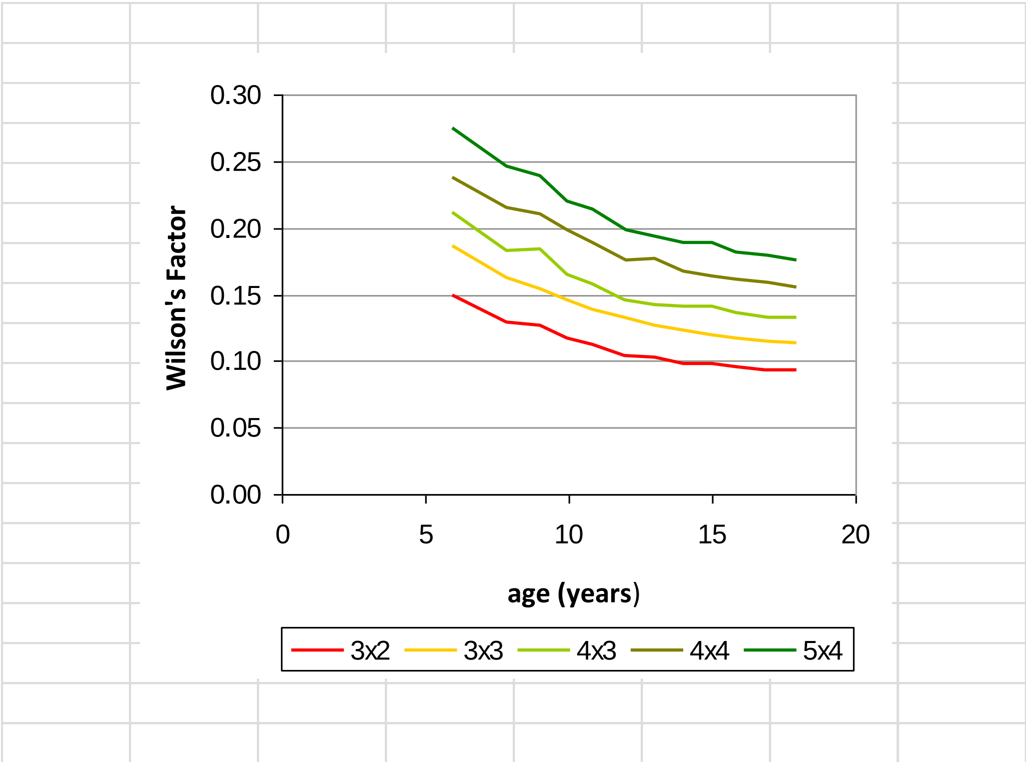
$$N = \frac{100^2}{hdom^2 FW^2}$$

The number of trees corresponding to a Wilson's factor of 0.25 and a dominant height of 16 m is determined by solving the expression for N:

$$N = \frac{100^2}{\left(16^2\ 0.25^2\right)} = 625$$

Thus, the thinning to be carried out will have to remove (756-625) =131 trees per ha.

Regardless of the site quality and initial stand age, all even-aged stands of a given species tend to show the same pattern for Wilson's factor evolution over time. Please note that, if the number of trees corresponding to the Wilson factor of 0.25 were 800 trees per ha, **no thinning would be required and this would not evidence the need to increase stand density by planting**.



The graphic shows the evolution of the Wilson's factor (FW) in Eucalyptus stands with different spacings. Initially, FW decreases rapidly the fastest as wider the spacing, but afterward it tends to asymptotically approach a minimum FW.

1.4.2.7 Spacing Coefficient

The spacing coefficient (Cspac) relates the average distance between trees and the mean crown width (cwm):

$$\text{Cspac} = \frac{\text{Average distance between trees}}{\text{cwm}}$$

This index is not immediately applicable because the tree crown diameters are not commonly measured in forest inventories given the high costs related to these measurements. However, it is a very interesting index for species characterized by wide crowns such as cork oak and umbrella pine. In practical applications, the crown diameters can be estimated using allometric equations that only use variables measured in the course of forest inventories.

Admitting a regular squared spacing, as assumed for the Wilson's factor, the average distance between trees is equal to the square root of 10000/N. Thus, the spacing coefficient is given by:

$$\text{Cspac} = \frac{100}{\text{cwm} \sqrt{N}}$$

There is a close relationship between this index and the percent crown cover (%CC). A Cspac of 1.2 corresponds to a %CC value of 58%, defined by Natividade (1950) as the maximum possible value that avoids strong competition between trees. Cspac values lower than 1.2 indicates that thinning is required. In order to calculate Cspac, all we need to do is calculating the average of the crown diameters (twice the value of the crown radius):

$$\text{Cspac} = \frac{100}{8.2\ \sqrt{110}} = 2.3$$

In the SUBER model (Tomé et al., 2004), a forest management model available for cork oak stands in Portugal, the need and intensity of a thinning are based on this index.

