Statistics and Experimental Design 2026-27

4 Experimental Design and Fixed-Effects Analysis of Variance Exercises

1. The aim is to study the effect of 4 calcium concentrations in the soil (C1, C2, C3 and C4) on root growth in cuttings of the Galega olive variety. For this purpose, 20 pots are available, each containing one plant. Assuming homogeneous external conditions, the technician responsible for the study will randomly assign each of the 4 treatments to 5 pots.

   (a) Identify the experimental units in the trial.

   (b) State the number of replicates for each treatment.

   (c) Identify the type of experimental design used in the trial.

   (d) Using the *agricolae* package, generate a possible randomization for this trial in R.

   (e) If the researcher concludes that it would be better to conduct the trial in a greenhouse, where there is a temperature gradient along the bench (lower temperatures near the entrance and higher temperatures further inside), how could this source of variation be controlled? Indicate a plausible type of experimental design, generate a possible randomization in R using the *agricolae* package, and present a possible layout for this experiment.

2. A study was conducted to investigate the effects of four types of fungicides (F1, F2, F3, F4) on potato yield in certain field plots. A control treatment without fungicide (C) was also included to provide a baseline for comparison. The field selected for the trial is heterogeneous, but it was possible to identify six different homogeneous plots of land. Which experimental design should be adopted? Generate a possible randomization in R using the *agricolae* package and present a possible layout for this experiment.

3. An experiment was conducted to investigate the effects of soil type (clay, G, and sand, R) and water availability (low, B, and high, A) on lupin growth. Four treatments were therefore applied, designated GB, GA, RB and RA, representing different combinations of soil type (G and R) and amount of water supplied (B and A). The experiment was conducted using 16 pots in a greenhouse, each pot containing 2 plants. The treatments were randomly assigned to the pots so that each treatment had the same number of replicates.

   (a) Identify the experimental units in the trial.

   (b) Does this experiment involve pseudoreplication?

   (c) What type of experimental design was used in this experiment?

   (d) How many replicates are there per treatment?

   (e) Generate a possible randomization in R using the *agricolae* package and present a layout for this experiment.

   (f) After a closer examination of the conditions in the greenhouse, it was found that there is a temperature gradient along the width of the bench and different light levels along its length. The rows and columns of pots should therefore be considered as control factors for these sources of variation.

      i. What type of experimental design does this experiment now involve?

      ii. Generate a possible randomization in R using the *agricolae* package and present a layout for this experiment.

4. An orchard has 5 consecutive rows, each containing 50 trees of one apple variety (therefore, 5 different varieties are being studied). The aim is to study the effect of variety on fruit production.
   (a) Do you consider that the conditions are suitable for achieving the objective of the study?
   (b) In your opinion, how should an experiment be designed to address the stated problem?

5. A field trial is to be established to study the competitive effects of four different weed species (A, B, C, D) on maize under two levels of water stress (presence, I, or absence of irrigation, NI). The water-stress treatments can only be applied to large areas of land, whereas the different weed species can be applied to small plots. The field where the trial will be established has 4 large patches of soil associated with different fertility levels. In your opinion, what type of experimental design should be adopted? Generate a possible randomization in R using the *agricolae* package and present a layout for this experiment.

6. The aim is to compare the rooting success of cuttings from 4 eucalyptus clones (E1, E2, E3, E4). There are 80 cuttings available from each clone. Technician A proposes placing all 80 cuttings from each clone in a single tray, whereas Technician B proposes using 8 trays per clone, each containing 10 cuttings.
   (a) Which technician proposes the correct procedure? Justify your answer.
   (b) Assuming the correct procedure is followed, the trays will be placed in a controlled-environment room. There are several shelves in the room, so light availability varies from shelf to shelf.
      i. Does this experiment involve pseudoreplication?
      ii. Identify the experimental units in the trial.
      iii. How many replicates does the experiment have?
      iv. Given the conditions described for carrying out the study, what experimental design would you consider appropriate for comparing the effect of the clone on rooting without bias? Generate a possible randomization in R using the *agricolae* package and present a layout for this experiment.

