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PBRAVO Model

BRIEF DESCRIPTION, EQUATIONS & SOFTWARE

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Background Introduction

Whole-stand models with diameter distribution, like general whole-stand models, not only include dominant height, basal area, and number of trees per hectare as principal variables, but some location variables of the diameter distribution, usually the minimum diameter and a percentile close to the upper limit of the distribution. All other stand variables - volumes and biomass - are estimated based on the simulation of the diameter distribution ie of the average tree in each diameter class.

The simulation of diameter distributions is almost always based on a ***probability density function*** (*pdf*) which, once the parameter values have been estimated, allows calculating the proportion of trees in each diameter class and, subsequently, by multiplying by the number of trees per hectare, the diameter distribution (ie. the number of trees per hectare in each class).

The PBRAVO model (Páscoa, 1987, 1990), the first growth and yield model built for Portugal, was developed using data collected from continuous forest inventories of the Leiria National Forest (MNL). These public forest, located in the central coastal region of Portugal, covers an area of 11,023 ha, has predominantly sandy soils mostly occupied by *Pinus pinaster* Ait.. The management objective in MNL is producing high-quality timber. The final harvest of the stands is traditionally scheduled for when the trees reach 80 years of age. Several thinnings are carried out during the rotation; the first thinning occurs around 20 years of age, with subsequent thinnings approximately every 5 years. Resin is harvested as a byproduct during the last 3 years of the stand's life (prior to final harvest).

MNL is divided into well-defined forest management units of approximately 35 ha; when homogeneity within the stands is not achieved, they are subdivided into smaller units—plots—with varying numbers and areas. MNL consists of 324 plots, most of which coincide with the stands.

Model Description

PBRAVO model is organized in several modules: initialization, growth, calculus and management.

PBRAVO initialization module

The PBRAVO model's initialization module is used when the objective is simulating a young stand that has not been measured (Table1). However, the user must provide information on the site index (S).

Table 1. Initialization equations for non-thinned young stands (in MNL the 1st thinning occurs between 10 and 15 years)

Variable	Inicialization equations
hdom (m)	$hdom = S\ 10^{0.380999-2.694076\ t^{-1/2}}$ (Site index curves)
dmin *(cm)*	$dmin = 12.302998\ hdom^{1.617991} N^{-0.687786}$
P90	$P90 = 2.754927\ hdom^{1.445365} N^{-0.161445}$
G *(m^2 ha^{-1})*	$G = 0.000984\ hdom^{2.103402} N^{0.797707} e^{-8.702327/t}$
N	$N = 1324.485\ hdom^{-0.382656} G^{0.231516} e^{5.407995/t}$

The PBRAVO growth module

The growth module integrates the principal variables including those required to simulate the diameter distribution. These are: dominant height (hdom); the number of trees per hectare (N); the minimum diameter (dmin); the basal area (G) and the 90th percentile (P90). The prediction equations for these variables are shown in Table 2.

Table 2. Growth functions in the growth module of PBRAVO model

Variable	Projection equations
hdom (m)	$hdom = S\ 10^{0.380999-2.694076\ t^{-1/2}}$ (Site index curves)
N	$N = Np^{(1-0.00400488\ t)}$ (*Young stands never thinned*) $N_{t2} = N_{t1}$ *(Stands previously thinned)*
G (*m*^{2} *ha*$^{-1}$)	$G_{t2} = G_{t1}^{t1/t2} e^{(1-t1/t2)(4.178774+0.039053\ hdom)}$
dmin (*cm*)	$dmin_{t2} = 44.99529 \left(\frac{G_{t1}^{t1/t2}}{N_{t1}}\right)^{0.5} e^{f(t1,t2,hdom)}$ $f(t1,t2,hdom) = 0.062039(t2-t1) + \left(1-\frac{t1}{t2}\right)(-6.721703+0.039053\ hdom)$
P90	$P90_{t2} = 154.155154 \left(\frac{G_{t1}^{t1/t2}}{N_{t1}}\right)^{0.5} e^{f(t1,t2,hdom)}$ $f(t1,t2,hdom) = -0.009172(t2-t1) + \left(1-\frac{t1}{t2}\right)(2.919335-0.034485\ hdom)$

The PBRAVO calculus module

PBRAVO calculus module includes the Weibull probability distribution function (eq 1):

$$F(x) = 1 - e^{-\left(\frac{x-a}{b}\right)^{c}} \qquad \text{(Eq. 1)}$$

Where x corresponds to the upper limit of the diameter class, and a, b and c are the Weibull parameters estimated with the following equations:

$$a = 0.9\ d_{min} \qquad b = \frac{(x_p - a)}{\left[(-\ln(1-P_{95}))^{1/c}\right]} = \frac{(P_{95} - a)}{\left[2.99573^{1/c}\right]}$$

Where d_{min} represents the minimum diameter of the stand/plot and P_{95} the percentile 95. After *a* and *b* are estimated, the *c* parameter is estimated by resolution using numerical methods following the non-linear equation.

$$a^2 + 2a\left(x_p - a\right)\frac{\Gamma\left(1+\frac{1}{c}\right)}{-\ln\left(1-p\right)^{1/c}} + \left(x_p - a\right)^2 \frac{\Gamma\left(1+\frac{2}{c}\right)}{-\ln\left(1-p\right)^{2/c}} - \mathrm{E}\left(x^2\right) = 0$$

The Weibull function has the inconvenience of having one of its parameters difficult to estimate (*c*). Thus, for practical applications the parameter is either provided or the user is recommended to use the PBRAVO software developed by (Pascoa xxxx). The model was programmed in *fortran* and a user friendly interface was developed. Before moving to a description of the interface let us look at an example of how these type of models work. The list of equations in this module can be found in Table 3.