1.5 Stand Heights

1.5.1 Mean Height of a Stand

For even-aged stands it is valid to admit that the heights of trees in one plot will have close values. Therefore, is makes sense to calculate the mean height as the average of the heights of all trees in the plot:

$$hm = \frac{\sum_{i=1}^{n} hi}{n}$$

According to the European school, it is a standard procedure to define the mean height as the height of the tree with a diameter = to the quadratic mean diameter and it is referred to as hg. This height can be obtained by two alternative processes

a) Measuring the height of trees with a diameter closer to the quadratic mean diameter

b) Estimating the height corresponding to the dg using a height-diameter curve fitted using the plot data

1.5.2 Dominant Height

It is common to define the mean height of the biggest trees in one stand as dominant height. In practice, there is the need to define which trees are the biggest and therefore accounted for the dominant height determination. Kramer (1959) classified the method available for assessing dominant height in two types: mathematical and biological. For the mathematical method, the number of trees considered for calculating hdom is determined by an objective and quantitative rule:

a) A fixed percentage of the total number of trees, 10 or 20%, selected according to their height or diameter;
b) A fixed number of trees per hectare, usually 50, 100 or 200, selected according to their height or diameter.

For the biological method, trees are selected based on their social classification being possible to select just the dominant trees or the dominant and codominant trees in the plot.

In Portugal the mean height of the 100 thickest trees (at diameter height) per hectare has been traditionally used to define hdom. This definition cannot be applied in cork oak or umbrella pine stands. In fact, most of these stands have stand densities smaller than 100 trees/ha. In the course of NFI 2005/2007, the mean height of the 25 thickest trees in the stand was used ($hdom_{25}$) for these stands.

Dominant height is an important variable because regardless of the method used to select the dominant trees, this stand variable is most of the times independent of the thinnings carried out being for this reason a good measure of the potential productivity of the stand.

1.5.3 Height-diameter Curves

Height-diameter curves, expressing the relationship between height and diameter for a given stand, have been introduced already when defining the ways to determine tree level variables.

Local height-diameter curves are fitted for a single stand using the diameter as the only independent variable, whereas generic or regional height-diameter curves are applicable to a region and use stand level variables that express stand density and site quality apart from diameter. Replacing the values of these variables by those of a given stand we obtain a height-diameter curve for the stand. Therefore, a height-diameter curve characterizes a given stand.

The concept of potential productivity of a forest system involves the influence of three types of factors that are determinant of growth: specific potential productivity, site quality and management intensity (Monteiro Alves, 1982).

The site quality of a given tree species refers to the potential present and future productivity of a species in a given site. According to the Society of American Foresters, the term site refers to an area considered for its environment insofar as it determines the type and quality of vegetation that the site can have (Avery and Burkhart, 1983). Site quality assessment is essential for an accurate characterization of a given stand and vital for growth prediction.

1.6 Site Quality

1.6.1 Site Quality Assessment

Avery e Burkhart (1983) grouped the methods for site quality assessment in two classes:

- *Direct assessment*: through the direct measurement of the environmental factors linked to growth;
- *Indirect assessment*: through the measurement of vegetation characteristics that express the impacts of those environmental factors.

Direct Assessment of Site Quality

Despite being theoretically possible to assess site quality through the factors affecting forest productivity (such as nutrient and soil water availability, weather, radiation, topography, etc.) it is extremely difficult to take into account all the factors and their interactions. Therefore it is common to decide for indirect assessment.

Marques (1987, 1991) developed a model to predict site index (defined in section 1.5.5):

$$S = 10.7214 + 0.780177\ X_1 + 0.0246574\ X_2 + 0.00672025\ X_3 - 0.00441198\ X_4$$

Where X_1 stands for the mean temperature in autumn (ºC), X_2 stands for the available potassium (moles m^{-2} in the soil profile), X_3 represents the total soil porosity ($dm^3\ m^{-2}$ in the soil profile) and X_4 the percentage of fine sand ($dm^3\ m^{-2}$ in the soil profile). A low R-square value (0.544) was obtained in the fit. However, site index estimates in young stands, age classes 5 and 10, have proven to be more precise than those obtained by indirect methods.

Indirect Assessment of Site Quality

Indicator plants, stand volume (mean annual increment for the age corresponding to the maximum growth) can be used in the indirect assessment.