7. A study aims to compare the total acidity of must from 3 grapevine varieties (Touriga Nacional, V1; Touriga FÃªmea, V2; Touriga Franca, V3) grown on a farm in the Douro region and arranged in the field according to an experimental design suitable for this comparison. For each variety, 6 samples of grapes (6 bags containing 100 grapes each) are taken to the winery laboratory. It is known that the accuracy of the total acidity measurement is affected by the operator. However, in order to obtain the results more quickly, the analyses will be carried out by 6 different technicians (A, B, C, D, E, F). Which experimental design would you consider appropriate for use in the winery laboratory? Generate a possible randomization in R using the *agricolae* package and present a layout for this experiment.

8. The aim is to compare the organoleptic properties of apple juice from 8 commercial brands (S1, S2, S3, S4, S5, S6, S7, S8). A tasting session will be organized with 10 tasters (P1, P2, P3, P4, P5, P6, P7, P8, P9, P10). Given the subjectivity of each taster's responses, which experimental design would you consider appropriate to ensure that the tasting results are not biased? Generate a possible randomization in R using the *agricolae* package.

9. In the breeding of traditional tomato varieties, an important characteristic is skin firmness. This characteristic was assessed in 6 tomato varieties in a trial involving 18 tomato plots. Specifically, each variety was randomly assigned to 3 tomato plots. Each individual observation consists of the mean fruit firmness for each plot (measured using a texturometer, in gram-force, *gf*). The values obtained for each plot are shown below (and are available in the file named `tomate.txt` on the course website), together with the means and variances for each variety and for all observations:

Variety	Observations			Mean	Variance
18	632.04	629.30	420.59	560.6433	14 713.08
28	253.00	219.34	252.11	241.4833	367.9434
29	223.71	374.48	274.66	290.9500	5881.921
40C	503.51	757.44	856.39	705.7800	33 132.64
Ace	375.18	376.81	379.77	377.2533	5.414433
Roma	333.05	324.82	338.45	332.1067	47.11163

- The overall mean of the observations is $\overline{y}_{..} = 418.0361$;
- the sample variance of all the observations is $s_y^2 = 34\,517.82$.

(a) Identify the type of experimental design involved. Specify the corresponding model and all additional assumptions required for the analysis of the problem under study.

(b) Construct the summary table for the corresponding analysis of variance,

i. using only a calculator and the information available in this statement;

ii. using the following R commands:
`tomate.aov <- aov(res.pel ~ variedade , data=tomate)`
`summary(tomate.aov)`.

(c) Formulate and perform the appropriate F-test for the problem described above, using a significance level of 5%. Can it be concluded that the mean skin firmness is not the same for all varieties?

(d) What is the largest significance level α for which your answer to the previous question would change? What is this value called?

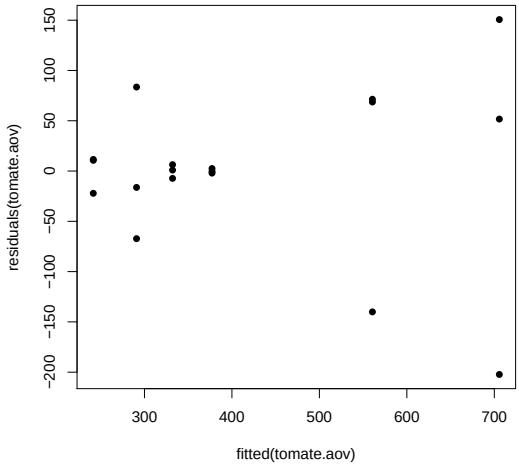
(e) Use the `model.matrix` command in R to inspect the structure of the model matrix, $\mathbf{X}$, in this context.

(f) Use the `fitted` command in R to identify the fitted values of the response variable in this Analysis of Variance.

(g) Calculate the parameter estimates of the model and interpret the values obtained.
Finally, use the commands:
`coef(tomate.aov)` and `lm(res.pel ~ variedade , data=tomate)`.