Table 3. Equations in PBRAVO calculus module (these are to be used to obtain estimates for the average tree in each diameter class)

Variable	Equations for the calculus of tree variables
h (m)	$h = 1.891036\ hdom^{0.890695} G^{-0.146749} N^{0.075548} e^{2.000723/t - 11.96184/d}$
di *(cm)*	$di = d\left(a - 2a\frac{hi}{h} + c\frac{hi^2}{h^2}\right)^{1/2}$
v *(m³)*	$v = \frac{\pi\ d^2 h}{40000}\ 0.336\ e^{0.94/h + 3.79/d}$
$v_{(h2-h1)}$ *(m³)*	$v_{(h2-h1)} = \frac{\pi\ d^2 h}{40000}\left[a\left(\frac{h2}{h} - \frac{h1}{h}\right) - a\left(\frac{h2^2}{h^2} - \frac{h1^2}{h^2}\right) + \frac{a}{3}\left(\frac{h2^3}{h^3} - \frac{h1^3}{h^3}\right)\right]$ $a = 1.0080\ e^{0.94/h + 3.79/d}$

where *v* is tree volume and $v_{(h2-h1)}$ is tree volume between heights h2 and h1

Knowing the values of the 3 Weibull parameters and the stand density (N), the accumulated probabilities of trees that occur under each dbh class can be estimated. Suppose **a=2.6**, **b=9.0**, **c=3.2** and N=878.

classe d	dcentral (cm)	dsup (cm)	Weibull P(d<=dsup)
[2.5, 7.5]	5	7.5	0.137
]7.5, 12.5]	10	12.5	0.751
]12.5, 17.5]	15	17.5	0.994
]17.5, 22.5]	20	22.5	1.000
]22.5, 27.5]	25	27.5	1.000
]27.5, 32.5]	30	32.5	1.000
]32.5, 37.5]	35	37.5	1.000

$$F(12.5) = 1 - e^{-\left(\frac{12.5-2.6}{9.0}\right)^{3.2}}$$

Weibull function to estimates the accumulated frequency of trees per dbh class

Nacum is obtained by multiplying the accumulated probabilities by N=878. The number of trees in each dbh class is obtained subtracting the Nacum between consecutive dbh classes and the corresponding G by dbh class is obtained as shown.

dcentral (cm)	Weibull P(d<=dsup)	Nacum (ha^{-1})	N (ha^{-1})	G (m^2ha^{-1})
5	0.137	120	120	0.2
10	0.751	659	539	4.2
15	0.994	873	214	3.8
20	1.000	878	5	0.2
25	1.000	878	0	0.0
30	1.000	878	0	0.0
35	1.000	878	0	0.0
		Stand	878	8.4

$$4.2 = \left(\frac{\pi}{4}\,(10/100)^2\right) 539$$

The next step is predicting the height and volume of the average tree of each dbh class, volume per dbh class is obtained by multiplying the average tree volume by the number of trees.

dcentral (cm)	N (ha^{-1})	h (m)	v (árvore)	V (m^3ha^{-1})
5.0	120	1.4	0.003848	0.5
10.0	539	4.6	0.021739	11.7
15.0	214	6.8	0.060050	12.8
20.0	5	8.4	0.119334	0.6
25.0	0	0.0	0.000000	0.0
30.0	0	0.0	0.000000	0.0
35.0	0	0.0	0.000000	0.0
	878			25.6

$$11.7 = (0.021739)\ 539$$

Stand volume is estimated by summing up these values

PBRAVO management module

PBRAVO thinning module allows two different types of thinning: mechanical thinning (young stands) and selective thinning (understory thinning).

The selective thinning is a thinning from below, meaning that trees are removed starting with the smallest diameter classes. The residual basal area is used as the criterion for determining the thinning weight. To simulate selective thinning, one must first update principal variables immediately after thinning, then simulate the diameter distribution after thinning, and subsequently the remaining variables in the stand table. Tables 4 and 5 present the equations used to update the principal variables after thinning.

Table 4. PBRAVO equations used to estimate principal variables after the 1st thinning

Variable	Equations to estimate residual variables (after thinning)
$hdom_r$ (m)	Dominant height is not affected when thinning from below
$dmin_r$ (cm)	$dmin_r$ =0.438034+1.047001 dmin
$P90_r$	$P90_r = 0.121887 + 1.000760\ P90$
G_r ($m^2\ ha^{-1}$)	Residual basal area is a user-defined input
N_r	$N_r = N\left(1-\left(1-\frac{G_r}{G}\right)^{0.896268}\right)^{0.882856}$

Table 5. PBRAVO equations used to estimate principal variables after the 2nd thinning

Variable	Equations to estimate residual variables (after thinning)
$hdom_r$ (m)	Dominant height is not affected by low thinning
$dmin_r$ (cm)	$dmin_r$ =0.684456+1.07904 dmin
$P90_r$	$P90_r = 1.22143 + 0.981559\ P90$
G_r ($m^2\ ha^{-1}$)	Residual basal area is given by the user
N_r	$N_r = N\left(1-\left(1-\frac{G_r}{G}\right)^{0.715054}\right)^{0.820574}$

According to the author (Páscoa, 1987), it is worth noting that the data used to develop the PBRAVO model include thinning operations carried out within the management approach followed in Leiria National Forest between 1971 and 1986 was characterized by spaced thinning over time with high weights with residual basal areas ranging between 15 and 25 m² ha⁻¹. For this reason, the simulations carried out with the PBRAVO model should not deviate significantly from these patterns to avoid anomalous values, or even failing to achieve convergence in the estimation of the Weibull function parameters.
Mechanical thinning is not supposed to alter the diameter distribution; therefore, the parameters of the Weibull distribution remain unchanged—only the number of trees changes. The criterion for defining the thinning intensity is the percentage of trees to be removed, so it is quite simple to calculate the number of remaining trees and, from there, the new diameter distribution and, consequently, the remaining stand variables.

