| | | | |
|---|---|---|---|
| 1) | $(k)' = 0$ | 10) | $(\text{tg } f)' = \sec^2 f \cdot f'$ |
| 2) | $(f^{\alpha})' = \alpha \cdot f^{\alpha-1} \cdot f', \qquad \alpha \in \mathbb{R}$ | | $(\sec f = \frac{1}{\cos f})$ |
| 3) | $(\mathrm{e}^{f})' = \mathrm{e}^{f} \cdot f'$ | 11) | $(\text{cotg } f)' = -\text{cosec}^2 f \cdot f'$ |
| 4) | $(\mathrm{a}^{f})' = \mathrm{a}^{f} \cdot f' \cdot \ln a, \qquad a \in \mathbb{R}^+$ | | $(\text{cotg } f = \frac{1}{\text{tg } f}$ e $\text{cosec } f = \frac{1}{\sin f})$ |
| 5) | $(\ln f)' = \frac{f'}{f}$ | 12) | $(\sec f)' = \sec f \cdot \text{tg } f \cdot f'$ |
| 6) | $(\log_a f)' = \frac{f'}{f \cdot \ln a}, \qquad a \in \mathbb{R}^+ \setminus \{1\}$ | 13) | $(\text{cosec } f)' = -\text{cosec } f \cdot \text{cotg } f \cdot f'$ |
| 7) | $(f^{g})' = g \cdot f^{g-1} \cdot f' + f^{g} \cdot \ln f \cdot g', \qquad f > 0$ | 14) | $(\arcsin f)' = \frac{f'}{\sqrt{1-f^2}}$ |
| | $(f^{g} = e^{g \ln f})$ | 15) | $(\arccos f)' = \frac{-f'}{\sqrt{1-f^2}}$ |
| 8) | $(\sin f)' = \cos f \cdot f'$ | 16) | $(\text{arctg } f)' = \frac{f'}{1+f^2}$ |
| 9) | $(\cos f)' = -\sin f \cdot f'$ | | |

Tabela 1.1: Derivadas de algumas funções elementares.

| | |
|---|---|
| 1) | $(f+g)' = f' + g'$ |
| 2) | $(\alpha f)' = \alpha f', \qquad \alpha \in \mathbb{R}$ |
| 3) | $(fg)' = f'g + fg'$ |
| 4) | $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ |
| 5) | $(g \ o \ f)'(x) = g'[f(x)]\, f'(x)$ |

Tabela 1.2: Regras de derivação.

| | | | |
|---|---|---|---|
| 1) | $\sin^2 x + \cos^2 x = 1$ | | |
| 2) | $1 + \text{tg}^2 x = \sec^2 x$ | 3) | $1 + \text{cotg}^2 x = \text{cosec}^2 x$ |
| 4) | $\sin(x+y) = \sin x \cos y + \sin y \cos x$ | 5) | $\cos(x+y) = \cos x \cos y - \sin x \sin y$ |
| 6) | $\sin(x-y) = \sin x \cos y - \sin y \cos x$ | 7) | $\cos(x-y) = \cos x \cos y + \sin x \sin y$ |
| 8) | $\sin 2x = 2 \sin x \cos x$ | 9) | $\cos 2x = \cos^2 x - \sin^2 x$ |
| 10) | $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ | 11) | $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ |
| 12) | $\sin x = \frac{2\text{tg } x/2}{1 + \text{tg}^2 \ x/2}$ | 13) | $\cos x = \frac{1 - \text{tg}^2 \ x/2}{1 + \text{tg}^2 \ x/2}$ |
| 14) | $\sin mx \cos nx = \frac{1}{2}[\sin(m+n)x + \sin(m-n)x]$ | | |
| 15) | $\cos mx \cos nx = \frac{1}{2}[\cos(m+n)x + \cos(m-n)x]$ | 16) | $\sin mx \sin nx = \frac{1}{2}[\cos(m-n)x + \cos(m+n)x]$ |

Tabela 2.4: Fórmulas trigonométricas que poderão ser úteis na primitivação.

| | Função a primitivar | Primitiva |
|---|---|---|
| 1) | $k, \ k \in \mathbb{R}$ | $kx$ |
| 2) | $f^{\alpha} \cdot f', \ \alpha \in \mathbb{R} \setminus \{-1\}$ | $\frac{f^{\alpha+1}}{\alpha+1}$ |
| | $x^{\alpha}, \ \alpha \in \mathbb{R} \setminus \{-1\}$ | $\frac{x^{\alpha+1}}{\alpha+1}$ |
| 3) | $\frac{f'}{f}$ | $\ln\lvert f\rvert$ |
| | $\frac{1}{x}$ | $\ln\lvert x\rvert$ |
| 4) | $\sin f \cdot f'$ | $-\cos f$ |
| 5) | $\cos f \cdot f'$ | $\sin f$ |
| 6) | $\text{tg } f \cdot f'$ | $-\ln\lvert\cos f\rvert$ |
| 7) | $\text{cotg } f \cdot f'$ | $\ln\lvert\sin f\rvert$ |
| 8) | $\sec^2 f \cdot f'$ | $\text{tg } f$ |
| 9) | $\text{cosec}^2 f \cdot f'$ | $-\text{cotg } f$ |
| 10) | $\sec f \cdot f'$ | $\ln\lvert\sec f + \text{tg } f\rvert$ |
| 11) | $\text{cosec } f \cdot f'$ | $\ln\lvert\text{cosec } f - \text{cotg } f\rvert$ |
| 12) | $\sec f \cdot \text{tg } f \cdot f'$ | $\sec f$ |
| 13) | $\text{cosec } f \cdot \text{cotg } f \cdot f'$ | $-\text{cosec } f$ |
| 14) | $a^{f} \cdot f', \ a \in \mathbb{R}^+ \setminus \{1\}$ | $\frac{a^{f}}{\ln a}$ |
| 15) | $e^{f} \cdot f'$ | $e^{f}$ |
| 16) | $\frac{f'}{\sqrt{1-f^2}}$ | $\arcsin f$ |
| 17) | $\frac{-f'}{\sqrt{1-f^2}}$ | $\arccos f$ |
| 18) | $\frac{f'}{1+f^2}$ | $\text{arctg } f$ |