<u>Indicator plants</u>

Sometimes, it is possible to relate the presence of some shrubs or herbaceous plants to site quality. The classification of site index based on indicator plants was developed by Cajander and his followers in Finland (Spurr, 1952; Vuokila, 1965). Never the less, this approach has disadvantages pointed out by Avery and Burkhart (1983) and Clutter et al. (1983):

- It only allows qualitative site quality assessment;
- Shrubs and herbaceous plants are often extremely sensitive to external factor such as fire or grazing;
- In most cases, indicator plants can only reflect the fertility of upper soil horizons, being the deeper ones vital for determining stand growth;
- Such an evaluation requires deep knowledge on ecology and systematics which makes its widespread application difficult.

Marques (1987, 1991) studied the relationship between site quality and the presence of indicator plants for maritime pine stands in the Tâmega valley, and the results have proven poorer than those obtained using site index curves.

Stand volume

Stand volume was sometimes used to evaluate site index in natural and un-thinned artificial stands or even in thinned stands where no more than 1/3 of the total volume had been removed (Assman, 1961, 1970). After heavy thinning became common, standing volume per hectare can suffer substantial reductions preventing this variable from being used with this purpose.

Stand height

The method traditionally used to define the quality of a site is certainly determining site index based on dominant height growth. In fact, dominant height is quite sensitive to differences in site quality, while at the same time seems to be little affected by stand density (please note that tree height is affected by stand density, just the height of the largest trees is not) and its composition. In Portugal, dominant height is defined as the mean height of the thickest trees in the stand in a proportion of one tree per 100 m^2. The use of the thickest trees intends to avoid the sensitivity to the type of thinning carried out.

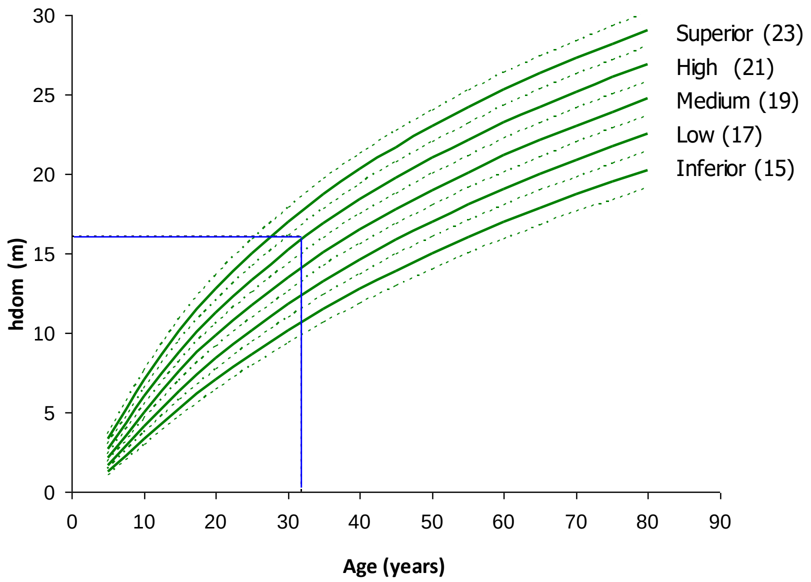
1.6.2 Site Index

Site index (S) can be defined as the dominant height a stand has, had or will have at a certain standard age. The standard age is species-specific and is defined as an age close to the stand rotation age. It is common to group the values that site index might take for a certain region in classes, the site index classes.

In practice, once the age of a stand and its dominant height are known for a given instant in time, three methods can be used to estimate site quality: a set of site index curves (graphical assessment); a dominant height growth equation and a site index prediction equation.

1.6.3 Site Index Curves

These curves represent the evolution of dominant height over age and are simultaneously represented in a graphic. The set of curves in the graphic represents the range of site indices that can be found for the region under study. Usually, 5 site index classes are represented: inferior, low, medium, high and superior; as for the curves fitted for maritime pine in Leiria National Forest by Falcão (1992). Considering a standard age of 50 years, the site indices corresponding to each of the classes are: 15, 17, 19, 21 and 23. Suppose a 32 years stand was measured presenting a height of 16 m. To which site index class does this stand correspond? The answer is to a high site index class (marked in blue in the graphic below).



1.6.4 Using Dominant Height Growth Functions to Estimate Site Index

Dominant height growth functions can be used to estimate the dominant height that corresponds to a given standard age and thus to estimate site index. Two types of dominant height growth functions can be used:

a) <u>Growth functions with S as an independent variable;</u>

One example of this type of function is the growth function that originates Oliveira (1985) site index curves for the mountainous and sub-mountainous regions of Portugal. In this situation, site index (S) is determined by solving the equation for S.