Using the PBRAVO model

Copy the folder PBRAVO from the memory stick, go to PBRAVO\PBRAVO-FPFP, then click on the setup (NOT on the SETUP1). After installing the setup, click on the PBRAVO application (**Figure 1**).

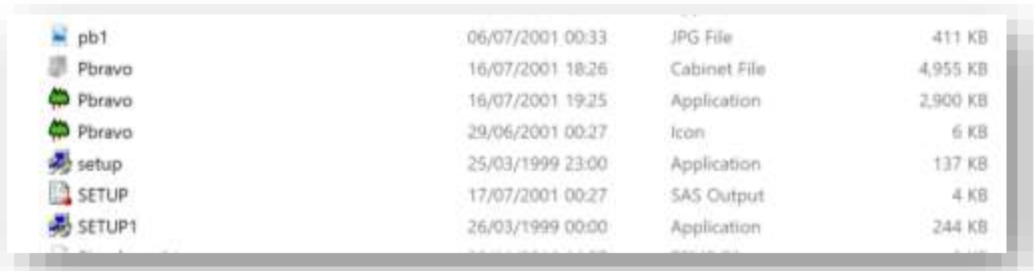

Figure 1. Location of the PBRAVO application within the folder.

Once installed, the PBRAVO model shows a very simple interface (**Figure 2**) with 4 buttons: 1) the Model Options (Opções do Modelo); 2) the Run (simulação); 3) The Help; and 4) the Exit (Sair).

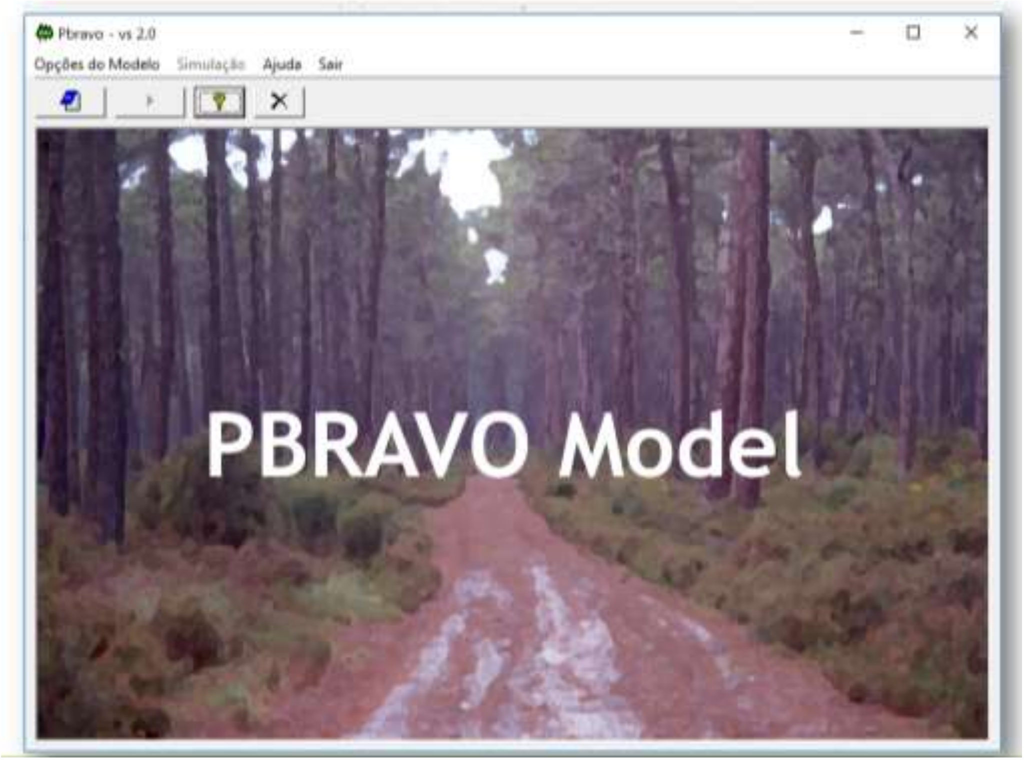

Figure 2. PBRAVO interface.

By Clicking on the Model Options button, a new window shows (**Figure 3**). Here the user is asked to provide some details about the simulation, such as:

- the current year, which should be consistent with the stand's age;
- whether to simulate underbark or overbark volume (since the bark of maritime pine trees represents 20-30% of stem volume);
- some details about the assortment dimensions (**Figure 4**).