(h) Which varieties have firmness values that differ significantly from that of variety 40C? Justify your answer using an appropriate test, at the level $\alpha = 0.05$.
Finally, execute the R command `library(agricolae)` and use the command
`HSD.test(tomate.aov, "variedade",console=TRUE)`.

(i) The plot of the (usual) residuals of the observations against the fitted values from the analysis of variance model is presented below. Comment on the plot and its possible implications. Identify the observation whose residual has the largest absolute value.



10. A study involving three coffee varieties, designated `CA`, `CL` and `PR`, was conducted according to a completely randomized design and focused on the lengths of the stomata on their leaves. Twelve plants were selected from each variety, and for each plant the mean length of the stomata on its leaves was measured under controlled environmental conditions (variable `Comprimento`, in μm). Only the means and variances of the 12 observations (plants) for each variety are available:

	CA	CL	PR
Mean	22.85833	19.49333	25.31583
Variance	13.69303	2.725424	9.388936

    (a) Describe in detail the ANOVA model appropriate for studying this problem.

    (b) Construct the ANOVA summary table corresponding to the model specified in the previous question.

    (c) What is the sample variance of stomatal lengths across all 36 observations?

    (d) Can it be concluded that, in the population, the mean stomatal length is the same for all three varieties, at a significance level of $\alpha = 0.05$? Provide a detailed answer.

    (e) Compare all pairs of means using a Tukey test ($\alpha = 0.05$). Comment on the results.

11. It is known that carbon dioxide has a critical effect on the growth of microbial populations; small amounts of CO_2 may stimulate the growth of some species, whereas, conversely, high concentrations generally have an inhibitory effect. This latter effect is used commercially to preserve stored food.

    A study was conducted to investigate the effect of different CO_2 concentrations on the growth rate of *Pseudomonas fragi*. The different levels (treatments) were fixed in advance, and the response variable measured was the percentage change in the mass of the cultures after one hour of growth under the respective conditions. The resulting data are shown in the following table.

CO_2 concentration				
0.0	.083	.29	.50	.86
62.6	50.9	45.5	29.5	24.9
59.6	44.3	41.1	22.8	17.2
64.5	47.5	29.8	19.2	7.8
59.3	49.5	38.3	20.6	10.5
58.6	48.5	40.2	29.2	17.8
64.6	50.4	38.5	24.1	22.1
50.9	35.2	30.2	22.6	22.6
56.2	49.9	27.0	32.7	16.8
52.3	42.6	40.0	24.4	15.9
62.8	41.6	33.9	29.6	8.8

These data are available on the course website in the file named `CO2.txt`. The CO_2 concentrations are repeated in two columns: one as a factor and the other as a numerical variable.

(a) The aim is to test the null hypothesis $H_0 : \mu_1 = \mu_2 = ... = \mu_5$, where μ_i denotes the expected growth rate for the i-th CO_2 concentration. The use of an Analysis of Variance is suggested. State the assumptions required in order to perform this test.

(b) Is there sufficient evidence to reject H_0 at a significance level of $\alpha = .05$?

(c) If you answered yes to the previous question, indicate which pairs of CO_2 concentrations have different effects on the growth of *Pseudomonas fragi*, using Tukey confidence intervals.

(d) Assess the validity of the assumptions of the ANOVA model.

(e) Given the nature of the predictor variable, a linear regression of the growth rates on carbon dioxide concentrations, treated as a numerical variable, could also be considered. Using the column containing the CO_2 concentrations as numerical values (that is, the `CO2.numerico` column), answer the following questions.

i. Construct a scatter plot of mass change against CO2 concentration.

ii. Fit the simple linear regression described above, plotting the regression line over the scatter plot. Comment on the result.

iii. Compare the results of the overall F-test obtained for the model with the factor `CO2` and for the model with the numerical predictor `CO2.numerico`.

12. The aim is to assess whether a green bean processing technique changes the zinc concentration in this food. There is particular interest in assessing the concentration at four stages of processing: at the beginning; immediately before the so-called blanching stage; immediately after blanching; and at the end of processing.

