

For 466W — Forest Resource Management
Lab 4: Real Price Changes and Optimal Rotations
Wednesday, February 3, 2016

You used the following equation in your first lab to calculate various measures of growth. It predicts the volume of wood, in thousands of board feet per acre, available for harvest from a good-site northern hardwood stand as a function of age.

$$Y(A) = Me^{-k/(A-d)} = 28e^{-80/(A-10)} \quad \text{for } A > d, 0 \text{ otherwise.}$$

In this lab, you will use the same equation, with different parameters, to conduct a marginal analysis of the rotation decision in even-aged stands. You will consider the relatively simple case where only financial benefits and costs are included. You will also assume constant interest rates, costs and prices. Under these assumptions, the financially optimal time to harvest an even-aged stand is the age where the *Land Expectation Value* (LEV) is at a maximum. This age is also the age where the marginal benefit of letting the stand grow for one more year becomes less than the marginal cost of letting the stand grow for one more year.

In this example, the **marginal benefit of waiting** is equal to the increase in the stand's harvest value during the year. This is the **value of the annual increment**, which, because we assume prices are constant in the example here, is just the price times the annual increment. The formula for the marginal benefit curve is (the subscript *a* on the marginal benefit and the annual increment is there to remind you that they are functions of (vary with) the stand age):

$$MB_a = Price \times Annual\ increment_a$$

In this example, there are three components to the **marginal cost of waiting** for one year. The first component is the **annual property taxes**. The second is the **land rent**. The rent should equal the opportunity cost of selling the land and investing the money elsewhere. This is equal to the best LEV that can be earned with the land times the interest rate ($LEV^* \times r$). (Note that because LEV^* is the best LEV it is not a function of the stand age; it is a constant.) The third and final component of the marginal cost of waiting to harvest the stand is the **interest on the inventory**, which is equal to the value of the inventory ($price \times yield$) times the interest rate (r) (i.e., $price \times yield \times r$). This is the opportunity cost of not putting the money that could have been earned by cutting the timber into an alternate investment. The following equation quantifies these three components:

$$MC_a = Ann.\ prop.\ tax + LEV^* \times r + Price \times Yield \times r$$

You will create two graphs. The first graph will show the yield and the LEV as a function of age. The second graph will show the marginal costs and benefits of delaying the harvest of the stand for one year as a function of stand age. **A demo** of the graphs you will create can be found by following the Rotation Demo link in this lab's folder on ANGEL.

As in the growth and yield lab, you will create a parameter block at the top of the new sheet where you can enter the economic parameters of the problem. After you have constructed the graphs, you will answer some questions about how the graphs are affected when you change those parameters.

Step 1. Set up your yield data.

Note: you may want to start this lab by opening your spreadsheet from Lab 1 and saving it as your Lab 4 spreadsheet. Step 1 basically repeats what you did in Lab 1, so you can skip this step if you start with your Lab 1 file. However, you will need to change the yield parameters, as shown below. Also, you should delete your old Answers sheet.

On a new sheet (label it “Yields”) create a parameter block at the top with the growth and yield parameters, and calculate age and yields in Columns A and B.

- a. In a block at the top left, indicate the following yield equation parameters:

	A	B	C
1	Parameter Symbol	Value	Description
2	<i>M</i>	19.64995	Maximum volume
3	<i>K</i>	80.31604	Rate parameter
4	<i>d</i>	5	Last age with yield = 0

- b. In a block below the parameter block create the following columns.

Column A. Title: **Age**. Fill in ages 0 to 120.

Column B. Title: **Yield**. For ages 1 to 120, enter the yield equation given on page 1 with references to the parameters in the parameter block at the top of the sheet. Use an *If function* to return a 0 if the age is less than or equal to parameter *d* (Last age with yield = 0) and to calculate the yield using the yield function on p. 1 if the age is greater than the parameter.

Column C. Title: **Growth**. Calculate the growth for age 1 to 120 (difference between the volume at that age and the volume one year earlier).

Column D. Title: **MAI**. Calculate the mean annual increment for ages 1 to 120.

- c. Use the *Max function* to determine the value of the maximum MAI. In column E, use the *If function* to return the age where the MAI equals the maximum MAI and to return a zero for all other ages. Sum this column.

- d. At the top of the sheet create a block of cells that looks like this:

	...	D	E
1	...	Description	Value
2	...	Maximum MAI	0.08487
3	...	Age of Max MAI	90

Reference the maximum MAI that you calculated at the bottom of column D and reference the CMAI (culmination of mean annual increment) from the sum at the bottom of column E.

Step 2. Set up your LEV and Marginal Analysis data.

On a new sheet in your spreadsheet calculate the LEV, marginal cost and the marginal benefit of waiting another year before harvesting, according to the instructions below. Label this sheet “LEV.”

- a. In a block at the top left, indicate the following economic parameters:

	A	B	C
1	Parameter Symbol	Value	Description
2	r	0.03	Real interest rate
3	<i>Stumpage price factor</i>	1	Used to shift all prices up or down
	E	120	Establishment cost (\$/ac)
4	A	2	Annual taxes (\$/ac·yr)

- b. Way to the right of the parameter block (go at least to column G), create a block like the one below, showing the proportions of the total yield and the price for each product.

	Red Oak	White Oak	Red Maple	Sugar Maple	Black Cherry	Other Hdwd	Conifer	Total/Average
Yield Proportion	0.21	0.14	0.12	0.21	0.01	0.28	0.03	100%
Price (\$)	\$437	\$333	\$835	\$258	\$356	\$89	\$26	?????