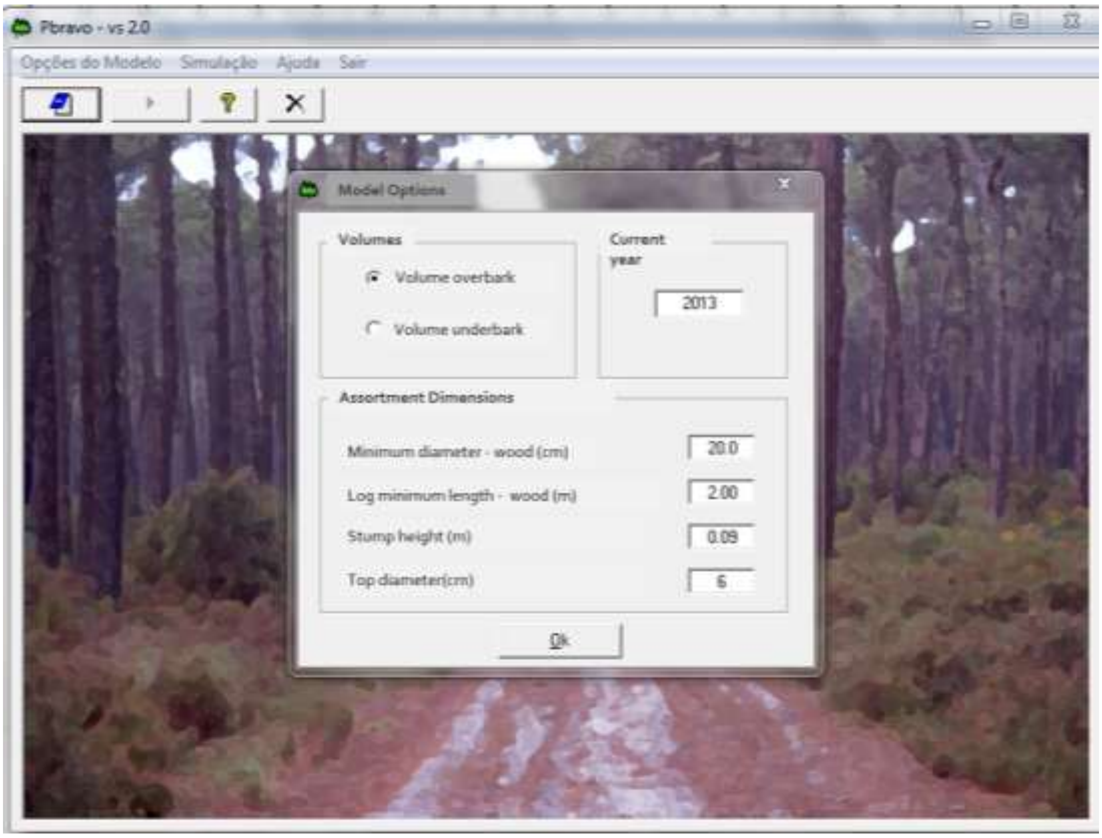

Figure 3. PBRAVO model options window 1.

For the example on the right, the inputs should be:

- Minimum diameter of wood (cm) = 20 cm
- Log minimum length (m) = 2 m
- Stump height (m) = 9 cm
- Top diameter (cm) = 6 cm

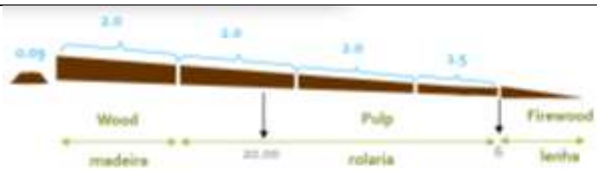

Figure 4. Example of the assortment dimensions requested by PBRAVO model.

Moving to the next step, the user will be asked to decide whether he wants to run the model for one of the 3 situations (**Figure 5**):

- stands that have the dominant height (hdom) measured, and then fill the hdom value;
- stands with no information on dominant height, and in this case the site index class is required;
- unthinned stands, ie young stands. Please note that, projections are not so good for old stands that have never been thinned.

Figure 5. Example of a 17 year old stand with dominant height measured.

For young stands, the user has to provide either the number of standing trees (ha^{-1}) or the number of trees planted (ha^{-1}). In the latest case, a mortality model is applied to express the death of trees due to competition in early stages of stand development. The model runs in 5-year time-steps, stopping at each step allowing the user to (re-)define the management for the next 5-years period. For older stands (already thinned), the user has to provide either the number of trees by diameter class or the required stand variables so that the Weibull parameters (a, b, c) can be obtained (**Figure 6**). Before filling in the information the user has to remember to check which of the 2 options he prefers.