Since it is known that different batches of beans differ in their initial zinc concentrations, it was decided to use nine different batches, measuring their respective concentrations at the four stages of processing indicated above. The measurements obtained are shown in the Table (and in the file available on the course *web* page named `zinco.txt`).

	Processing stages			
Batch	1	2	3	4
1	2.23	3.71	2.53	5.46
2	2.20	4.67	2.87	5.19
3	2.44	3.45	2.83	5.51
4	2.11	2.73	2.33	4.82
5	2.30	2.58	2.19	6.63
6	1.72	1.85	1.80	2.39
7	1.78	1.81	1.75	2.09
8	2.36	2.32	1.83	2.27
9	2.91	2.50	1.97	2.39

(a) Identify the experimental design used and describe the corresponding (fixed-effects) ANOVA model.

(b) Construct the Analysis of Variance table for this study.

(c) Perform the F-test to determine whether the different processing stages affect the mean zinc concentrations ($\alpha = 0.05$). Comment.

(d) Calculate the fitted value for the zinc concentration at the beginning of processing, for the first batch. Compare it with the observed value and comment.

(e) By mistake, an analyst assumed that there was only one factor in the study, corresponding to the different processing stages. Compare the ANOVA summary tables obtained in each case and comment.

13. In 2015, a trial was conducted in Vila Real involving 6 clones of the Aragonez grape variety, also known in the Douro region as Tinta Roriz, as part of a study by ISA and PORVID (Portuguese Association for Grapevine Diversity). The aim was to measure the so-called Potential Fertility Index (variable `if`, defined as the number of grape clusters divided by the number of buds that have burst) and determine whether it differs according to the genotypes studied. Since fertility is also known to depend on soil type, six plots with different soils (`terreno`) were selected for the trial. Each plot was divided into six subplots, with each subplot randomly assigned to a different genotype. The data file is available on the course *web* page (`rzif.txt`). An ANOVA model was fitted, with the following results:

```
            Df  Sum Sq  Mean Sq  F value
clone       ??      ??       ??   4.204
terreno     ??      ??       ??   2.691
Residuals   ??      ??   0.3660
```

(a) State the type of experimental design used, describing in detail the ANOVA model corresponding to the summary table above.

(b) Complete the summary table, indicating how the missing values are obtained.

(c) Next, fit the corresponding analysis of variance model in R.

(d) State, formally justifying your answer, which effects are significant at the $\alpha = 0.01$ level.

(e) An analyst who was only familiar with one-factor ANOVAs decided to study the effects of genotype using the same data. Construct the resulting summary table and state what conclusion would be reached (at the same $\alpha = 0.01$ level). Comment.

(f) Another analyst claims that some genotypes perform particularly well in certain plots of land under study, and suggests that this possibility should be taken into account by the model. Based on the experiment conducted, is it possible to predict this eventuality?

14. The aim is to compare the yield obtained from four wheat varieties (`variedade`). Thirteen plots of land (`terrenos`) with different soil characteristics were identified, corresponding to the types of land on which the crops are to be grown. The 13 plots are then divided into four plots of equal size. In each plot of land, one subplot is randomly assigned to each of the four varieties. After harvesting, the yields obtained (`rend` in t/ha) are recorded in the table (and are available on the course *web* page in the file `terrenos.txt`).

(a) Identify the experimental design adopted and fit the corresponding analysis of variance model in R.

(b) The sample means for each variety suggest that some varieties perform better than others. But are these differences significant? To answer this question, perform an appropriate Analysis of Variance, constructing the corresponding summary table. Comment on your conclusions.

(c) Test whether there are significant differences between plots of land, as might be expected. Comment.

(d) Calculate the fitted yield for variety A in plot I. Compare it with the observed yield and comment.