Consider a stand with 22 years and a dominant height of 16 m.
Determine its site index for a standard age of 40 years

Oliveira (1985) $$hdom = S\ e^{-14.2234\left(\frac{1}{t}-\frac{1}{40}\right)}$$

1) solve the equation for S

$$S = hdom\ e^{14.2234\left(\frac{1}{t}-\frac{1}{40}\right)}$$

2) Calculate site index

$$S = 16\ e^{14.2234\left(\frac{1}{22}-\frac{1}{40}\right)}$$

S= **16.3** m

b) Growth functions formulated as difference equation functions, meaning that dominant height in instant 2 ($hdom_2$) is estimated based on dominant height in instant 1 ($hdom_1$) and on the age in both instants (t_2 and t_1).

This type of functions can be found in the GLOBULUS model (Tomé et al., 2001) where a collection of regionalized dominant height growth functions formulated as difference equations can be used to estimate site index for eucalyptus stands in Portugal. Difference equations, if correctly developed, should be invertible, which means that the function when solved for $hdom_1$ gets the same expression simply shifting the indices 1 with 2. Take the growth function for planted stands in the Coastal Central area of Portugal as an example:

$$hdom_2 = 61.1371\left(\frac{hdom_1}{61.1371}\right)^{\left(\frac{t_1}{t_2}\right)^{0.4805}} \qquad hdom_1 = 61.1371\left(\frac{hdom_2}{61.1371}\right)^{\left(\frac{t_2}{t_1}\right)^{0.4805}}$$

To estimate site index using this type of equations simply assume t_2 (or t_1, depending whether the stand age is lower or higher than the standard age) is equal to the standard age.

1) Consider a stand with 4 years and a dominant height of 8.2 m.

If the age of the stand is lower than the standard age:

$hdom_1$ = 8.2 t_1 = 4

$hdom_2$ = ? (S) t_2 = 10

$$hdom_2 = 61.1371 \left(\frac{hdom_1}{61.1371}\right)^{\left(\frac{t_1}{t_2}\right)^{0.4805}} \qquad S = 61.1371 \left(\frac{hdom_1}{61.1371}\right)^{\left(\frac{t_1}{10}\right)^{0.4805}}$$

S = **16.8** m

2) Consider a stand with 14 years and a dominant height of 24.1 m.

If the age of the stand is lower than the standard age:

$hdom_1$ = ? (S) t_1 = 10

$hdom_2$ = 24.1 t_2 = 14

$$hdom_1 = 61.1371 \left(\frac{hdom_2}{61.1371}\right)^{\left(\frac{t_2}{t_1}\right)^{0.4805}} \qquad S = 61.1371 \left(\frac{hdom_2}{61.1371}\right)^{\left(\frac{t_2}{10}\right)^{0.4805}}$$

S = **20.5** m

Note that the expressions presented for estimating site index for stands younger or older than the standard age are equivalent as a consequence of the dominant height growth functions being invertible. This property has to be verified before using these functions, if it is proven they are not invertible then two alternative expressions need to be found and applied depending on stand age being lower or higher than the standard age.

For a better idea on the variety of functions available for estimating site index, **Table 5** shows some dominant height growth functions for the main Portuguese tree species.

Table 5. Some dominant height growth functions available for Portugal.

Region and reference	Expression	tp[1]
Maritime pine		
Mountainous and sub-mountainous regions Oliveira (1985)	$hdom = S_{40}\ e^{-14.2234\left(\frac{1}{t}-\frac{1}{40}\right)}$	40
Tâmega Valley Marques (1987)	$hdom = e^{4.04764-\frac{8.75819}{t^{0.56087}}} + 1.19874\left(1-e^{-0.081\ t}\right)^{2.99578}(lqe - 17.38)$	35
Páscoa (1987)	$hdom = S_{50}\ 10^{0.380999-2.694076\,t^{-1/2}}$	50
Leiria National Forest Falcão (1992)	$hdom = 103.7\left(\frac{S_{50}}{103.7}\right)^{\left(\frac{50}{t}\right)^{0.3593}}$	50 (qq)
Portugal (PAMAF 8165) Tomé et al. (2001)	$hdom_2 = 69\left(\frac{hdom_1}{69}\right)^{\left(\frac{t_1}{t_2}\right)^{0.4582}}$	qq
Eucalyptus		
Soporcel Litoral Stand Amaro et al. (1990)	$hdom_2 = 35.7^{\left(1-\frac{\ln(1-e^{-0.0917\ t_2})}{\ln(1-e^{-0.0917\ t_1})}\right)}\ hdom_1^{\frac{\ln(1-e^{-0.0917\ t_2})}{\ln(1-e^{-0.0917\ t_1})}}$	10 qq
Centre Coastal Portugal 1st rotation[2] Tomé et al. (2001)	$hdom_2 = 61.1371\left(\frac{hdom_1}{61.1371}\right)^{\left(\frac{t_1}{t_2}\right)^{0.4805}}$	10 qq
Cork oak		
Spain (Catalunia and Huelva) Gonzalez et al. (2005)	$hdom_2 = 16.2954\cdot\left\{1-\left[1-\left(\frac{hdom_1}{16.2954}\right)^{1-0.3282}\right]^{\frac{t2}{t1}}\right\}^{\frac{1}{1-0.3282}}$	80 qq