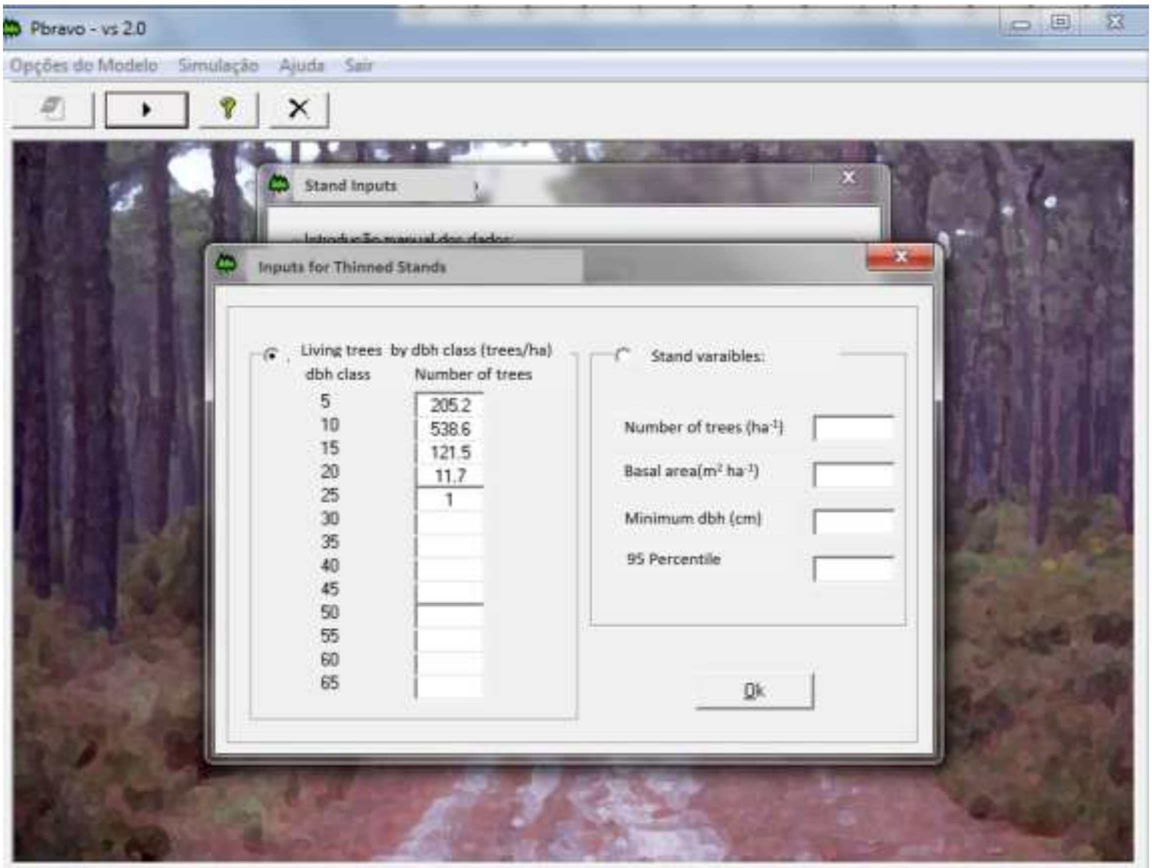

Figure 6. Example of a simulation run for which it was chosen to provide the number of trees by diameter class instead of the stand variables.

Making a bridge to what has been previously explained; the dbh class value represents the midpoint of the diameter class. In case the stand has trees with dbh values greater than 67.5 cm these should be grouped under the 65 class. Likewise, dbh class 5 includes not only the trees with dbh [2.5, 7.5[but also those with dbh < 2.5 cm.

After the stand has been described, by clicking the OK button a new window showing a summary table with the estimated diameter distributions is displayed (**Figure 6**). Among the information provided, the user will see the values of the Weibull parameters and the volume by assortments for each dbh class. To move forward the user will have to choose whether to carry out a thinning or not. If he chooses not to thin, then growth is projected by dimeter class for a 5 years period and an updated table is displayed (**Figure 7**).

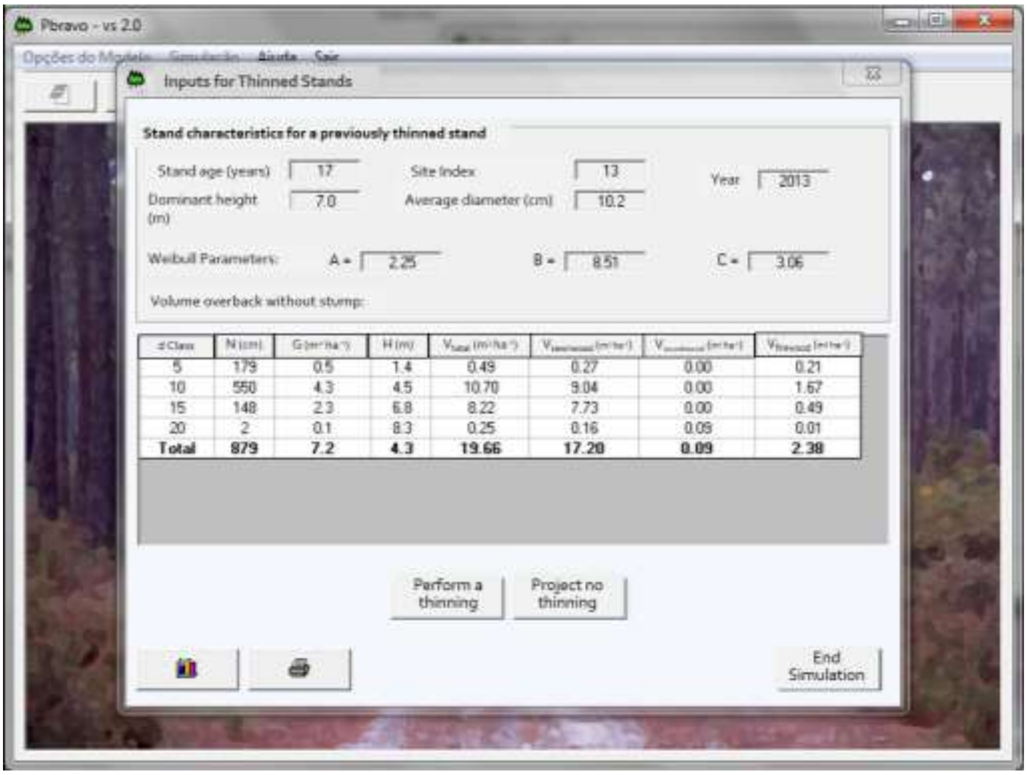

Figure 6. Example of a summary table characterizing the stand at the beginning of simulation (following the examples in previous figures).

