TOPOLOGICAL SIGNIFICANCE OF STREAM LABELING METHODS

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ABSTRACT

Several stream ordering and labeling methods have been proposed in order to describe some aspects of the geometry of river networks; namely, Horton's, Strahler's, Milton-Ollier's, Scheidegger's ordering techniques, and the STORET location coding system. This paper analyzes the topological significance of each of these methods, that is, the amount of information on the topological structure of the net that they can yield. Horton's and Strahler's ordering methods give only numerical information on the distribution of channels among different classes (orders); Milton-Ollier's and Scheidegger's methods give more information, from the topological point of view, as the former assigns a unique label to each stream segment in a network, and the latter takes into account all junctions; the STORET system labels interconnections between channels, but does not use the concept of order and is therefore more suitable for other purposes than for the theoretical study of river nets.

1. INTRODUCTION

The aim of the present paper is to discuss the topological significance of the different stream labeling methods which have been proposed in connection with the study of river networks. These methods have not been devised with the main purpose of giving a complete description of the topology of river nets; they nevertheless contain some interesting information of this sort, and it is of interest to see how much of this topological information can be retrieved from them.

In the course of the discussion, some expressions, as "river network", "stream net", "channel network", "stream segment", "link", "base river", will be frequently employed. Therefore, we define the meaning we attach to these expressions.

A "river network" (or "channel network", "stream net", et sim.) is the interrelated drainage pattern formed by a set of streams in a certain area, from any number of sources down to the mouth, or root point, of the net. A "stream segment" is that stretch of channel along which the dimensionless parameter called "order" (in the Strahler sense, cf. Strahler, 1957) associated with it remains constant. A "link" is the unbroken stretch of channel along which no junctions occur (i.e., it is that portion of a channel between two junctions, or between the source and the first junction, or between the last junction and the mouth, always going downstream). A "base river" of a given net or subnet is that river which receives only lower (Strahler) order tributaries. The words "channel", "river", and "stream" will be employed in the following in their general sense, and no specialized meaning will be attached to them.

The topological structure of river networks can be examined, of course, from topographic maps or other pictorial material; but it is clear that such a procedure per se does not give the amount of information needed. On the other hand, several stream labeling systems have been proposed in the literature; we shall examine each one of them, discuss their topological significance, and then compare the amount of topological information that they can yield.

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2. Stream Labeling Methods

All but one of the five stream labeling methods that we shall consider (namely, the STORET location code) employ, in one form or another, the concept of "stream order". This is a dimensionless parameter which, along with several other "quantitative physiographic factors" was introduced by Horton (1945) in order to describe and study quantitatively the hydrological and geomorphological characteristics of stream nets.

The fundamental idea of stream ordering, as expressed by Horton, consists in distinguishing among different classes (i.e., orders) of streams, and in assigning to each stream of a certain class a given dimensionless number; in this fashion, the label attached to each stream indicates the class which the stream under consideration belongs to, that is, its order. The detailed procedure for doing so is described by Horton (1945). It is clear that the concept of order, in Horton's sense, applies to complete streams, and not to stream segments or links, since the order of any channel remains unchanged from the source down to the point where such a channel "dies" by joining a higher-order river, or by reaching the mouth of the network. It must be pointed out also that stream order is evidently affected by map scale and map quality, since some first-order streams (fingertip tributaries) may possibly be lost in a map which is not detailed enough or which has too small a scale; on the other hand, the possibility of neglecting streams because they are not marked on the maps is present in any stream labeling method, whenever maps are used for the analysis of river networks. It is therefore advisable to specify the fashion in which information has been gathered (maps, aerial photographs, field survey, et sim.), and, when topographic maps are employed, to indicate their scale and type; in this case, it is common practice to consider as first-order streams those which appear to have no tributaries on these maps. Furthermore, in Horton's method, an additional difficulty arises, since it is not always clear which fingertip channels are to be considered true first-order streams, and which ones are simply upstream extensions of higher-order rivers. Figure 1 gives an example of Horton's stream ordering method applied to a hypothetical river net.