1 tp indicates the standard age
2 for other regions and rotations see the original publication
qq indicates the function can be used for any standard age, although authors recommend the suggested age

1.6.5 Using Site Index Prediction Equations

The previous methods are based on functions fitted to estimate dominant height growth for any age (not for the standard age in particular). However, some authors have preferred to develop functions specifically for estimating site index.

Marques (1987, 1991) developed for the Tâmega Valley region separate equations (despite using the same data set) for estimating site index and dominant height growth:

$$S = 17.38\ f_1(t)\ e^{f_2(t)} + f_1(t)\ \text{hdom}$$

$$f_1(t) = 0.865685 - 0.00804747\ t + 0.000994305\ t^2 - 0.0000187066\ t^3$$

$$f_2(t) = 4.04764 - 8.75819\ t^{-0.560870}$$

The problem with this lays on the compatibility between site index and dominant height functions being taken into account. The two functions are defined as compatible if the site index estimates obtained for the same age with both of them is the same.

1.7 Volume

As mentioned for tree volume, stand volume also refers to over bark volume including the stump. The volume of the plot or stand is obtained. Depending on the volume considered for the trees (for example, if the over bark volume without stump was determined for each tree, the corresponding plot/stand volume will also be over bark volume without stump). Stand volume can be determined using alternative approaches. In the case of volume, several alternative approaches can be applied: direct and indirect, based on complete enumeration or diameter distribution.

Direct assessment of volume

In theory it is possible to calculate the cubic volume of each of the trees in a plot using direct assessment. However, in practice this is rarely done because the gain in precision is not counterbalanced for the time and resources consumption. The exception could be the measurement of some permanent plots at the time of harvesting.

1.7.1 Estimation of Volume using Equations

1.7.1.1 Stand Volume Equations

A volume equation is a mathematical expression (fitted by regression) that allows obtaining stand volume based on the values of other stand variables. Volume is usually determined as a function of stand basal area ($m^2\ ha^{-1}$) and of a stand height, usually dominant height. **Table 6** presents some examples of stand volume equations available in Portugal.

Table 6. Stand volume equations for the main tree species in Portugal.

Region and reference	Expression
Maritime pine	
Leiria Nacional Forest Azevedo Gomes (1952)	$V = 55.745 + 0.3873\ G\ hdom$ V ($m^3\ ha^{-1}$) G ($m^2\ ha^{-1}$) hdom (m)
Tâmega Valley (Moreira e Fonseca, 2002)	$V = 0.2768\ G + 0.4376\ G\ h_{dom}$ V ($m^3\ ha^{-1}$) G ($m^2\ ha^{-1}$) hdom (m) (principal stand before thinning)
Eucalyptus	
Centre Coastal Portugal 1st rotation[1] Tomé et al. (2001)	$V = \left(0.4886 - 0.1348\ \frac{100}{S\ \sqrt{Npl}}\right) t^{0.0655} hdom^{0.8839}\ G^{1.0263}$ V ($m^3\ ha^{-1}$) G ($m^2\ ha^{-1}$) hdom (m)

1 for other regions and rotations see original publication

Consider a stand with a basal area of 24.7 $m^2\ ha^{-1}$ and a dominant height of 22 m.			
Determine stand volume			
Leiria Nacional Forest Azevedo Gomes (1952)	$V = \beta_0 + \beta_1\ G\ hdom$ V ($m^3\ ha^{-1}$) G ($m^2\ ha^{-1}$) hdom (m)	b0 =	55.745
		b1 =	0.3873
G = 24.7	V = **266.20** $m^3\ ha^{-1}$		
hdom = 22			

A forest technician must always use an appropriate stand volume equation. In order to achieve this he can either fit his own stand volume equation in case the stand area and the wood market value justify doing so or he can pick an existing equation from bibliography.