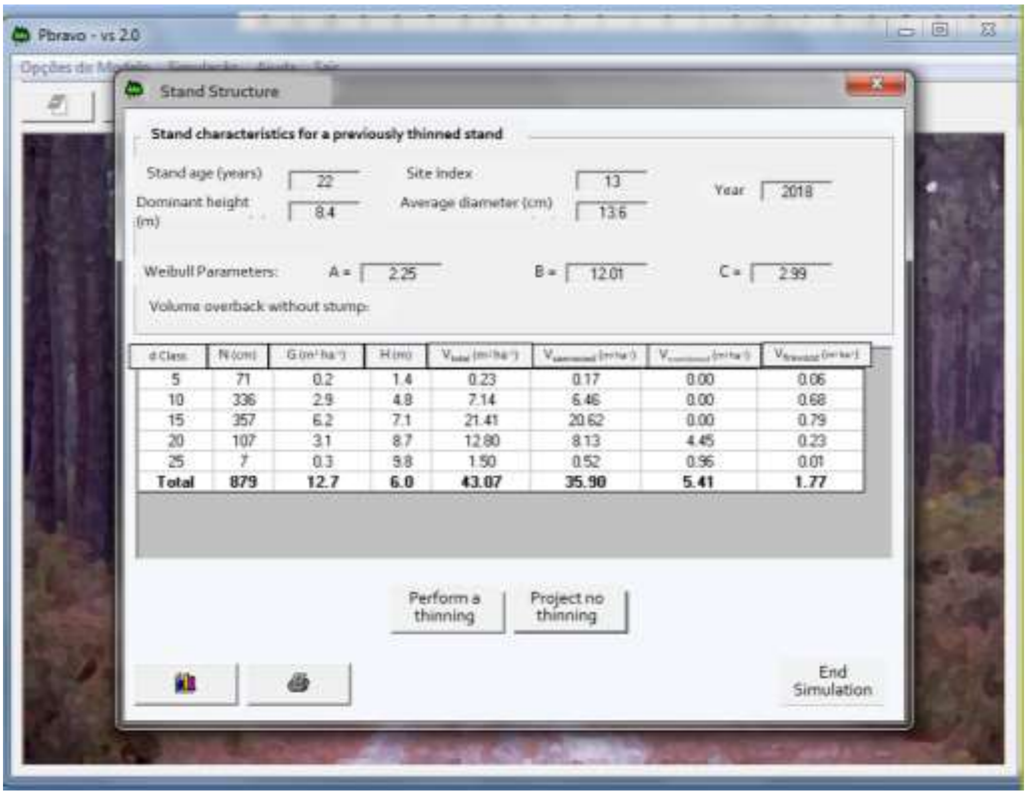

Figure 7. Example of the summary table characterizing the stand after the first 5-years simulation run.

On the other hand, if the user chooses to thin, he has to click on "Project thinning" so that a new window requesting the thinning details is displayed (**Figure 8**). In this window the user has to choose whether to thin by removing a percentage of trees or defining the residual basal area (the standing basal area).

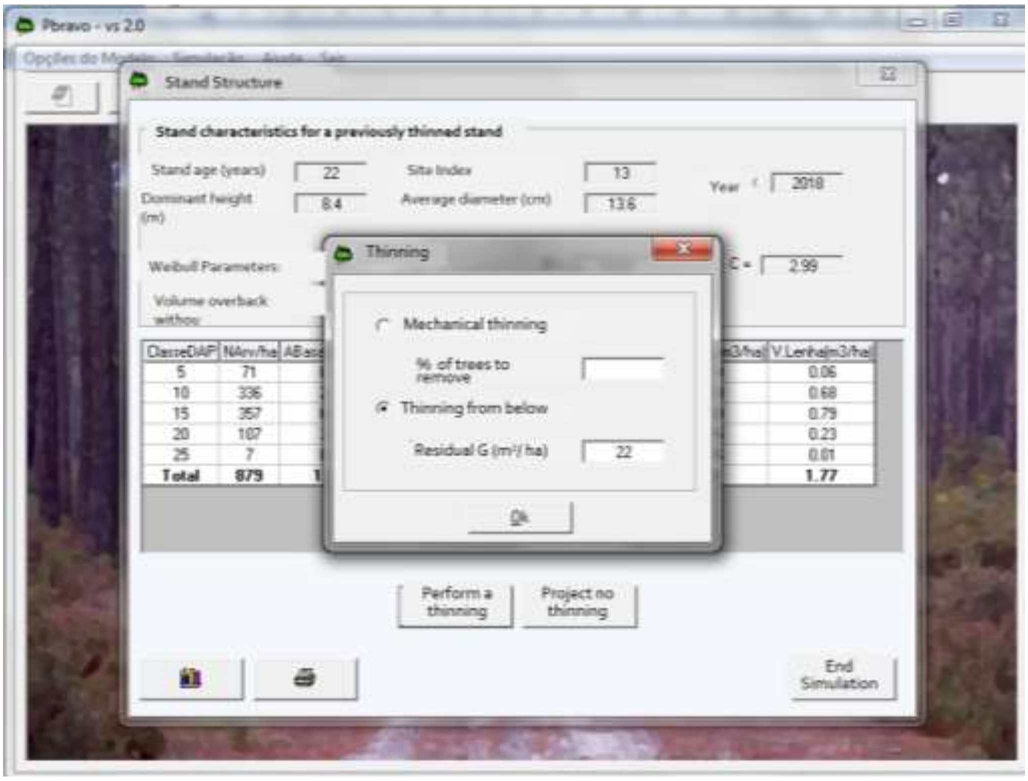

Figure 8. Example of the thinning details window assuming a thinning from below leaving a basal area of 22 m^2ha^{-1}.