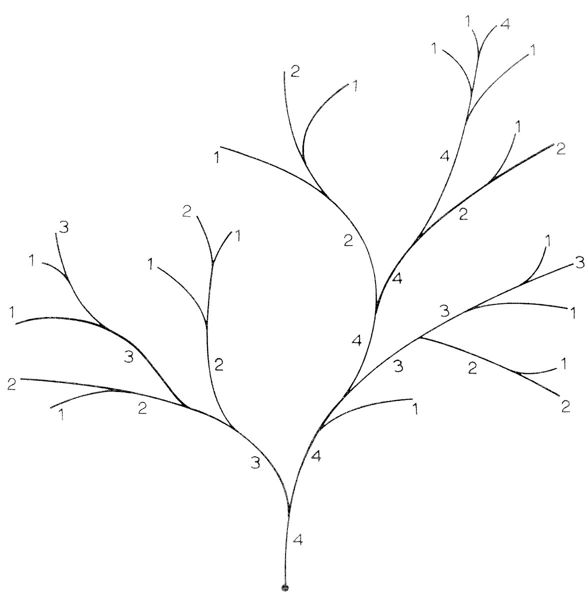

Fig. 1 — Horton Stream Orders.

Strahler (1957) proposed a slightly modified way of stream ordering. In his method, the difficulty of the choice of true first-order streams is avoided by regarding all fingertip tributaries as segments of order 1. When two channels of order n, m join, the resulting channel downstream is assigned the order according to the rule,

$$n * m = n + 1 \quad \text{if} \quad n = m$$
$$n * m = n \quad \text{if} \quad n > m$$

where a "star" symbolizes a junction. The examination of these formulas shows that lower-order channels which join a given base stream simply "get lost" in Strahler's ordering procedure, in the sense that junctions of lower-order segments do not modify the order of the base stream. On the other hand, it is clear that the concept of Strahler order, by definition, applies to stream segments, and the order may well change along the same river. Figure 2 gives an example of Strahler's stream ordering method. When the analysis of a natural river network is carried out, Horton and Strahler orders are equivalent only in the very particular case in which the net is structurally regular in all its parts (Scheidegger, 1968); otherwise, it is necessary to state at the outset of the analysis which ordering technique is being employed.

Another stream labeling method which basically employs the concept of Strahler order has been devised by Milton and Ollier (1965). This method enables one to assign code numbers and letters to stream segments and junctions in such a fashion that each segment in the net is given a unique label, and, moreover, makes it possible to gather some information on the structure of the network directly from the code employed. An example of Milton-Ollier's stream ordering procedure is shown in figure 3. In a given net or subnet, the main river is labeled by assigning to it its Strahler order; the in-flowing tributaries are given a label consisting of both order and a letter (a, b, c, etc.) which takes into account the order of junctions (looking upstream). Then, the labeling goes on to the lower-order subnets; the procedure is the same. In this fashion, each stream segment in a given channel network has a unique code label.

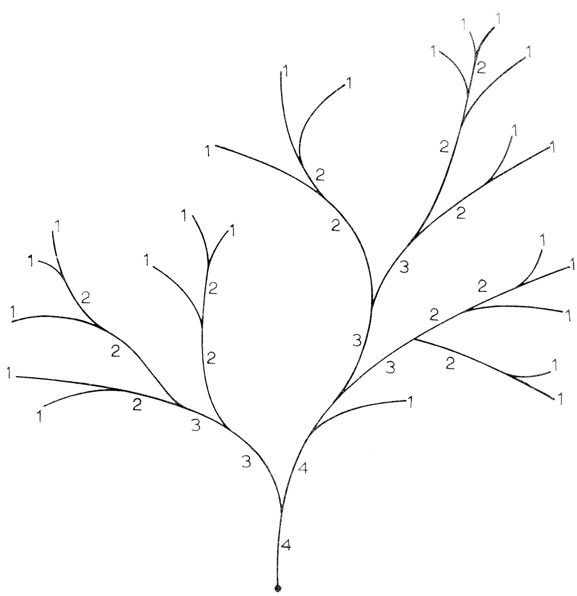

Fig. 2 — Strahler Stream Orders

We have seen that, in Strahler's algebra of stream orders, a lower-order segment joining a higher-order one does not change the order of the latter; moreover, neither Strahler's nor, consequently, Milton-Ollier's stream ordering methods generate an algebra of stream orders in which the associative law holds. Actually, if again a "star" is used to symbolize a junction, and n is the order of a given segment, we have, following Strahler's procedure,

$$n * [(n-1) * (n-1)] = n+1$$
$$[n * (n-1)] * (n-1) = n$$

that is, the associative law does not hold. It is conceivable, however, to assume that in nature, under similar climatic and geographic conditions, the discharge and other hydrological