Plot of land (`terreno`)	Variety (`variedade`) A	B	C	D
I	1.800	2.457	0.722	0.789
II	1.709	1.839	1.546	1.304
III	1.277	1.293	1.515	1.273
IV	1.675	1.745	0.800	0.846
V	1.814	1.833	1.678	1.732
VI	1.896	1.203	1.192	1.580
VII	1.078	1.689	1.583	1.168
VIII	1.740	1.518	1.050	1.305
IX	1.200	1.133	0.778	1.033
X	1.500	0.722	0.636	0.925
XI	1.932	1.700	1.203	0.850
XII	1.169	1.209	1.112	0.986
XIII	1.438	1.577	1.355	1.525
Mean	1.556	1.532	1.167	1.178
Variance	0.0879	0.1855	0.1266	0.0934

15. In a study on the growth characteristics of stone pine, conducted in Sines and Tavira by the National Institute for Agrarian and Veterinary Research (INIAV), the mean height of pine trees from five different provenances (Morocco-`Marrocos`, Greece-`Grecia`, Portugal and two provenances from Italy-`Italia`) was assessed two years after planting. In both Sines and Tavira, six plots of trees from each provenance were planted, thus generating $n = 60$ height values (variable `alt2`, in cm), whose sample variance is $s^2 = 34.49584$ (data available on the course *web* page in the file `pmanso.txt`). Some of the resulting means are shown below.

```
prov
Grecia Italia-1 Italia-2 Marrocos Portugal
 28.81    32.75    30.23    35.13    31.90
```

```
local
Sines Tavira
28.14  35.38
```

```
  prov:local
            local
prov        Sines  Tavira
  Grecia    22.52   35.10
  Italia-1  31.03   34.46
  Italia-2  26.91   33.56
  Marrocos  31.16   39.09
  Portugal  29.09   34.70
```

```
Grand mean
  31.76298
```

(a) Identify the experimental design used and the appropriate ANOVA model. Describe the model in detail.

(b) Knowing that the Residual Mean Square is 16.59 and that the Sum of Squares associated with the five different provenances is 280.61, construct the summary table for the appropriate ANOVA model.

(c) Fit the analysis of variance model described in part a) in R.

(d) Use an F-test to assess whether there are interaction effects between the provenance of the pine trees and the location of the trial. Comment on your conclusions. Briefly indicate which other types of effects should be considered significant. Use $\alpha = 0.05$.

(e) In the sample, the greatest mean height in Sines is lower than the smallest mean height in Tavira. Regardless of your answers to the previous parts, use Tukey's test to determine whether the same statement can be extended to the population. Comment. Finally, run the R commands required to obtain the results of Tukey's test.

16. An experiment aims to study the yield of a wheat variety as a function of different ways of applying two fertilizers, one containing phosphorus (a phosphate fertilizer), and the other containing potassium. Three application rates are considered for the phosphate fertilizer (`fosforo`), designated Low (`Baixa`)), Medium (`Media`)) and High (`Alta`)). Likewise, three application rates are considered for the potassium (`potassio`) fertilizer, also designated (`Baixa`)), Medium (`Media`)) and High (`Alta`)). The experiment is carried out on a field with 27 plots of equal size. Three plots are assigned completely at random to each combination of application rates of the two fertilizers. The results obtained (in t/ha) were as follows:

		potassio (K)									Mean	Variance
		Baixa			Media			Elevada				
fosforo (P)	Baixa	4.6	4.9	4.3	6.3	6.1	6.4	6.6	6.7	6.9	5.8667	0.9775
	Media	5.4	5.6	5.2	6.8	5.7	/ 6.7	7.5	8.0	7.3	6.4667	1.045
	Elevada	5.3	5.7	5.1	7.5	7.0	7.2	7.1	7.4	6.1	6.4889	0.88861
Mean		5.1222			6.6333			7.0667				
Variance		0.20944			0.3200			0.3175				

The observed means for each combination of application rates of the two types of fertilizer were as follows:

		potassio		
		Baixa	Media	Elevada
fosforo	Baixa	4.600	6.267	6.733
	Media	5.400	6.400	7.600
	Elevada	5.367	7.233	6.867

The Summary Table associated with this experiment is as follows:

```
                 Df Sum of Sq  Mean Sq  F Value      Pr(F)
fosforo           ?   2.24296 1.121481    ?     0.00366530
potassio          2  18.75630    ?        ?     0.00000001
fosforo:potassio  ?   1.93926 0.484815  3.36504 0.03187154
Residuals        18   2.59333    ?
```

Answer the following questions using the information provided in the exercise statement.