1.7.1.2 Using Tree Volume Equations

There are three methods based on tree volume equations: complete enumeration of diameters and heights; using sample trees and local height-diameter curves and using generic height-diameter curves.

Using complete enumeration of diameters and heights

Once the most appropriate tree volume equation has been selected, tree volumes are estimated, summed up and reported to the hectare.

Property: *Fernão Ferro*
Plot: *74*
Date: *1997*

Tree Number	d (cm)	height (m)	g (m^2 ha^{-1})	volume estimate
16	30.6	*14.2*	0.0735	0.48138
30	27.5	*19.2*	0.0594	0.52460
3	26.4	*14.9*	0.0547	0.37855
17	26.0	*14.5*	0.0531	0.35797
21	25.7	*15.2*	0.0519	0.36635
1	25.4	*13.4*	0.0507	0.31604
2	25.4	*12.9*	0.0507	0.30464
4	25.2	*13.3*	0.0499	0.31030
22	24.2	*13.1*	0.0460	0.28246
5	24.1	*13.1*	0.0456	0.27976
15	24.1	*13.1*	0.0456	0.27976
14	23.5	*12.8*	0.0434	0.26186
31	23.4	*12.9*	0.0430	0.26120
18	23.2	*12.8*	0.0423	0.25603
32	23.2	*14.5*	0.0423	0.28742
19	22.7	*13.4*	0.0405	0.25516
20	22.7	*13.4*	0.0405	0.25516
26	22.4	*13.3*	0.0394	0.24740
8	22.1	*13.4*	0.0384	0.24288
27	21.7	*13.5*	0.0370	0.23646
29	21.6	*18.5*	0.0366	0.31662
6	21.5	*13.5*	0.0363	0.23144
7	21.1	*13.3*	0.0350	0.22157
25	20.3	*13.1*	0.0324	0.20255
9	19.6	*16.9*	0.0302	0.24107
10	18.2	*12.4*	0.0260	0.15707
11	17.1	*11.1*	0.0230	0.12642
13	14.6	*9.4*	0.0167	0.08254
12	14.5	*9.4*	0.0165	0.08157
28	13.9	*9.4*	0.0152	0.07592
24	11.5	*10.5*	0.0104	0.06081
23	11.1	*8.6*	0.0097	0.04938

G_p = 1.2356 m^2 V_p = 8.0323 m^3

G = **24.71** m^2 ha^{-1} V = **160.65** m^3 ha^{-1}

With g calculated using the expression: PI()/40000*d^2

(with d representing DBH)

With v calculated using the expression: 0.01177+0.000035319*d^2*h

(with d representing DBH)

With G_p as the sum of g and G as G_p times the expansion factor that in this case is 20 (10000/500)

With V_p as the sum of v and V as V_p times the expansion factor that in this case is 20 (10000/500)

Using Sample Trees Combined with Height-diameter Curves

If the plot is big enough (1000 m^2 or more) a local height-diameter curve can be fitted to the data plot data. However, if the plot area is small, but the plot is in a homogeneous stand for which several plots have been measured it is possible to fit a local height-diameter curve using the data from all the plots.

Property: *Fernão Ferro*
Plot: *74*
Date: *1997*

Tree Number	d (cm)	height (m)	g (m^2 ha^{-1})	height estimate	volume estimate
16	30.6	*14.20*	0.0735	14.20	0.48138
30	27.5	*19.2*	0.0594	19.20	0.52460
3	26.4	*14.90*	0.0547	14.90	0.37855
17	26.0	*14.50*	0.0531	14.50	0.35797
21	25.7	*15.2*	0.0519	15.20	0.36635
1	25.4		0.0507	13.60	0.32173
2	25.4		0.0507	13.60	0.32173
4	25.2		0.0499	13.56	0.31591
22	24.2		0.0460	13.34	0.28764
5	24.1		0.0456	13.31	0.28489
15	24.1		0.0456	13.31	0.28489
14	23.5		0.0434	13.17	0.26869
31	23.4		0.0430	13.15	0.26604
18	23.2		0.0423	13.10	0.26078
32	23.2	*14.5*	0.0423	14.50	0.28742
19	22.7		0.0405	12.97	0.24788
20	22.7		0.0405	12.97	0.24788
26	22.4		0.0394	12.90	0.24031
8	22.1		0.0384	12.82	0.23287
27	21.7		0.0370	12.71	0.22315
29	21.6	*17.7*	0.0366	17.70	0.30344
6	21.5		0.0363	12.65	0.21838
7	21.1		0.0350	12.54	0.20899
25	20.3		0.0324	12.31	0.19090
9	19.6	*16.1*	0.0302	16.10	0.23022
10	18.2		0.0260	11.62	0.14771
11	17.1		0.0230	11.21	0.12756
13	14.6	*9.40*	0.0167	9.40	0.08254
12	14.5		0.0165	10.08	0.08665
28	13.9		0.0152	9.78	0.07854
24	11.5	*10.5*	0.0104	10.50	0.06081
23	11.1		0.0097	8.14	0.04720

hdom=	G_p =	1.2356	m^2	V_p =	7.9836 m^3
15.60	G =	**24.71**	m^2 ha^{-1}	V =	**159.67** m^3 ha^{-1}

