

1) $(k)' = 0$	10) $(\operatorname{tg} f)' = \sec^2 f \cdot f'$ $(\sec f = \frac{1}{\cos f})$
2) $(f^\alpha)' = \alpha \cdot f^{\alpha-1} \cdot f'$, $\alpha \in \mathbb{R}$	11) $(\operatorname{cotg} f)' = -\operatorname{cosec}^2 f \cdot f'$ $(\operatorname{cotg} f = \frac{1}{\operatorname{tg} f}$ e $\operatorname{cosec} f = \frac{1}{\sin f}$)
3) $(e^f)' = e^f \cdot f'$	12) $(\sec f)' = \sec f \cdot \operatorname{tg} f \cdot f'$
4) $(a^f)' = a^f \cdot f' \cdot \ln a$, $a \in \mathbb{R}^+$	13) $(\operatorname{cosec} f)' = -\operatorname{cosec} f \cdot \operatorname{cotg} f \cdot f'$
5) $(\ln f)' = \frac{f'}{f}$	14) $(\arcsin f)' = \frac{f'}{\sqrt{1-f^2}}$
6) $(\log_a f)' = \frac{f'}{f \cdot \ln a}$, $a \in \mathbb{R}^+ \setminus \{1\}$	15) $(\arccos f)' = \frac{-f'}{\sqrt{1-f^2}}$
7) $(f^g)' = g \cdot f^{g-1} \cdot f' + f^g \cdot \ln f \cdot g'$, $f > 0$ $(f^g = e^{g \ln f})$	16) $(\operatorname{arctg} f)' = \frac{f'}{1+f^2}$
8) $(\sin f)' = \cos f \cdot f'$	
9) $(\cos f)' = -\sin f \cdot f'$	

Tabela 1.1: Derivadas de algumas funções elementares.

1) $(f + g)' = f' + g'$
2) $(\alpha f)' = \alpha f'$, $\alpha \in \mathbb{R}$
3) $(fg)' = f'g + fg'$
4) $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
5) $(g \circ f)'(x) = g'[f(x)] f'(x)$

Tabela 1.2: Regras de derivação.

1) $\sin^2 x + \cos^2 x = 1$	3) $1 + \operatorname{cotg}^2 x = \operatorname{cosec}^2 x$
2) $1 + \operatorname{tg}^2 x = \sec^2 x$	5) $\cos(x + y) = \cos x \cos y - \sin x \sin y$
4) $\sin(x + y) = \sin x \cos y + \sin y \cos x$	7) $\cos(x - y) = \cos x \cos y + \sin x \sin y$
6) $\sin(x - y) = \sin x \cos y - \sin y \cos x$	9) $\cos 2x = \cos^2 x - \sin^2 x$
8) $\sin 2x = 2 \sin x \cos x$	11) $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
10) $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$	13) $\cos x = \frac{1 - \operatorname{tg}^2 x/2}{1 + \operatorname{tg}^2 x/2}$
12) $\sin x = \frac{2 \operatorname{tg} x/2}{1 + \operatorname{tg}^2 x/2}$	
14) $\sin mx \cos nx = \frac{1}{2}[\sin(m+n)x + \sin(m-n)x]$	16) $\sin mx \sin nx = \frac{1}{2}[\cos(m-n)x - \cos(m+n)x]$
15) $\cos mx \cos nx = \frac{1}{2}[\cos(m+n)x + \cos(m-n)x]$	

Tabela 2.4: Fórmulas trigonométricas que poderão ser úteis na primitivação.

Função a primitivar		Primitiva
1)	k , $k \in \mathbb{R}$	kx
2)	$f^\alpha \cdot f'$, $\alpha \in \mathbb{R} \setminus \{-1\}$ x^α , $\alpha \in \mathbb{R} \setminus \{-1\}$	$\frac{f^{\alpha+1}}{\alpha+1}$ $\frac{x^{\alpha+1}}{\alpha+1}$
3)	$\frac{f'}{f}$ $\frac{1}{x}$	$\ln f $ $\ln x $
4)	$\sin f \cdot f'$	$-\cos f$
5)	$\cos f \cdot f'$	$\sin f$
6)	$\operatorname{tg} f \cdot f'$	$-\ln \cos f $
7)	$\operatorname{cotg} f \cdot f'$	$\ln \sin f $
8)	$\sec^2 f \cdot f'$	$\operatorname{tg} f$
9)	$\operatorname{cosec}^2 f \cdot f'$	$-\operatorname{cotg} f$
10)	$\sec f \cdot f'$	$\ln \sec f + \operatorname{tg} f $
11)	$\operatorname{cosec} f \cdot f'$	$\ln \operatorname{cosec} f - \operatorname{cotg} f $
12)	$\sec f \cdot \operatorname{tg} f \cdot f'$	$\sec f$
13)	$\operatorname{cosec} f \cdot \operatorname{cotg} f \cdot f'$	$-\operatorname{cosec} f$
14)	$a^f \cdot f'$, $a \in \mathbb{R}^+ \setminus \{1\}$	$\frac{a^f}{\ln a}$
15)	$e^f \cdot f'$	e^f
16)	$\frac{f'}{\sqrt{1-f^2}}$	$\arcsin f$
17)	$\frac{-f'}{\sqrt{1-f^2}}$	$\arccos f$
18)	$\frac{f'}{1+f^2}$	$\operatorname{arctg} f$