Calculate the weighted average price in the cell in the bottom right corner (where it shows the ????) using Excel’s *SumProduct* function. The *SumProduct* function has the following format: *SumProduct(Array1, Array2)*. In this case, *Array1* should reference the row of yield proportions, and *Array2* should reference the row of prices. The function takes each price and multiplies it by the corresponding proportion and sums up the resulting products. (This is called vector multiplication, for those of you that have studied some matrix algebra.) The weighted average price, multiplied by the price factor in the parameter block you created in a, above, is the price you will use for your LEV calculations.

- c. Below the parameter block you created in part a, create the following columns.

Column A. Title: **Age**. Fill in ages 0 to 120.

Column B. Title: **LEV**. Set the LEV value for age 0 to 0. For ages 1 to 120, calculate the land expectation value using the following formula:

$$LEV = \frac{-E(1+r)^R + Y_R P}{(1+r)^R - 1} - \frac{A}{r}$$

Here, the values of E , r , and A should come from fixed references to your parameter block, the value of P should be obtained by fixed references that multiply 1) the weighted average price you calculated in b by 2) the price factor in the parameter block, and the values of R and Y_R should come from variable (movable) references to Column A and Column B in the Yield sheet. (Note: the

LEV will be negative for some ages – especially very short and very long rotations.)

Column C. **Annual Increment Value.** This is the annual increment (reference the value in Column C in the Yield sheet) times the price (obtained by multiplying the weighted average price by the price factor – these references should be fixed).

Column D. Title: **Interest on the Inventory.** This is the yield (reference the value in Column B in the Yield sheet) times the price (again, obtained by multiplying the weighted average price by the price factor) times the interest rate (the interest rate should reference the value in the parameter block at the top of this sheet).

Column E. Title: **Land Rent.** This is the interest rate times the maximum LEV (i.e., $r \times LEV^*$). Use the Max function at the bottom of the LEV column to identify the best LEV. The value LEV^* should come from a fixed reference to this cell. The value of r should come from a fixed reference to the parameter block at the top. Note: the land rent will be the same for all rotation ages.

Column F. Title: **Marginal Cost of Delaying Harvest.** This column is calculated by summing the Interest on the Inventory (Column D), the Land Rent (Column E), and the annual property tax (from the reference block).

- d. Create a second block at the top of the spreadsheet that will show the maximum LEV and the rotation age that maximizes the LEV. This will allow you to identify immediately how the LEV and the optimal rotation change when you change the value of an economic parameter.

Create a block of cells that looks like this (of course, the values should be based on your calculations, but these are the correct numbers):

	...	E	F
1	...	Description	Value
2	...	Optimal LEV	\$95.12
3	...	Optimal rotation	54

At the bottom of Column B, use the =Max() function to calculate the value of the optimal LEV. Then reference this cell to show the maximum LEV in the block at the top of the spreadsheet. To calculate the age where the LEV is maximized (the optimal rotation), you will need to use the same technique you used in Lab 1 to identify the age where the MAI was maximized (i.e., use the *If function* to return the age where the LEV equals the maximum LEV and to return a zero for all other ages. Sum this column, and reference this sum in the optimal rotation cell).

Step 3. Create the Graphs

- a. Create a graph showing the yield (Column B from the yield data sheet) and the LEV (Column B in the LEV data sheet) vs. age. Since the two series have different scales and different units, graph the LEV against the secondary y-axis. In order to focus on the positive values of the LEV, fix the range of the secondary y-axis to only show values between \$0 and \$300 per acre. Be sure to title your graph, label the axes, and create a graph legend. Make this graph an object in the LEV data sheet.
- b. Create a graph of the marginal benefit of delaying harvest (i.e., Annual Increment Value) and the marginal cost of delaying harvest (Column F) vs. age. The intersection of the annual

increment value curve and the marginal cost of delaying harvest curve should occur at the rotation that maximizes the LEV. You should be able to explain how and why the optimal rotation shifts in response to changes in the economic parameters by understanding how the change in the parameter shifts these curves (or does not shift them, as the case may be). Fix the maximum value of the y-axis in this graph to \$200 per acre. As always, give the graph an appropriate title, label the axes and create a graph legend. Make this graph an object in the LEV data sheet.

Step 4. Answer these questions.

Label a new sheet “Answers,” put your names on it, and answer these questions.

- a. Why is the land rent the same for all rotation ages?
- b. In your marginal cost/marginal benefit figure, which curve(s) change if the interest rate is increased to 3.5%? How do they change? How does the LEV change (increase, decrease, no change – be specific; e.g., “The best LEV decreased from \$x to \$y.”)? How does the optimal rotation change? What happens to the LEV when the interest rate is 4%?
- c. Reset the interest rate to 3%. Now, how do the MC/MB curves change if the stumpage price factor is increased to 1.5 (representing a 50% increase in all prices)? How do they change? How does the LEV change? How does the optimal rotation change?
- d. Reset the stumpage price factor to 1. How do the MC/MB curves change if the stand establishment cost is raised to \$180/acre? How do they change? How does the LEV change? How does the optimal rotation change?
- e. Reset the stand establishment cost to \$120. How do the MC/MB curves change if the annual land tax is increased to \$3/ac? How do they change? How does the LEV change? How does the optimal rotation change?
- f. How does the optimal rotation under the base economic assumptions compare with the rotation that maximizes the MAI (i.e., which rotation is longer/shorter, by how much)?
- g. What would you say about the general profitability of this stand? Do you think the assumed real interest rate is realistic? Is the assumed stand establishment cost realistic? Are these tax rates realistic? To what degree does the profitability of the stand depend on the species composition?

Upload your spreadsheet.

Upload your lab spreadsheet into your group folder in ANGEL. The lab is due at the end of next Wednesday’s lab.