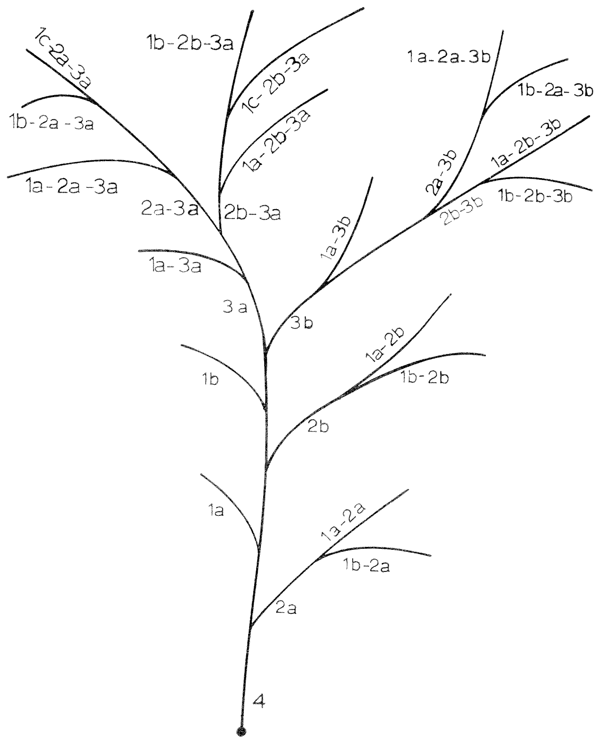

Fig. 3 — Milton-Ollier Stream Labeling System.

characteristics of a channel depend, on the average, upon the number of streams whose water ultimately flows through it, regardless of the pattern of their interconnections. On the other hand, from the topological point of view, this very pattern is of interest. Scheidegger (1965) devised a stream ordering method in which no stream "gets lost", and in which the associative law holds; this method has been called "consistent" stream ordering. The definition of consistent order applies to links rather than to segments, since each junction changes the order of the channel. The consistent order N of any link formed by junction of two links of order r, s is given by the logarithmic composition law

$$N = \log_2(2^r + 2^s)$$

which expresses the fact that all junctions in a network are taken into account, i.e., the effect of all tributaries on the order of the main stream is considered (cf. Scheidegger, 1965).

Consistent orders are not necessarily integers; therefore it is sometimes more practical to employ "associated integers", $\nu = 2^N$, obtained by a summation procedure after giving the first (Strahler) order stream segments the associated integer 2. The procedure is shown in figure 4. It is clear that, by dividing the associated integer of any link by 2, one finds the number of first order segments which "make up" the link under consideration; in other words, one finds the number of sources whose water ultimately flows through the link. The number ζ, defined by

$$\zeta = 2^{(N-1)}$$

is sometimes called the "equivalent integer". This concept was first introduced by Scheidegger (1966); Shreve (1967) has introduced a similar parameter which he calls the "magnitude" of a link. The terms "equivalent integer" and "magnitude" are synonymous.