If the user decides to perform a thinning leaving a residual basal area greater than the stand basal area, a message is displayed (**Figure 9**). Please note that the stand basal area was 12.7 m^2ha^{-1} (**Figure 7**) and when the thinning was defined a residual basal area of 22 m^2ha^{-1} (**Figure 8**). In such situation, the user is left with the option to project growth without thinning until the stand basal area is greater than the predefined value or decreasing the residual basal area. However the user should be aware that the PBRAVO model was programmed with a restriction that only allows thinning to take place if the amount of basal area removed is above a certain threshold to avoid that the thinned wood would pay off the for the thinning costs. For this reason, even after reaching a basal area of 23.6 m^2ha^{-1}, a warning message is still displayed for a residual basal area of 22 m^2ha^{-1} (**Figure 10**). Whenever a thinning takes place two tables are displayed characterizing the stand before and after thinning. At any point of the simulation the user can click on the graphs button on the lower left part of the window and a smaller window opens displaying a summary of stand characteristics and a graphic comparing the situation before and after thinning (**Figure 11**). Alternatively, the user can choose to print the results.

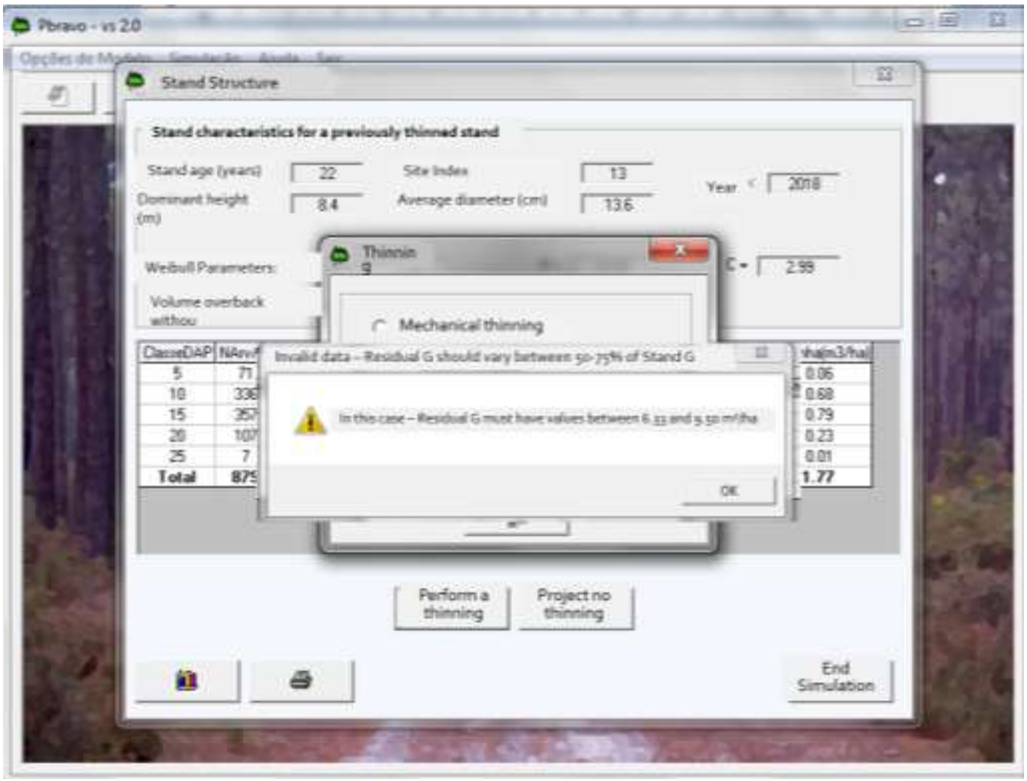

Figure 9. Warning message stating the interval of residual basal area values that would allow a thinning to take place for this stand.