(a) Complete the Summary Table, indicating how each of the missing values is obtained.

(b) Which *types* of effects in the model associated with this case should be considered significant? Justify your answer by explicitly stating the hypotheses of the tests performed, the test statistics and significance levels used, as well as the nature of the critical regions, the values obtained and the conclusions.

(c) Do you think that the yields justify choosing between the following two options: (i) a High application rate of potassium with a Low application rate of phosphorus; and (ii) a High application rate of potassium with a Medium application rate of phosphorus? Justify your answer.

(d) Now fit a two-factor model that does not include interaction effects. Construct the corresponding summary table. Identify the differences between this table and the one shown above (associated with the model that includes interaction effects). Comment on the differences and identify the consequences of failing to include interaction effects when such effects appear to exist in r

17. An agricultural engineer wishes to select wheat cultivars for the four farms for which she is responsible, located (`localidade`) in Elvas, Evora, Benavila and Revilheira. On each of these farms, 36 plots of land are defined, with four plots randomly assigned to each of nine cultivars: Celta, Helvio, TE9006, TE9007, TE9008, TE9110, TE9115, TE9204 and Trovador. Yield, in kg/ha, was measured in each plot. The variance of all the observed yields is $s^2 = 1\,714\,242$.

(a) Specify the experimental design used and describe in detail the appropriate ANOVA model for this experiment.

(b) An ANOVA model was fitted using the `R` program. Partial results from this fit are given below.

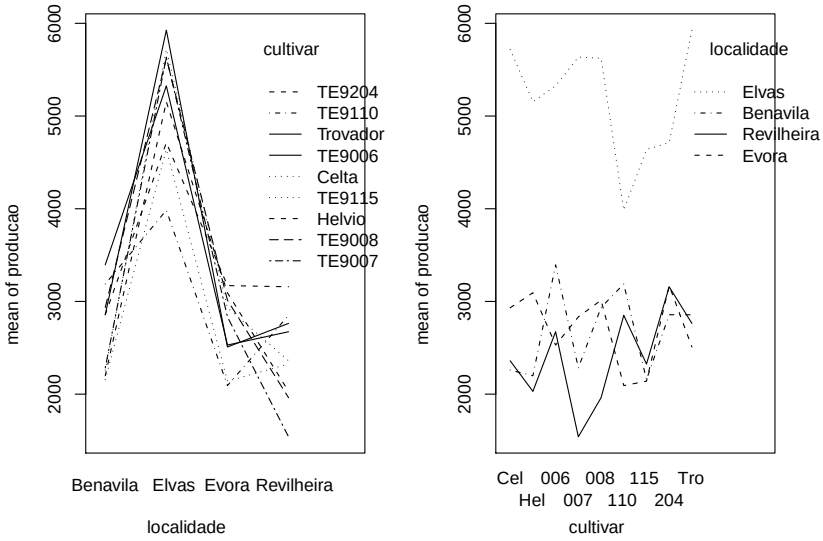
	Df	Sum Sq	Mean Sq	F value
localidade	???	183759916	???	234.9531
cultivar	???	???	964060	???
localidade:cultivar	???	???	???	4.0768
Residuals	???	28156076	260704	

i. Complete the table, indicating how each of the missing values is obtained.

ii. What is the estimated variance of the random errors in the model, and what are its units of measurement?

iii. Formally test (at a significance level of $\alpha = 0.01$) which types of effects in the model should be considered significant. Describe one test in detail and discuss the others briefly.

iv. The interaction plots associated with this experiment are shown below. Comment on them, relating your conclusions from the previous parts to the plots.



v. Discuss the effect of changing the measurement units of the response variable from kg/ha to tonnes per hectare. Which values in the table change, and which remain the same? What effect does the change of units have on the conclusions of the F-tests?