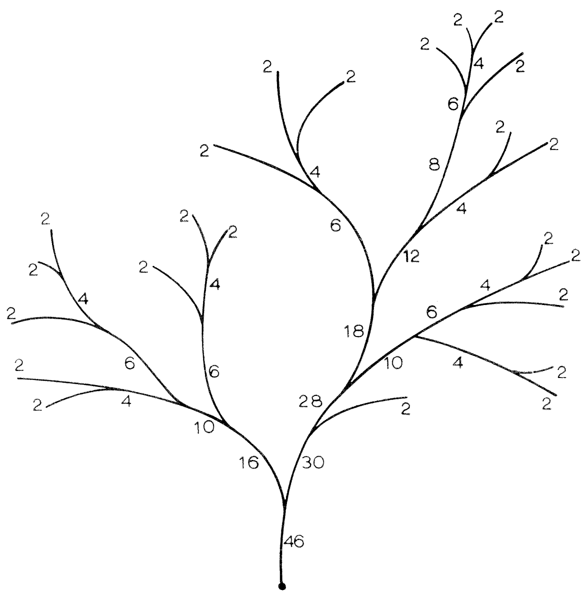

Fig. 4 — Consistent Associated Integers.

Another procedure for labeling basins, rivers, and junctions has been devised by the Division of Water Supply and Pollution Control of the U.S. Public Health Service (cf. for instance Green *et al.*, 1966), as part of a system which takes into account not only the relative location of streams and junctions in a network (both metrically and topologically), but also the location of control stations, and various parameters describing physical and chemical characteristics of water. This system has been devised with the main purpose of allowing a unified and fast handling of information in water quality and pollution control problems with the help of electronic machines, and has been named STORET (Water Quality Data *Sto*rage and *Ret*rieval System). From the topological point of view, what concerns us most is the location coding of streams and junctions in a network.

In the STORET system location code, streams are labeled using the concept of "stream level". The main river (flowing into an ocean, sea, or great lake) of a given channel network is assigned the level 1, all tributaries to the main river are assigned the level 2, all tributaries to the level 2 rivers are assigned the level 3, and so on. Clearly, the concept of stream level applies to complete streams, and not to segments or links. Once the levels have been assigned, tributaries of the same level along a given river (and thereby their junctions with this river) are assigned increasing "stream index" numbers (going upstream) which also allow one to distinguish between tributaries entering from the left and tributaries entering from the right (looking upstream). Mileages are also given: they refer to the distance from the junction with the closest lower-level stream, or from the mouth. The detailed procedure may be found, for instance, in Green *et al.* (1966). Figure 5 gives an example of STORET location coding. As it can be seen, each junction is labeled. Its identification is given by a fraction whose numerator gives topological information (the first digit indicates the stream level, the others indicate the stream index number), and whose denominator gives metric information (mileage). In this fashion all junctions and streams in a network are labeled.

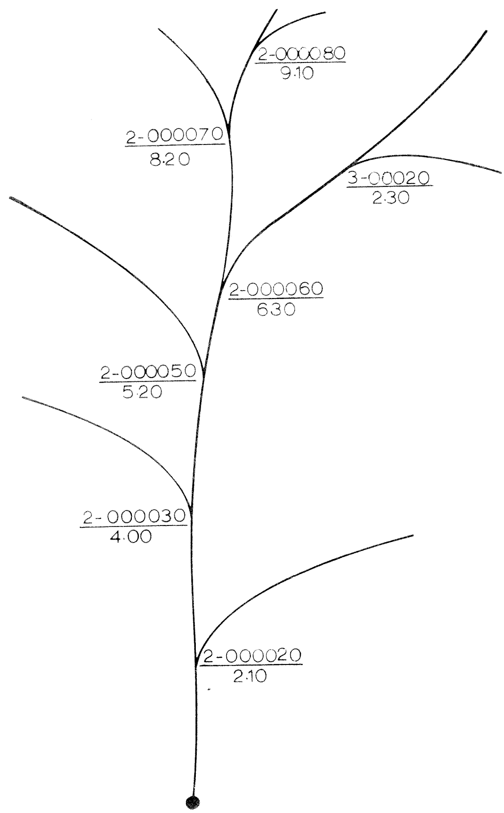

Fig. 5 — STORET Location Code.