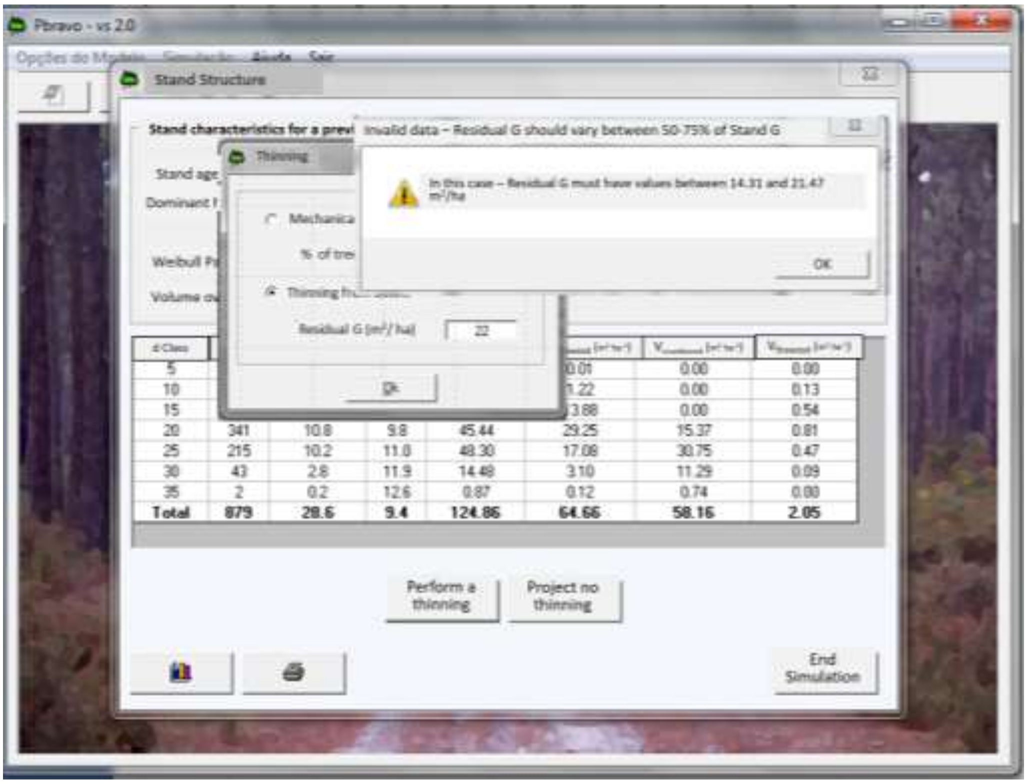

Figure 10. Warning message stating the interval of residual basal area values that would allow a thinning to take for this stand even when the stand has a basal area greater than the residual basal area.

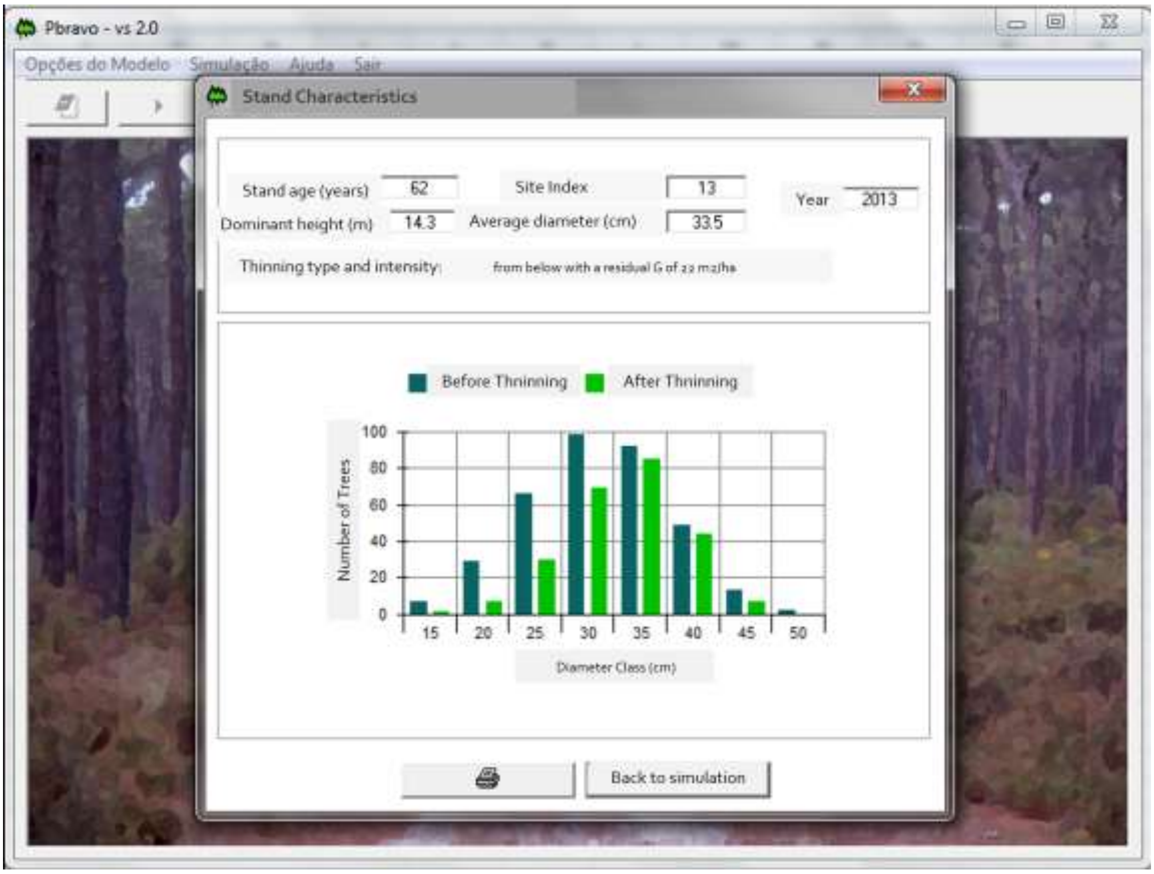

Figure 11. Graphical display of stand characteristics comparing the before and after thinning situations.

When the simulation is terminated, the user can click on the “End Simulation” button on the lower right and the last window is displayed. At this stage, the user can choose to visualize results in graphical or tabular form, save the simulation results in a text file, print the simulation outputs or carry put another simulation (**Figure 12**).

Figure 12.Interface giving the user the option to run a new simulation, visualising, printing or saving results.

References

Páscoa, F., 1987. Estrutura, crescimento e produção em povoamentos de pinheiro bravo. Um modelo de simulação. Tese de Doutoramento, Instituto Superior de Agronomia, Lisboa, Portugal.

Páscoa, F., 1990. Using forest inventory data to build growth and yield stand models. In: L. C. Wensel and G. S. Biging (eds), Forest Simulation Systems, Proc. of the IUFRO Conf., Bull. 1927, Division of Agricultural and Natural Resources, University of California, pp. 279-286.