(c) The mean yields of the four plots corresponding to each combination of location and cultivar were as follows.

```
            cultivar
localidade   Celta Helvio TE9006 TE9007 TE9008 TE9110 TE9115 TE9204 Trovador
  Benavila   2260  2198   3395   2283   2935   3188   2157   2857   2857
  Elvas      5719  5154   5326   5636   5624   3984   4640   4713   5927
  Evora      2932  3094   2530   2833   3013   2092   2140   3171   2509
  Revilheira 2360  2030   2673   1540   1962   2850   2325   3158   2763
```

Using Tukey's test, indicate which cultivars in Elvas do not differ significantly from the highest yield observed at that location ($\alpha = 0.10$). Comment.

18. The aim is to analyse changes in the tannin content of sapodilla pulp (the fruit of the sapodilla tree, *Manilkara achras*) caused by storage temperature (high/low) `alta/baixa)` and storage time (0, 3, 6 or 9 days). The study produced the following data:

Temperature	Time							
	0 days		3 days		6 days		9 days	
high	20.8	19.7	26.5	27.5	26.5	26.4	26.5	26.9
(`alta`)	18.0	19.5	27.0	26.4	27.0	24.0	25.9	26.3
low	32.3	34.1	20.8	20.5	16.4	15.7	10.3	9.7
(`baixa`)	30.7	31.8	21.0	20.9	15.9	16.0	7.8	9.8

The mean and variance of the set of 32 observations are 22.14375 and 47.83222, respectively. The means associated with each storage time, each temperature and each combination of time and temperature are:

```
Tables of means
tempo
     0      3      6      9
25.862 23.825 20.987 17.900

temperatura
  alta  baixa
24.681 19.606
```

```
tempo:temperatura
      temperatura
tempo  alta baixa
    0 19.50 32.23
    3 26.85 20.80
    6 25.97 16.00
    9 26.40  9.40
```

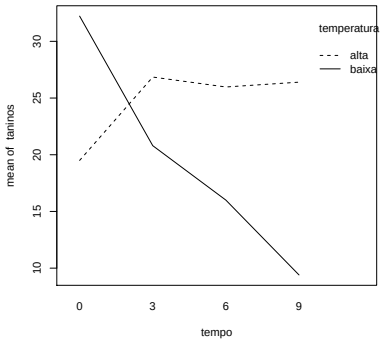


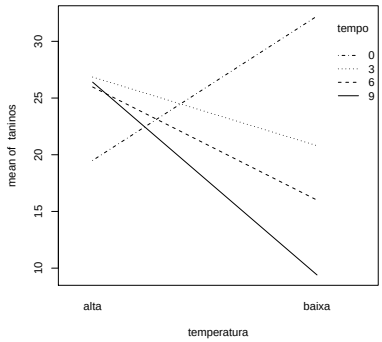
(a) Identify the experimental design used in the study and describe in detail the best ANOVA model associated with it.

(b) Knowing that the Residual Sum of Squares is 20.72 and that the Mean Square associated with the different storage times is 96.01, construct the ANOVA Summary Table associated with this experiment.

(c) Can it be concluded that the different storage times affect the tannin content in the pulp of these fruits? Answer this question using hypothesis tests.

(d) Interpret the following interaction plots and compare the conclusions obtained with the results in the ANOVA Summary Table.





(e) A researcher claims that if the storage temperature is high, storage time does not affect the mean tannin content in the sapodilla pulp. Perform a statistical analysis that allows you to state whether or not you agree with this claim.

19. In a study of a genotype of a grapevine variety (Tinta Francisca), yield (in kg/plant) was studied at two different locations (Tabuaco and Regua), and in different years. The data available for Tabuaco refer to the years 1999 and 2003, and for Regua to the years 1999, 2000 and 2002. At each location and in each year, 8 experimental units (plots) were observed, and the results shown in the table below were obtained. These data are also available in the file `TabRegua.txt`.

Tabuaco		Regua		
1999	2003	1999	2000	2002
1.292	1.188	0.181	0.992	1.594
1.025	0.458	0.917	0.881	2.315
0.958	0.258	0.435	0.835	2.125
0.625	0.892	0.195	0.758	0.835
0.625	0.225	0.142	0.488	1.675
0.125	0.244	0.235	0.925	0.995
0.425	0.444	0.029	0.488	0.679
0.092	0.092	0.019	0.125	0.594

The aim is to determine whether there are location effects and/or year effects on the yield of this genotype.