3. Topological Significance

The concepts of Horton and Strahler orders, along with other parameters, are useful in the study of several aspects of river networks. For example, in order to analyze the composition of the net, one has to introduce the concept of stream number (i.e., number of channels of a certain order); combining the set of stream numbers with the sequence of stream orders, one obtains a representation of the composition of the net, from which the empirical relationship known as "Horton's law of stream numbers" has been inferred (cf. Horton, 1945, and Strahler, 1957). From the point of view of the present paper, however, it must be pointed out that, even when the complete set of stream numbers and the orders are known, one knows the distribution of channels among different classes (i.e., among different orders) in the network under consideration, but nothing about its topological structure, because information on the pattern of interconnections is lacking. Consequently, the amount of information on the topological structure of the network which can be retrieved from the code itself, without making use of pictorial material, is indeed limited in both methods.

In the method proposed by Milton and Ollier, on the other hand, each stream segment in a given network has a unique label; this fact is useful in determining its relative location. The topological information yielded by the code is not complete, as it is apparent from two considerations. First, the order of junctions is actually specified only for segments of the same order; for instance, with reference to figure 3, one can infer from the code employed that segment $1a$ joins the base river downstream of segment $1b$, but nothing can be deduced from the code itself about the relative position of segment $2a$ with respect to, say, segment $1a$. Second, there is no distinction (apart from the particular case when a base river bifurcates) between tributaries entering from the left and tributaries entering from the right. These considerations are important, if the topological structure of the net is to be studied, since two networks identically labeled following Milton-Ollier's method migth be topologically different.

Scheidegger's "consistent" stream labeling method neglects no junctions; moreover, the concept of equivalent integer, or magnitude, as pointed out also by Shreve (1967), characterizes a network more precisely than stream order does; in effect, it is reasonable to assume that the magnitude of a link is related to the hydrological and geomorphological characteristics of that link more closely than a stream order is related to the corresponding stream segment. As pointed out above, the discharge of a channel, in a network submitted to the same geographic and climatic controls, is indeed likely to be statistically related to the number of sources whose water ultimately flows through that channel; note that the expression "geologic controls" has been purposely omitted, since such controls do not seem to be very relevant in determining the development of a network, as shown by the remarkable uniformity of net structure under different geologic conditions. It is clear that the topological structure of a network is more clearly recorded in a labeling method which takes into consideration all junctions present in the network, than in one which neglects some of them. On the other hand, there is no way, in Scheidegger's method, of distinguishing between tributaries entering from the left and tributaries entering from the right along a given base stream.

The STORET system, as seen above, does not make use of the concepts of stream order or magnitude; in its location code the basic dimensionless parameter is the stream *level*, which applies to "complete" rivers, i.e., to the whole length of a stream, from its source down to the junction at which it "dies" by joining another (more important) stream, or by reaching the mouth of the network. Naturally, this implies a certain degree of arbitrariness in determining, at each junction, which channel "dies" there and which one is the more important; this difficulty is practically overcome by following either empirical criteria (discharge, headwater extension, etc.), or convention (names on maps).

The numerator of the fraction identifying each junction in the STORET system, i.e., the stream level and the stream index number, is of topological significance. It is necessary to point out, however, that stream level and stream index do not give a complete topological identification of a junction (and thereby of the stream which "dies" there); since generally in a network there are more than one k-th level streams, $k > 1$, the topological location of a junction is uniquely determined by the ordered sequence of junction labels from the mouth up to the junction under consideration. For example, in the river net of figure 6, the two sequences

2–000010, 3–00020, 4–0010

and 2–000020, 3–00010, 4–0010

indicate, respectively, the (topological) location in the network of junctions (a) and (b), whereas the notation 4–0010 by itself is ambiguous. Therefore an ordered sequence is necessary to determine uniquely the location of any junction (stream); in the particular case of the identification of a second level tributary, this sequence consists of one term only. The stream index number yields additional information, as it allows one to determine whether a given tributary enters the lower level river from the left or from the right.