Tabela 2.1: Primitivas de algumas funções.

1)	$P(f + g) = Pf + Pg$
2)	$P\lambda f = \lambda Pf$, com $\lambda \in \mathbb{R}$
3)	$P(fg) = Fg - P(Fg')$, com $F' = Pf$
4)	$Pf(x) = P(f(\varphi(t))\varphi'(t))$, com $x = \varphi(t)$

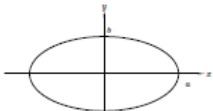
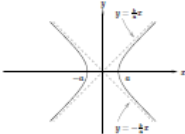
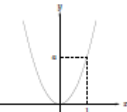
Tabela 2.3: Propriedades das primitivas.

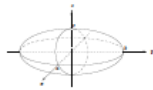
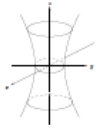






Fracção simples a primitivar	Primitiva
$\frac{Ax+B}{(x-p)^2+q^2}^k$	
1) $k = 1$	$\frac{Ap+B}{q} \arctg \frac{x-p}{q} + \frac{A}{2} \ln((x-p)^2 + q^2)$
2) $k = 2$	$\frac{Ap+B}{2q^2} \left(\frac{q(x-p)}{(x-p)^2+q^2} + \arctg \frac{x-p}{q} \right) - \frac{A}{2((x-p)^2+q^2)}$

Tabela 2.2: Primitivas de algumas fracções simples.

Função a primitivar	Substituição
1) $R(x, \sqrt{a^2 - b^2 x^2})$	$x = \frac{a}{b} \sin t$
2) $R(x, \sqrt{a^2 + b^2 x^2})$	$x = \frac{a}{b} \operatorname{tg} t$
3) $R(x, \sqrt{b^2 x^2 - a^2})$	$x = \frac{a}{b} \sec t$
4) $R(x, x^{\frac{p}{q}}, \dots, x^{\frac{r}{s}})$ com $p, q, \dots, r, s \in \mathbb{Z} \setminus \{0\}$	$x = t^k, k = m.m.c.(q, \dots, s)$
5) $R\left(x, \left(\frac{ax+b}{cx+d}\right)^{\frac{p}{q}}, \dots, \left(\frac{ax+b}{cx+d}\right)^{\frac{r}{s}}\right)$ com $p, q, \dots, r, s \in \mathbb{Z} \setminus \{0\}$	$\frac{ax+b}{cx+d} = t^k, k = m.m.c.(q, \dots, s)$ $\left(x = \frac{dt^k - b}{a - ct^k}\right)$
7) $R(\sin x, \cos x)$	$t = \operatorname{tg} \frac{x}{2}$ $\left(\sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}\right)$
8) $R(e^x)$	$x = \ln t$

Tabela 2.5: Substituições sugeridas para racionalizar $R(v_1, \dots, v_m) = \frac{P(v_1, \dots, v_m)}{Q(v_1, \dots, v_m)}$, em que $P(v_1, \dots, v_m)$ e $Q(v_1, \dots, v_m)$ são polinómios nas variáveis v_1, \dots, v_m .

Cónicas definidas por equações na forma reduzida		
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	ELIPSE	
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	HIPÉRBOLE	
$y = ax^2$	PARÁBOLA	

Quádricas definidas por equações na forma reduzida		
$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	ELIPSOIDE	
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$	HIPERBOLOIDE DE 1 FOLHA	
$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	HIPERBOLOIDE DE 2 FOLHAS	
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$	CONE	
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$	PARABOLOIDE ELÍPTICO	
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c}$	PARABOLOIDE HIPERBÓLICO	
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	CILINDRO ELÍPTICO	
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	CILINDRO HIPERBÓLICO	
$ax^2 = y$	CILINDRO PARABÓLICO	