With g calculated using the expression: PI()/40000*d^2

(with d representing DBH)

With h estimated using the expression: EXP(3.2545-0.0895*ln(hdom)-10.1175/d)

With v calculated using the expression: 0.01177+0.000035319*d^2*h

(with d representing DBH)

With G_p as the sum of g and G as G_p times the expansion factor that in this case is 20 (10000/500)

With V_p as the sum of v and V as V_p times the expansion factor that in this case is 20 (10000/500)

On the other hand, if the plot area is small and the plot is the only representative of the stand a generic height-diameter curve can be used (either by fitting an equation to the measured data either by searching the literature). This alternative does not require that all trees are measured; only those necessary to the application of the height-diameter curve such as dominant trees (needed to obtain dominant height).

Afterward, trees' heights are estimated and volume determined as for the complete enumeration.

Using the Diameter Distribution

Volume can be determined using data aggregated in k diameter classes. In this case, the volume calculus is based on the diameter class mid-point (or central diameter - vc_j) and on the average of the heights measured under each diameter class (hm_j) when choosing not to use height-diameter curves.

$$V = \sum_{j=1}^{k} n_j \; vc_j \qquad \text{with} \qquad vc_j = f\,(dc_j, hm_j)$$

Where vc_j is the volume corresponding to the mid-point diameter in class j determined using a volume equation function of dc_j and hm_j.

However, if only the dominant trees were measured, a height-diameter curve needs to be used as shown below.

Property: *Fernão Ferro*
Plot: *74*
Date: *1997*

d Classes	central diameter	fj (n_p)	n	hmj (m)	vcj	Vcj
7.5-12.4	10	2	40	7.37	0.037786	1.511433
12.5-17.4	15	4	80	10.32	0.093784	7.50269
17.5-22.4	20	9	180	12.22	0.184352	33.18343
22.5-27.4	25	15	300	13.52	0.310141	93.04221
27.5-32.4	30	2	40	14.46	0.471404	18.85615

hdom = 15.6   N = 640   V = 154.0959 $m^2\,ha^{-1}$

Withmean h calculated using the expression: EXP(3.2545-0.0895*LN(hdom)-10.1175/"central diameter"); vcj calculated with 0.01177+0.000035319*"central diameter"^2*"mean h" and Vcj as the product of vcj * n

V results from summing up the Vcj values in each diameter class

Some Considerations when using Volume Equations

Any situation that affects the tree or stand level volume estimates, namely defects at diameter at breast height and/or the top of the trees will affect the precision of the estimates depending on how severe and frequent they are. Volume equations provide estimates for healthy normal average trees, therefore the correction of abnormalities must be carried out carefully otherwise it might lead to severe errors either by over or under estimating volume. Moreover, when the inventory objective is other than commercial the correction of abnormalities can be disregarded.

As long as a local height-diameter curve fitted using a dataset of trees representative of the population, or a generic height-diameter curve well adapted to the site is used, it is possible to obtain volume estimates with errors below 10%, or even less (Azevedo Gomes, 1952).

1.8 Leaf Area Index

The Leaf Area Index (LAI) results from the sum of the leaf area of all trees in the stand (adimensional measure). This stand variable is not easy to determine. Three alternative approaches can be used: using sample trees, estimation using allometric equations or through indirect assessment of intercepted light.

The second approach requires the use of an allometric equation to estimate LAI for each tree. The problem in this approach relates to the fact that sometimes the independent variables used in these allometric equations have not been measured for all trees (total height and height to the base of the crown) and requires fitting new allometric equations using the measurements conducted in that stand or simply estimating the independent variables so that the allometric equation can be used (for example, if total tree height is missing, estimate it using a height-diameter curve). Finally, the third approach is based on the close relation between LAI and light intercepted. A device, named ceptometer, can be used to measure intercepted light in all possible situations: above crown, undercover in transition areas and in open areas. However, the details on approaches one and three are not covered by this course.