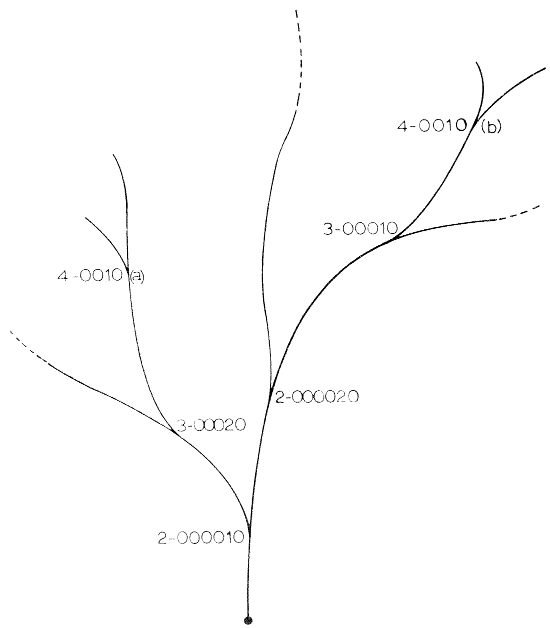

Fig. 6 — Junction Identification in the STORET System.

It is therefore clear that, once a stream network has been completely labeled according to the STORET system location code, one obtains a set of complete sequences (up to the fingertip tributaries) which contain all topological information. On the other hand, there is no unique correlation between stream order and stream level, neither between magnitude and stream level; order (except Horton's) and magnitude change along a river, while the level remains unchanged. Also the Horton order remains unchanged along a river, until it joins a higher-order channel, but the procedure of assigning orders is completely different from the procedure of assigning levels, and channels of the same Horton order may have different levels, and vice-versa.

First Strahler (or consistent) order stream segments (or links; segment and link are synonymous in case of first order channels) can be identified using a complete set of STORET data; since the topological location of a junction is uniquely determined by a code sequence, that junction which is indicated by a complete sequence (no more junctions upstream) represents a first (Strahler or consistent) order tributary. This, however, is no longer applicable in case of higher order channels, which are indicated by incomplete sequences; the order of the channel cannot be inferred only from the sequence which identifies the junction where the channel under consideration "dies". It should be noted, however, that the process of finding the magnitude of a link is merely a counting procedure: all one has to do is to count the number of first order segments (i.e., all fingertip tributaries) whose water flows through the link under consideration. This information is contained in a complete set of STORET codes, along with information on the number and location of junctions; this suggests the possibility of the utilization of STORET data for topological purposes. However, the conversion of stream level into stream order or magnitude, that is, the expression of the STORET location code in terms of the classical ordering methods of theoretical geomorphology, is a rather cumbersome procedure, even on a computer (tapes must be searched for sequences, etc.), since the STORET system was not designed for geomorphological purposes.

4. Conclusion

In recent years there has been a growing interest in the topological structure of river networks. However, no stream labeling method giving complete information on such structure is available. All existing stream ordering procedures (Horton's, Strahler's, Milton-Ollier's, Scheidegger's) were not primarily devised for dealing with the problem considered in the present paper. Some of these procedures contain more topological information than some others, but none of them is complete in this respect. The STORET system location code, on the other hand, employs the concept of stream level, which is not very suitable for geomorphological purposes.

REFERENCES

Green, R.S., Dubois, D.P., and Tutwiler, C.W., Data Handling Systems in Water Pollution Control, *J. San. Eng. Div. ASCE, 92(SA1)*, *Proc. Paper 4652*, pp. 55-67, 1966.

Horton, R.E., Erosional Development of Streams and their Drainage Basins; Hydrophysical Approach to Quantitative Morphology, *Geol, Soc. Am. Bull., 56*, pp. 275-370, 1945.

Milton, L.E., and Ollier, C.D., A Code for Labeling Streams, Basins, and Junctions in a Drainage Net, *J. Hydr., 3*, pp. 66-68, 1965.

Scheidegger, A.E., The Algebra of Stream Order Numbers, *U.S. Geol. Survey Prof. Paper 525-B*, pp. B187-B189, 1965.

—, Statistical Description of River Networks, *Water Resources Research, 2(4)*, pp. 785-790, 1966.

—, Horton's Law of Stream Order Numbers and a Temperature Analog in River Nets, *Water Resources Research, 4(1)* pp. 167-171, 1968.

Shreve, R.L., Infinite Topologically Random Channel Networks, *J. Geol., 75(2)*, pp. 178-186, 1967.

Strahler, A.N., Quantitative Analysis of Watershed Geomorphology, *Am. Geophys. Un Trans., 38*, pp. 913-920, 1957.