(a) State the type of experimental design used and an appropriate ANOVA model for this experiment.

(b) Perform the necessary tests to determine whether you consider there to be location effects and/or year effects.

(c) Use Tukey's tests to study pairs of combinations of location and year that have significantly different mean yields.

20. Interest in introducing grape varieties from outside the region into Palmela led to the organisation (in collaboration between ISA and PORVID) of a trial involving two grape varieties: Malvasia Fina (MF) and AntÃ£o Vaz (AN). It was decided to work with four genotypes of each variety (named MF201, MF1035, MF1420 and MF1426 in the case of Malvasia Fina, and AN105, AN142, AN145 and AN510 in the case of AntÃ£o Vaz), with the aim of selecting the variety and genotype combinations associated with the highest yields in the region. A balanced design was used, with eight replicates in each experimental condition. The mean yield across all observations was $4.467625\,kg/plant$, and the corresponding sample variance was $5.389415\,(kg/plant)^2$.

(a) Describe in detail the most appropriate ANOVA model for the experiment now described.

(b) Construct the summary table corresponding to this model, knowing that the Residual Mean Square is 2.873782 and that the Sum of Squares associated with the varieties is 79.73597.

(c) Was it important to account for the possibility that yields differ as a result of genotype effects? Justify your answer using a formal test.

(d) In addition to possible genotype effects, can differences between the two varieties be identified? Formally justify your answer.

(e) Knowing that the mean yields in the eight experimental conditions are given below, test whether it is possible to conclude that the mean yield of genotype MF201 differs from all the others.

AN105	AN142	AN145	AN510	MF1035	MF1420	MF1426	MF201
2.925	2.208	3.593	4.680	3.451	5.367	5.839	7.678

21. Following a trial to select garlic varieties (in collaboration between ISA and INIAV), a farmer from Braga was advised to use a genotype marketed under the designation V7088. Knowing that the study leading to this recommendation had been conducted in Elvas in 2000 and 2004, the farmer decided to repeat the trial on his own land, under conditions similar to those of the initial trial. Over three years (from 2007 to 2009), he randomly selected four plots and used the aforementioned genotype. As in the initial study, the farmer chose to measure the commercially important variable of garlic bulb weight. The mean weights (in g) obtained from the four plots in each of the situations described were as follows.

```
1   Elvas Ano2000 13.9200
2   Elvas Ano2004 29.8200
3   Braga Ano2007 15.4500
4   Braga Ano2008 19.9325
5   Braga Ano2009 12.9425
```

Having requested statistical support, the farmer was presented with the following fitted ANOVA model:

```
> summary(aov(peso.bolbo ~ Local/Ano, data=exame300113))
            Df Sum Sq Mean Sq F value    Pr(>F)
Local        1 159.34 159.345  13.072  0.002543
Local:Ano    3 605.94 201.980  16.570 5.005e-05
Residuals   15 182.84  12.189
```

(a) Indicate the type of experimental design used in the experiment, and describe in detail the ANOVA model that was fitted, explaining the meaning of all the terms in the model equation, as well as the additional assumptions made.

(b) Perform the F-tests that you consider appropriate. Comment on the results and, based on these tests, state whether it is possible to conclude that the mean bulb weights of this genotype differ between Braga and Elvas.

(c) Are there significant differences in the mean annual bulb weights in Braga? Justify your answer using a Tukey test ($\alpha = 0.05$).

(d) Confused by the results, the farmer, who had attended an introductory Statistics course in which only one-way Analysis of Variance had been taught, decided to fit a model of that type, using all the available data but ignoring the years to which the data refer.

i. Construct, with justification, the summary ANOVA table obtained by the farmer.

ii. Briefly describe the results obtained from the farmer's ANOVA ($\alpha = 0.05$). Compare these results with those obtained from the analysis of the previous model and comment on them.