1.9 Stand biomass

Stand biomass (usually represented by W) is defined as the sum of biomass of all the trees in the stand reported to the hectare. Similarly to tree biomass, stand biomass has several components that need to be assessed separately. It is common to distinguish between aboveground and belowground biomass. Aboveground biomass (Wa) comprises the biomasses of wood (Ww), bark (Wb), branches (Wbr), leaves/needles (Wl), flowers and fruits (Wf). Belowground biomass (Wr) can be differentiated in main root, coarse roots and fine roots. Stand biomass can be determined by the following approaches: using sample trees and estimation either using tree or stand level biomass equations.

1.9.1 Using Sample Trees

The method is similar to what has been described for volume and can also present the same troubles described for LAI concerning the availability of some of the independent variables. Given the difficulties of correctly assessing root biomass, this variable is not determined for all sample trees, even for aboveground biomass it is common to adjust the number of sample trees to minimize the number of trees selected for destructive sampling.

1.9.2 Using Tree and stand Biomass Equations

When estimating biomass using tree level equations it is possible that some of the independent variables such as the height of the base of the crown are not available. This problem will have to be solved as described for LAI. On the other hand, if stand level biomass equations are available, as it happens for eucalyptus (**Table 7**) biomass estimates per hectare can be directly obtained.

Table 7. Stand level biomass equations for eucalyptus in GLOBULUS 3.0 model (Tomé et al., 2007)

Eucalyptus					
Wi	Expression				
Ww	$Ww = a_W\, G^{b_W}\, hdom^{c_W}$ $a_W = 0.0967$ $b_W = 1.0547 - 0.0018 \times rot - 0.0065 \times \left(\frac{N}{1000}\right) - 0.5198 \times \left(\frac{S}{1000}\right) - 1.2105 \times \left(\frac{t}{1000}\right)$ $c_W = 1.1886$				
Wb	$Wb = a_b\, G^{b_b}\, hdom^{c_b}$ $a_b = 0.03636$ $b_b = 1.1691 - 0.0083 \times rot - 0.0459 \times \left(\frac{N}{1000}\right) + 3.2289 \times \left(\frac{S}{1000}\right) + 2.0880 \times \left(\frac{t}{1000}\right)$ $c_b = 0.6710$				
Wl	$Wl = a_l\, G^{b_l}\, hdom^{c_l}$ $a_l = 1.0440$ $b_l = 1.0971 - 0.0112 \times \left(\frac{N}{1000}\right) - 1.2207 \times \left(\frac{S}{1000}\right) - 6.2807 \times \left(\frac{t}{1000}\right)$ $c_l = -0.3129$				
Wbr	$Wbr = a_{br}\, G^{b_{br}}\, hdom^{c_{br}}$ $a_{br} = 0.3972$ $b_{br} = 1.0005 - 0.0192 \times \left(\frac{N}{1000}\right) + 3.3170 \times \left(\frac{S}{1000}\right) - 1.2747 \times \left(\frac{t}{1000}\right)$ $c_{br} = -0.0160$				
Wa	Wa = Ww + Wb + Wl + Wbr	Wr	Wr = 0,2487 Wa	Wt	W= Wa + Wr

Unit: all biomass is in Mg ha^{-1}, G in m^2 ha^{-1}, N in ha^{-1} and t in years.

Tabela 8. Tree level biomass equations for maritime pine

Model (1) $wi = \beta_0\ d^{\beta_1}\ h^{\beta_2}$ (i = s, b) (2) $wi = \beta_0\ d^{\beta_1} \left(\frac{h}{d}\right)^{\beta_2}$ (i = br, l) (3) $wr = \beta_0 wa$					
Component	**Model**	**β_0**	**β_1**	**β_2**	**Source**
Stem (ws)	1	0.0146	1.94687	1.106577	Tomé et al., 2007d
Bark (wb)	1	0.0114	1.8728	0.6694	Tomé et al., 2007d
Branches (wbr)	2	0.00308	2.75761	-0.39381	Tomé et al., 2007d
Leaves (wl)	2	0.09980	1.39252	-0,71962	Tomé et al., 2007d
Aboveground (wa)	wa = ws + wbr + wl				
Roots (wr)	3	0.2756	-	-	Tomé et al., 2007d
d – tree diameter measured at 1.30 m (cm); h – total tree height (m); w_i – biomass component of tree i (kg); wa – tree aboveground biomass (kg)					

1.10 Estimating carbon stocks

Carbon stock estimation has become more important over the years. Usually refers to trees, but can also be estimated for the understory vegetation, litter-fall and soil. Except for the soil carbon stock, all others are assessed based on biomass and multiplied by the carbon content, which is close to 0.5 for all tree components.