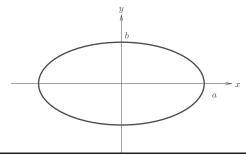
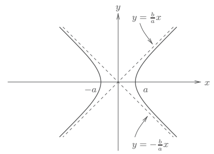
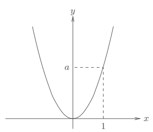
Tabela 2.1: Primitivas de algumas funções.

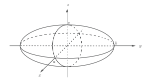
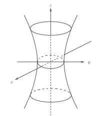
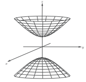
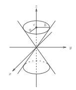
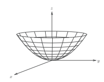
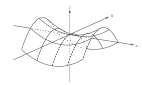
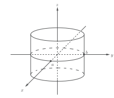
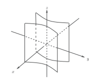
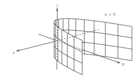
| | |
|---|---|
| 1) | $P(f+g) = Pf + Pg$ |
| 2) | $P\lambda f = \lambda P f, \ $ com $\lambda \in \mathbb{R}$ |
| 3) | $P(fg) = Fg - P(Fg'), \ $ com $F = Pf$ |
| 4) | $Pf(x) = P(f(\varphi(t))\varphi'(t)), \ $ com $x = \varphi(t)$ |

Tabela 2.3: Propriedades das primitivas.

| Função a primitivar | Substituição |
|---|---|
| 1) $R\left(x, \sqrt{a^2 - b^2x^2}\right)$ | $x = \frac{a}{b}\sin t$ |
| 2) $R\left(x, \sqrt{a^2 + b^2x^2}\right)$ | $x = \frac{a}{b}\operatorname{tg} t$ |
| 3) $R\left(x, \sqrt{b^2x^2 - a^2}\right)$ | $x = \frac{a}{b}\sec t$ |
| 4) $R\left(x, x^{\frac{p}{q}}, \ldots, x^{\frac{r}{s}}\right)$ com $p, q, \ldots, r, s \in \mathbb{Z} \setminus \{0\}$ | $x = t^k,\ k = m.m.c.(q, \ldots, s)$ |
| 5) $R\left(x, \left(\frac{ax+b}{cx+d}\right)^{\frac{p}{q}}, \ldots, \left(\frac{ax+b}{cx+d}\right)^{\frac{r}{s}}\right)$ | $\frac{ax+b}{cx+d} = t^k,\ k = m.m.c.(q, \ldots, s)$ |
| com $p, q, \ldots, r, s \in \mathbb{Z} \setminus \{0\}$ | $\left(x = \frac{dt^k - b}{a - ct^k}\right)$ |
| 7) $R(\sin x, \cos x)$ | $t = \operatorname{tg}\frac{x}{2}$ |
| | $\left(\sin x = \frac{2t}{1+t^2},\ \cos x = \frac{1-t^2}{1+t^2}\right)$ |
| 8) $R(e^x)$ | $x = \ln t$ |

Tabela 2.5: Substituições sugeridas para racionalizar $R(v_1, \ldots, v_m) = \dfrac{P(v_1, \ldots, v_m)}{Q(v_1, \ldots, v_m)}$, em que $P(v_1, \ldots, v_m)$ e $Q(v_1, \ldots, v_m)$ são polinómios nas variáveis $v_1, \ldots, v_m$.

| **Cónicas definidas por equações na forma reduzida** | | |
|---|---|---|
| $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ | Elipse |  |
| $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ | Hipérbole |  |
| $y = a\,x^2$ | Parábola |  |

| **Quádricas definidas por equações na forma reduzida** | | |
|---|---|---|
| $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ | Elipsoide |  |
| $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ | Hiperboloide de 1 folha |  |
| $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ | Hiperboloide de 2 folhas |  |
| $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$ | Cone |  |
| $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$ | Paraboloide Elíptico |  |
| $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c}$ | Paraboloide Hiperbólico |  |
| $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ | Cilindro Elíptico |  |
| $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ | Cilindro Hiperbólico |  |
| $a\,x^2 = y$ | Cilindro Parabólico |  |